Глава 4. ОСНОВНІ АЛГОРИТМИ ОБРОБКИ ІНФОРМАЦІЇ В ПІЛОТАЖНО – НАВІГАЦІЙНИХ КОМПЛЕКСАХ

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4. 1 Схема компенсації

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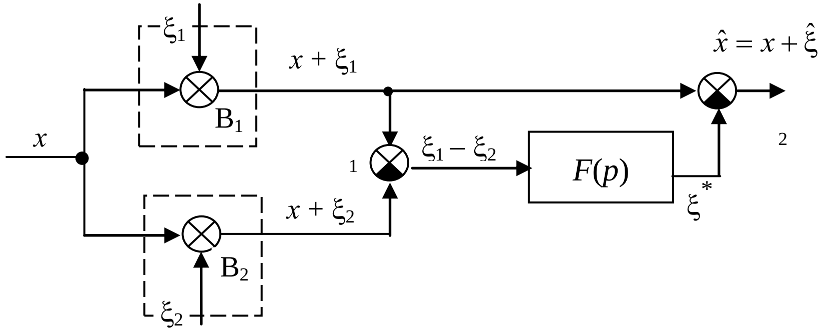


1 2

x ξ_1 ξ_2,

1

$\xi = (x + \xi_1) - (x + \xi_2) = \xi_1 - \xi_2$.

$F(p)$

2

$$\hat{x} = x + \xi_1 - F(p)[\xi_1 - \xi_2]$$

$$\hat{x} = x + [1 - F(p)]\xi_1 + F(p)\xi_2 = x + \hat{\xi}, \qquad (4.1)$$

$\hat{\xi} = [1 - F(p)]\xi_1 + F(p)\xi_2$ –

$F(p$

ξ_1 ξ_2

$\hat{\xi}$

ξ_1

ξ_2.

$F(p$

ξ_1 2

x

$$\hat{x} = x + \xi_1 - \xi_1 = x .$$

ξ_1 ξ_2

x $\hat{\xi}$.

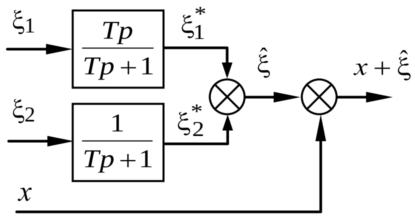
$F(p)$

$[1 - F(p)]$

$$F(p) = \frac{1}{Tp + 1}$$

T –

$$[1 - F(p)] = \frac{Tp}{Tp + 1}$$



$$\hat{x} = x + \hat{\xi}$$

ξ_1 ξ_2 –

$S_{\xi_1}(\omega)$ $S_{\xi_2}(\omega)$

ξ_1^* ξ_2^* (D $(\xi^*{}_1)$

D $(\xi^*{}_2)$

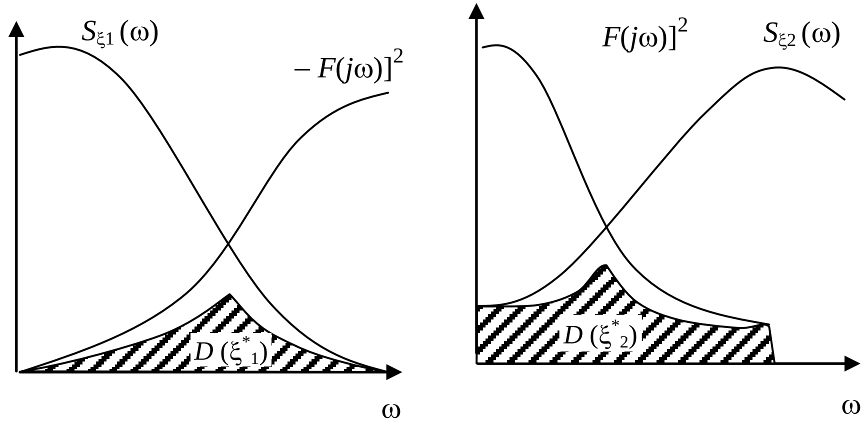
-

$[1 - F(j\omega)]$ $F(j\omega)$.

