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Digital Orthogonal Filtration of Spatiotemporal Signals in Multichannel Correlation Systems

The problem of justification of different processes simulation, particularly RC-noise, is being considered. The research paper describes the simplest models of noise with discrete time, peculiarities and characteristics of such models and represents the charts of correlation function estimate and spectral power density of RC process.

Key words: spatiotemporal signal, convolution, distribution function, correlation function, generating process, RC-noise, shaping filter.

The given paper considers mathematical models of noise that play an important role as initial models in simulation of more complex processes, for example a bandpass process. Depending on the field of science in which simulation method is applied models can be physical, biological, astronomic, mathematical ones. This research paper is devoted to mathematical models. Mathematical models are usually divided into analog ones (with continuous time) and digital ones (with discrete time). The research paper is aimed to describe exhaustively the simplest noise models with discrete time, justify the formulas of their technical parameters, graphically represent the formulas and describe the peculiarities and characteristics of such models.

The signals can be the objects of theoretical research and practical analysis only if the way of their mathematical formulation represents mathematical model of a signal. Mathematical formulation allows abstracting from the physical nature of the signal and the physical form of its carrier, conducting classification of signals, performing comparison of the signals, defining identity level, simulating of data processing systems. As a rule, description of a signal is set by the functional dependence of a particular information parameter of a signal from an independent variable. The choice of the mathematical tool for description is defined by the ease and convenience of its use while analyzing and processing information signals [1].

While recording in the detector the signals that carry destination information for the given types of measurement, all the parastic signals (noise and disturbances of different types) are recorded along with the main signal. Distortions of desired signals in the detection of different destabilizing factors on the measuring processes can also be viewed as disturbances.

Extraction of useful components from the total amount of the recorded signals or maximum decrease of noise and disturbances in the information signal while saving its useful components is one of the main tasks of initial signals processing (results of observation). It is worth to say that the separation of signals into desired signals and noise signals is relative. The sources of noise signals can be specific processes, phenomena, objects. After defining the nature of noise signals they can be transformed into information ones [1].

As a rule, signals received from the output of different sensing elements contain the desired signal and different disturbances the power of which exceeds the power of the useful signal. That is the reason for improvement of reception systems designed to perform in high-intensity noise. Hence, the development of automated data processing systems with advanced noise immunity is very important and has essential applied significance.

It is worth saying that the tasks of this type are frequent for hydroacoustics, radioastronomy, theory of automatic control, prospecting seismology, geodesy and other fields of science and technology.

Consider realizations of algorithms of spatiotemporal signal processing. To describe the signals polycharmonic signal model with finite cardinality can be used. A polycharmonic signal can be related to as determinate function of time, which is periodic one with a finite number of harmonics placed in a limited frequency band [3,5]:

$$Q(t)=\sum_{m=1}^{n} A_m \cos(\omega_m t+\theta_m), \tag{1}$$

where, t represents time, that can be continuous or discrete, $\omega_m \in (\omega_н, \omega_в)$ is the frequency of harmonics placed in a limited band, A_m and θ_m, $m \in \left[\overline{1,n}\right]$ are, respectively, amplitude-frequency and phase-frequency characteristics of a signal $Q(t)$, which differ from zero in a finite number of values $\left\{\omega_m, \quad m \in \left[\overline{1,n}\right]\right\}$.

Interconvertion of two polycharmonic signals $Q_1(t)$ and $Q_2(t)$ is defined in the following way

$$r_{12}(\tau)=\frac{1}{2}\sum_{m=1}^{n} A_m^{(1)} A_m^{(2)} \cos(\omega_m \tau+\Delta\theta_m),$$

where, $\left\{A_m^{(1)} A_m^{(2)}, \quad m \in \left[\overline{1,n}\right]\right\}$ is mutual amplitude-frequency specter and $\left\{\Delta\theta_m=\theta_m^{(2)}-\theta_m^{(1)}, \quad m \in \left[\overline{1,n}\right]\right\}$ is mutual phase-frequency specter.

A lot of phenomena in electrical engineering, physics, and mechanics can be described with the model (1). Such a model profoundly describes useful presented waves and rugged waves artificially created with vibrator. All the other waves that block extraction and processing of useful noise and measuring their parameters are considered as disturbances. Generally, the division of signals received into useful ones and disturbances is relative and depends on the method of signal reception and the purpose of its use.

Often, while researching mutual amplitude-frequency and phase-frequency characteristics of waves, deterministic polycharmonic processes where the categories of disturbances represent random signals can be viewed as useful signals. Due to widely used digital registration and spatiotemporal signals processing the necessity of elaboration of models with discrete time is beyond any doubt. The authors have received spectral and correlation characteristics of discrete white, colored [2, 4], RC and RLC noise [3]. The given paper considers only discrete RC noise, which is widely occurred in practice.