ξ_1 ξ_2

$S_{\xi_1}(\omega)$

$S_{\xi_2}(\omega)$,



$$D(\xi_1) = \sigma^2_{\xi_1} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{\xi_1}(\omega)d\omega, \quad D(\xi_2) = \sigma^2_{\xi_2} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{\xi_2}(\omega)d\omega,$$

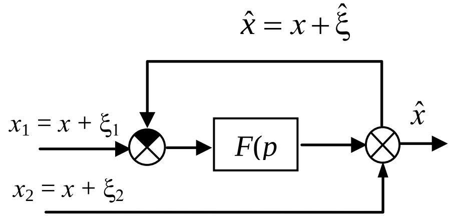
$\sigma_{\xi 1}, \sigma_{\xi 2}$ –

$$\sigma^2_{\hat{\xi}} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{[1 - F(j\omega)]^2 S_{\xi_1}(\omega) + [F(j\omega)]^2 S_{\xi_2}(\omega)\right\} d\omega$$

$\sigma^2_{\hat{\xi}}$

$F(j$

$x(t)$.



$$\hat{x} = [(x+\xi_1) - \hat{x}]F(p) + x + \xi_2 ,$$

$$\hat{x} = x + \frac{F(p)}{1+F(p)}\xi_1 + \frac{1}{1+F(p)}\xi_2 = x + \hat{\xi},$$

$$\hat{\xi} = \frac{F(p)}{1+F(p)}\xi_1 + \frac{1}{1+F(p)}\xi_2 .$$

$F(p) = Tp$

4. 2 Схема фільтрації

$$\hat{x} = \ _1(p)(x+\xi_1) + \ _2(p)(x+\xi_2),$$

$$\hat{x} = [\ _1(p) + \ _2(p)]x + \ _1(p)\xi_1 + \ _2(p)\xi_2 .$$

$_1(p \quad _2(p) = 1.$

$$\hat{x} = x + [1 - \ _2(p)]\xi_1 + \ _2(p)\xi_2 = x + \hat{\xi}, \qquad (4.2)$$

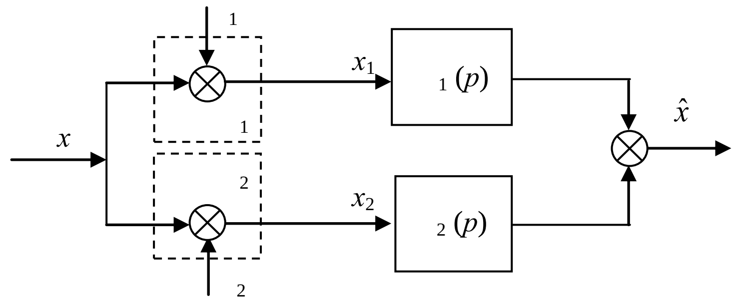
$$\hat{\xi} = [1 - \ _2(p)]\xi_1 + \ _2(p)\xi_2 .$$

$_2(p) = F(p$

Приклад 1.

$= \xi$

$$K_{\xi}\ (\tau) = \sigma^2\ e^{-\alpha\ |\tau|}$$



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ξ

$$K_{\xi}\ (\tau) = \sigma^2\ e^{-\alpha\ |\tau|}$$

$_0$,

$_0$ – $D\ _0$],

$= \xi \ + \xi$.

$\gg$

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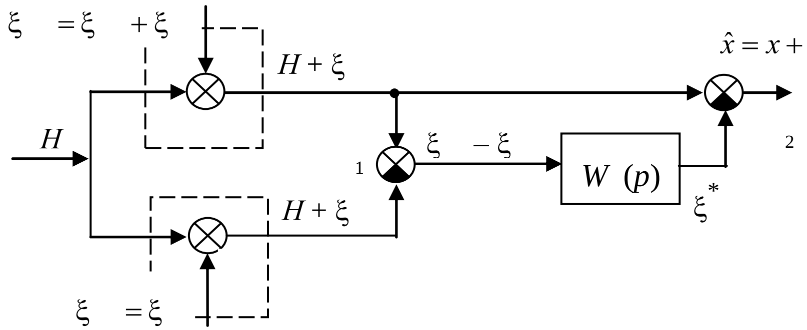