Discrete RC noise is the process with discrete time and low-frequency power spectrum. It is formed from the white noise by means of a e filter with unite-impulse response:

$$\varphi(t)=ue^{-\alpha t}U(t),$$

where, u is the filter transfer constant, $t \in \left(\overline{-\infty,\infty}\right)$, $\alpha>0$, $U(t)$ is unit function, that is

$$U(t)=\begin{cases}1, & t>0,\\ 0, & t\le 0.\end{cases}$$

A complex transfer characteristic is described as follows:

$$Y(\omega)=\frac{e^{\alpha}u}{e^{\alpha}-e^{-i\omega}}, \quad \omega\in[-\pi,\pi).$$

By means of such impulse-frequency characteristics most of real low-frequency filters can be described. Then, discrete RC noise can be represented as moving total [3]:

$$\xi(t)=\sum_{\tau=0}^{\infty} u e^{-\alpha\tau}\zeta(t-\tau), \quad t\in\left(\overline{-\infty,\infty}\right). \tag{2}$$

Its assembly average is

$$\mathbf{M}\xi(t)=\varkappa_1 \frac{u}{1-e^{-\alpha}}, \tag{3}$$

and u-parameter (transfer constant of shaping filter) is better to consider equal to 1 or $u=1-e^{-\alpha}$. In this case $u=1-e^{-\alpha}$ is taken from the condition $\sum_{t=-\infty}^{\infty}\varphi(t)=1$.

Correlation function is represented as

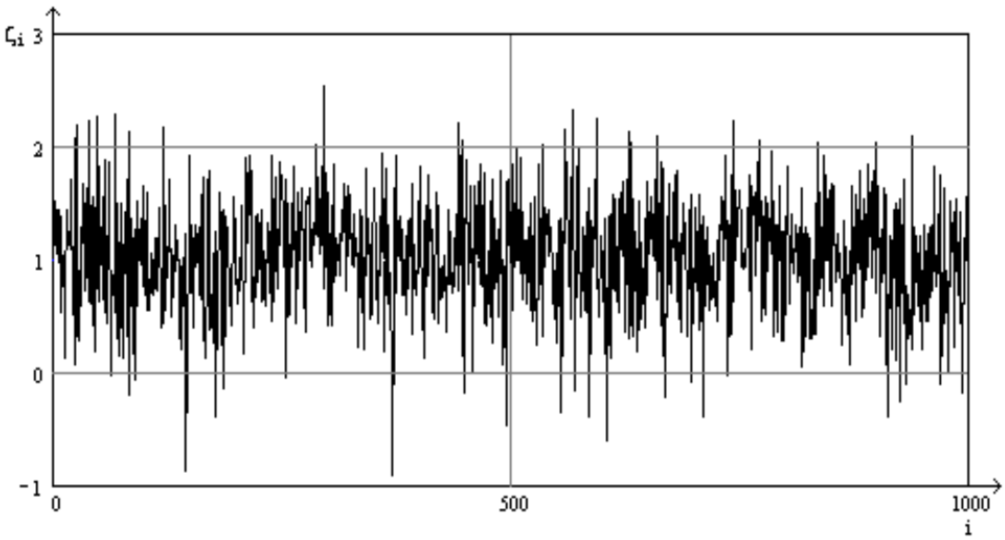
$$R(s)=\varkappa_2 \frac{u^2 e^{-\alpha|s|}}{1-e^{-2\alpha}}=R(0)e^{-\alpha|s|}; \quad s\in\left(\overline{-\infty,\infty}\right); \quad \alpha>0. \tag{4}$$

Power spectrum density of discrete RC noise can be defined according to he formula

$$S(\omega)=\frac{\varkappa_2 u^2 e^{-a}}{2(ch\alpha-\cos\omega)}, \quad \omega\in[-\pi,\pi). \tag{5}$$

View the practical part of the paper. Consider simulation of random process [3] for which the program has been developed and the following realizations and values has been received.

One realization of a standard random process [1] has been received. The value of independent functions ζ_i, $i=1,2,...,N$ of ζ -random value is shown in picture 1. For this case we can take sample size equal $N=1000$.



Pic.1 Realization of standard random process ζ_l.

Evaluation of assembly average and dispersion of input process with non-correlated values is defined according to the formulas:

$$\varkappa_1 = \frac{1}{n}\sum_{j=1}^{n}\zeta_j\,, \qquad \varkappa_2 = \frac{1}{n-1}\sum_{j=1}^{n}\left(\zeta_j - \nu_1\right)^2.$$

Hence, $\varkappa_1 = 0.996$, $\varkappa_2 = 0.256$.