$$W\ (p) = \frac{1}{1+Tp}\ ,$$

T

$$S_{\xi}\ (\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \sigma^2 \quad -\alpha^2\ |\tau| \quad -j\omega t\, d\tau = \frac{\sigma^2\ \alpha}{\pi(\alpha^2\ + \omega^2)}$$

$$S_{\xi}\ (\omega) = \frac{\sigma^2\ \alpha}{\pi(\alpha^2\ + \omega^2)}\ .$$

$\gg$ -

$$S_{\xi}\ (\omega) = S_{\xi}\ (0) = \frac{\sigma^2}{\alpha\ \ \pi}\ .$$

$D_i[\varepsilon$

$$D_i[\varepsilon] = \int_{-\infty}^{+\infty} |W_i(j\omega)|^2\ S_{\varepsilon_i}(\omega)d\omega \qquad (4.3)$$

$[1 - W\ (p)]$

- $W\ (p$

$$D_1[\varepsilon] = \int_{-\infty}^{+\infty} \frac{\sigma^2}{\alpha\ \ \pi}\left|\frac{1}{1+j\omega T}\right|^2 d\omega = \frac{\sigma^2}{\alpha\ \ T}$$

$$D_2[\varepsilon] = \int_{-\infty}^{+\infty} \frac{\sigma^2\ \alpha}{\pi(\alpha^2\ + \omega^2)}\left|1 - \frac{1}{1+j\omega T}\right|^2 d\omega = \frac{\sigma^2\ \alpha\ \ T}{\alpha\ \ T+1}$$

$$D_{\Sigma}[\varepsilon] = \sum_{i=1}^{2} D_i[\varepsilon] = \frac{\sigma^2}{\alpha\ \ T} + \frac{\sigma^2\ \alpha\ \ T}{\alpha\ \ T+1} \qquad (4.4)$$

$T.$

T

$$T = z, \qquad \frac{\sigma^2\ \alpha}{\sigma^2\ \alpha} = m^2\ . \qquad (4.5)$$

$$D_{\Sigma}[\varepsilon] = \sigma^2\left(\frac{1}{m^2 z} + \frac{z}{1+z}\right) = \psi(z)\ , \qquad z > 0. \qquad (4.6)$$

z

$$\frac{d\psi(z)}{dz} = 0\ , \qquad \frac{d^2\psi(z)}{dz^2} > 0\ . \qquad (4.7)$$

z

$$z^2\left(m^2 - 1\right) - 2z - 1 = 0\ ,$$

$$z_1 = \frac{1}{m-1}\ ; \qquad z_2 = \frac{-1}{m+1}\ .$$

z_2 z.

– z_1

m σ^2 σ^2

$\gg$

z_1.

$$\frac{d^2\psi(z)}{dz^2}\Big|_{z=(m-1)^{-1}} = \frac{2\sigma^2}{m^2}(m-1)^3\left(1-\frac{1}{m}\right) > 0,$$

$$z = \alpha \quad T = \frac{1}{m-1}$$

z

T

$$T = \frac{1}{\alpha \ (m-1)}.$$

4. 3. Методи оптимальної обробки інформації в ПНК

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$$\dot{\mathbf{X}}(t) = \mathbf{A}(t)\mathbf{X}(t) + \mathbf{B}(t)\mathbf{V}_x(t), \tag{4.8}$$

$\dot{\mathbf{X}}(t)$ – n- $\mathbf{A}(t)$ –

$n \times n$

$\mathbf{V}_x(t)$ – k-

$\mathbf{B}(t)$ –

$\dot{\mathbf{V}}_x(t)$

$$M\,[\mathbf{V}_x(t)] =$$

$$\mathbf{R}_x(t) = M[\mathbf{V}_x(t), \mathbf{V}_x^{\mathrm{T}}(t)].$$

$$M[\mathbf{V}_x(t)] = 0;$$

$$M[\mathbf{V}_x(t), \mathbf{V}_x^{\mathrm{T}}(t)] = \mathbf{R}_x(t)\delta(t-\tau),$$

M – $\delta(t-\tau)$ – -

$\mathbf{X}(t$

$$\mathbf{Y}(t) = \mathbf{H}(t)\mathbf{X}(t) \tag{4.9}$$

$\mathbf{Y}(t$ m.

$\mathbf{H}(t)$

$\mathbf{X}(t$

$\mathbf{Y}(t)$.

$\mathbf{X}(t$

$t \in [t_1, t_2]$, $\mathbf{Y}(t$ $t \in [t_1, t_2$

$\mathbf{X}(t$ $t \in [t_1, t_2$

$m \le n$.

$\mathbf{Y}(t$

$$\mathbf{Z}(t) = \mathbf{Y}(t) + \mathbf{V}_z(t),$$

$\mathbf{V}_z(t)$ –

$\mathbf{V}_z(t$

$\mathbf{V}_z(t$

$\mathbf{V}_x(t)$, $\mathbf{R}_z(t$

$m \times n$

$$M\,[\mathbf{V}_z(t)] = 0,$$

$$M\,[(\mathbf{V}_z(t), \mathbf{V}_z^{\mathrm{T}}(t)] = \mathbf{R}_z(t)\delta(t-\tau).$$

$\mathbf{V}_x(t)$ $\mathbf{V}_z(t$ -

$$\begin{aligned}\dot{\mathbf{X}}(t) &= \mathbf{A}(t)\mathbf{X}(t) + \mathbf{B}(t)\mathbf{V}_x(t);\\ \mathbf{Z}(t) &= \mathbf{H}(t)\mathbf{X}(t) + \mathbf{V}_z(t).\end{aligned} \tag{4.10}$$

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$\hat{\mathbf{X}}(t)$

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–

Алгоритм оцінювання за методом найменших квадратів

m $\mathbf{X}$

$$\mathbf{Z} = \mathbf{H}\mathbf{X} + \mathbf{V}_Z\,, \qquad (4.11)$$

$\mathbf{H}$ – $\mathbf{Z}$, $\mathbf{X}$ $\mathbf{V}_Z$ –

$\mathbf{Z}_i$, $\mathbf{X}_i$

$\mathbf{V}_{z_i}$ $i = \overline{1, m}$.

$\mathbf{Z}$

$\mathbf{H}$ $\mathbf{X}$.

$$J = \sum_{i=1}^{m} \mathbf{V}_{z_i}^{\mathrm{T}} \mathbf{V}_{z_i}$$

$\mathbf{V}_{z_i}$.

$$J = |V_{z_1} V_{z_2} \ldots V_{z_m}| \begin{vmatrix} V_{z_1} \\ V_{z_2} \\ \vdots \\ V_{z_m} \end{vmatrix}$$

$$J = (\mathbf{Z} - \mathbf{H}\mathbf{X})^{\mathrm{T}} (\mathbf{Z} - \mathbf{H}\mathbf{X}) \qquad (4.12)$$

$\hat{\mathbf{X}}$ $\mathbf{X}$

$$\frac{\partial J}{\partial \mathbf{X}} = 0\,.$$

$$\mathbf{H}^{\mathrm{T}}(\mathbf{Z} - \mathbf{H}\hat{\mathbf{X}}) + (\mathbf{Z} - \mathbf{H}\hat{\mathbf{X}})^{\mathrm{T}}\mathbf{H} = 0 \qquad (4.13)$$

$\mathbf{H}$ i $(\mathbf{Z} - \mathbf{H}\hat{\mathbf{X}}$

$$\mathbf{H}\ (\mathbf{Z} - \mathbf{H}\hat{\mathbf{X}}) = 0,$$

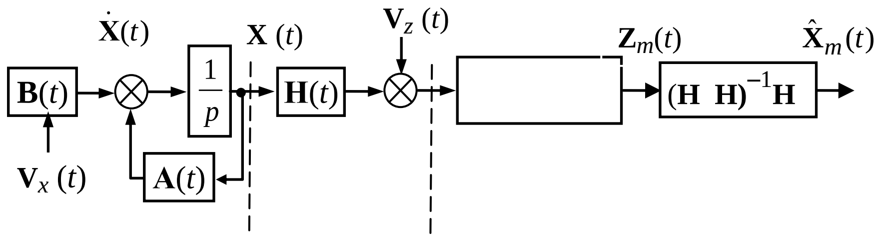
$\mathbf{H}\ \mathbf{Z} - \mathbf{H}\ \mathbf{H}\hat{\mathbf{X}}$ $\hat{\mathbf{X}} = (\mathbf{H}\ \mathbf{H})^{-1}\mathbf{H}\ \mathbf{Z}$.

$\hat{\mathbf{X}}$ $\mathbf{X}$

– Z_i, $i = \overline{1, m}$;

– $\mathbf{H}$;

– $\mathbf{H}\ \mathbf{H}$ $|\mathbf{H}\ \mathbf{H}| \neq 0$.



$\hat{\mathbf{X}}$

$\mathbf{Z}_m$

-

Приклад 2.

$$\begin{cases} z_1 = \gamma + \xi_1; \\ z_2 = \gamma + \xi_2; \\ z_3 = \gamma + \xi_3, \end{cases}$$

– ξ_1, ξ_2 ξ_3 –

$\mathbf{V}_z$).

$$\mathbf{Z} = \mathbf{HX} + \mathbf{V}_z,$$

$\mathbf{Z} = |z_1\ z_2\ z_3|$, $\mathbf{H} = |1 1 1|^{\mathrm{T}}$, $\mathbf{X}$ $\mathbf{V}_z = |\xi_1\ \xi_2\ \xi_3|$

$\mathbf{Z}$

$\mathbf{H}$ $\mathbf{X}$.

$$\hat{\mathbf{X}} = (\mathbf{H}\ \ \mathbf{H})^{-1}\mathbf{H}\ \ \mathbf{Z}$$

$$\mathbf{H}\ \ \mathbf{H} = \begin{vmatrix} 1 & 1 & 1 \end{vmatrix}\begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} = 1+1+1 = 3;$$

$$(\mathbf{H}\ \ \mathbf{H})^{-1} = \frac{1}{3};$$

$$(\mathbf{H}\ \ \mathbf{H})^{-1}\mathbf{H}\ \ = \frac{1}{3}\begin{vmatrix} 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix}.$$

$$\hat{\mathbf{X}} = (\mathbf{H}\ \ \mathbf{H})^{-1}\mathbf{H}\ \ \mathbf{Z} = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix}\begin{vmatrix} z_1 \\ z_2 \\ z_3 \end{vmatrix} = \frac{z_1 + z_2 + z_3}{3}.$$

Алгоритм оцінювання за методом максимуму правдоподібності

$\mathbf{V}_{z_m}$

$$P\left(\mathbf{V}_{z_m}\right) = \frac{1}{\sqrt{(2\pi)^m |\mathbf{R}_z|}} \exp\left[-\frac{1}{2}\mathbf{V}_{z_m}^{\mathrm{T}}\mathbf{R}_z^{-1}\mathbf{V}_{z_m}\right], \qquad (4.14)$$

$\mathbf{R}_z$ $\mathbf{R}_z$ |

$\mathbf{R}_z$.

$\mathbf{R}_z$ | ≠

$\mathbf{R}_z$

$$\psi(\mathbf{X}) = \frac{1}{\sqrt{(2\pi)^m |\mathbf{R}_z|}} \exp\left[-\frac{1}{2}(\mathbf{Z}_m - \mathbf{H}\mathbf{X}_m)^{\mathrm{T}}\mathbf{R}_z^{-1}(\mathbf{Z}_m - \mathbf{H}\mathbf{X}_m)\right],$$

$\hat{\mathbf{X}}_m$,

ψ ($\mathbf{X}$

$\mathbf{X}$

$$\frac{\partial \psi(\mathbf{X})}{\partial \mathbf{X}} = 0.$$

$$\mathrm{Ln}\,\psi(\mathbf{X}) = \mathrm{Ln}\frac{1}{\sqrt{(2\pi)^m |\mathbf{R}_z|}} - \frac{1}{2}(\mathbf{Z}_m - \mathbf{H}\mathbf{X}_m)^{\mathrm{T}}\mathbf{R}_z^{-1}(\mathbf{Z}_m - \mathbf{H}\mathbf{X}_m). \qquad (4.15)$$

$\mathbf{X}_m$

:

$$\frac{1}{2}\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}(\mathbf{Z}_m-\mathbf{H}\hat{\mathbf{X}}_m)+\frac{1}{2}\mathbf{H}\mathbf{R}_z^{-1}(\mathbf{Z}_m-\mathbf{H}\hat{\mathbf{X}}_m)^{\mathrm T}=0. \qquad (4.16)$$

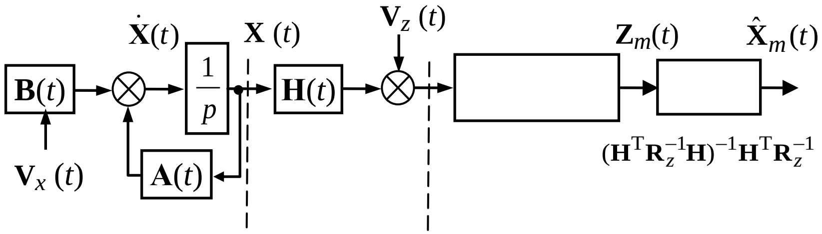
$$\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}(\mathbf{Z}_m-\mathbf{H}\hat{\mathbf{X}}_m)=0,$$

$$\hat{\mathbf{X}}_m=(\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{H})^{-1}\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{Z}_m. \qquad (4.17)$$

– m – $\mathbf{Z}_m$;

– $\mathbf{R}_z$

– $\mathbf{H}$.



$\hat{\mathbf{X}}_m$

$\mathbf{Z}_m$

Приклад 3. φ

$\sigma^2 = (6')^2$, $\sigma^2 = (3')^2$.

0°24′ 50 °30′.

φ

$$\hat\varphi=\left(\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{Z}.$$

$$\mathbf{H}=\begin{vmatrix}1\\1\end{vmatrix};\quad \mathbf{R}_z=\begin{vmatrix}\sigma^2 & 0\\ 0 & \sigma^2\end{vmatrix};\quad \mathbf{Z}=\begin{vmatrix}z\\z\end{vmatrix};$$

$$\mathbf{H}\ \mathbf{R}_z^{-1}=\begin{vmatrix}\frac{1}{\sigma^2} & \frac{1}{\sigma^2}\end{vmatrix};\quad \mathbf{H}\ \mathbf{R}_z^{-1}\mathbf{H}=\begin{vmatrix}\frac{1}{\sigma^2} & \frac{1}{\sigma^2}\end{vmatrix}\begin{vmatrix}1\\1\end{vmatrix}=\frac{\sigma^2+\sigma^2}{\sigma^2\ \sigma^2};$$

$$(\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{H})^{-1}=\frac{\sigma^2\ \sigma^2}{\sigma^2+\sigma^2};$$

$$(\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{H})^{-1}\mathbf{H}^{\mathrm T}=\frac{\sigma^2\ \sigma^2}{\sigma^2+\sigma^2}\begin{vmatrix}1 & 1\end{vmatrix}=\begin{vmatrix}\frac{\sigma^2\ \sigma^2}{\sigma^2+\sigma^2} & \frac{\sigma^2\ \sigma^2}{\sigma^2+\sigma^2}\end{vmatrix};$$

$$(\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{H})^{-1}\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}=\begin{vmatrix}\frac{\sigma^2}{\sigma^2+\sigma^2} & \frac{\sigma^2}{\sigma^2+\sigma^2}\end{vmatrix}.$$

φ

$$\hat\varphi=\left(\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{Z}=\left(\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm T}\mathbf{R}_z^{-1}\begin{vmatrix}z\\z\end{vmatrix}.$$

$$\hat\varphi=\frac{\sigma^2}{\sigma^2+\sigma^2}z+\frac{\sigma^2}{\sigma^2+\sigma^2}z,$$

$$\hat\varphi=\frac{3'}{6'+3'}50°24'+\frac{6'}{6'+3'}50°30'=50°28'.$$