Filter received random process by means of (1). The theoretical evaluation of the assembly average and dispersion of RC- noise can be found:

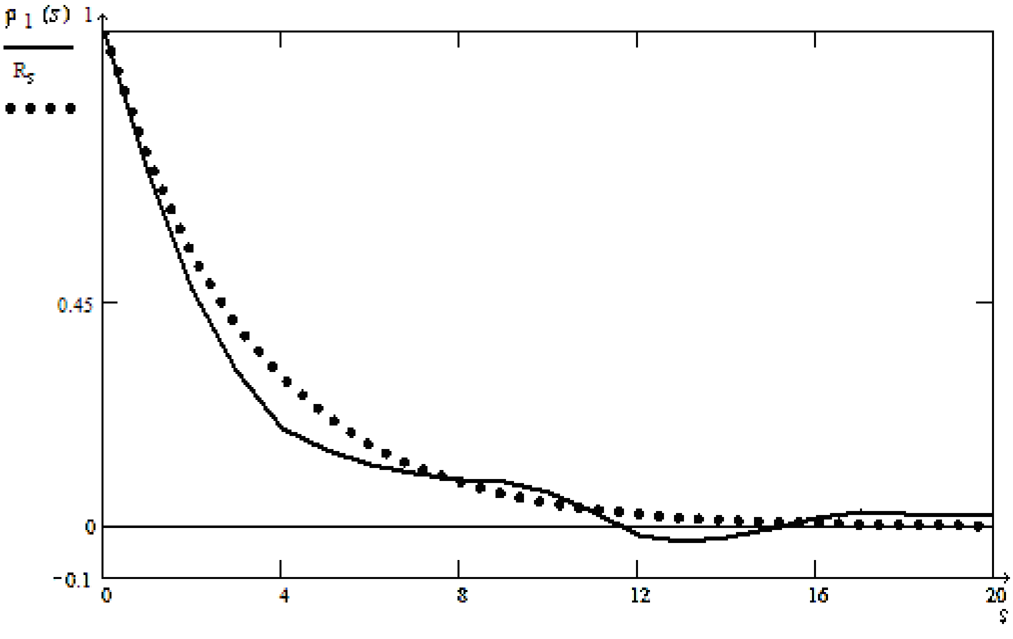
$$\nu_1 = \frac{1}{n}\sum_{j=1}^{n}\xi_j\,, \quad \mu_2 = \frac{1}{n-1}\sum_{j=1}^{n}\left(\xi_j - \nu_1\right)^2.$$

Hence, $\nu_1 = 3.84$, $\mu_2 = 0.523$.

The practical evaluation of the assembly average and dispersion of RC-noise can be obtained from (3): $\nu_1 = 3.843$, $\mu_2 = 0.566$.

Evaluate correlation function at realization of filtered process. Using MathCAD the graphs of theoretical curve of correlation function $R(s)$ evaluation can be built according to (4), and its practical realization according to the formula :

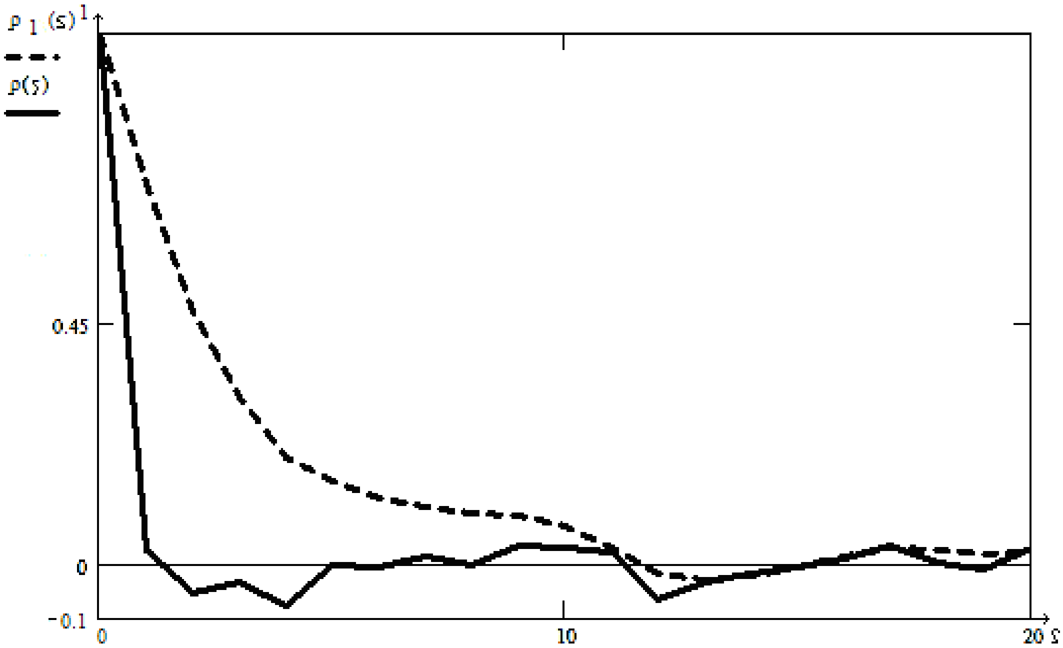
$$\rho_1(s) = \frac{1}{(n-1)\mu_2}\left[\sum_{i=0}^{n-s}(\xi_{i+s} - \nu_1)(\xi_i - \nu_1)\right].$$



Pic. 2. The Graph of correlation function $R(s)$ and $\rho_1(s)$ evaluation.

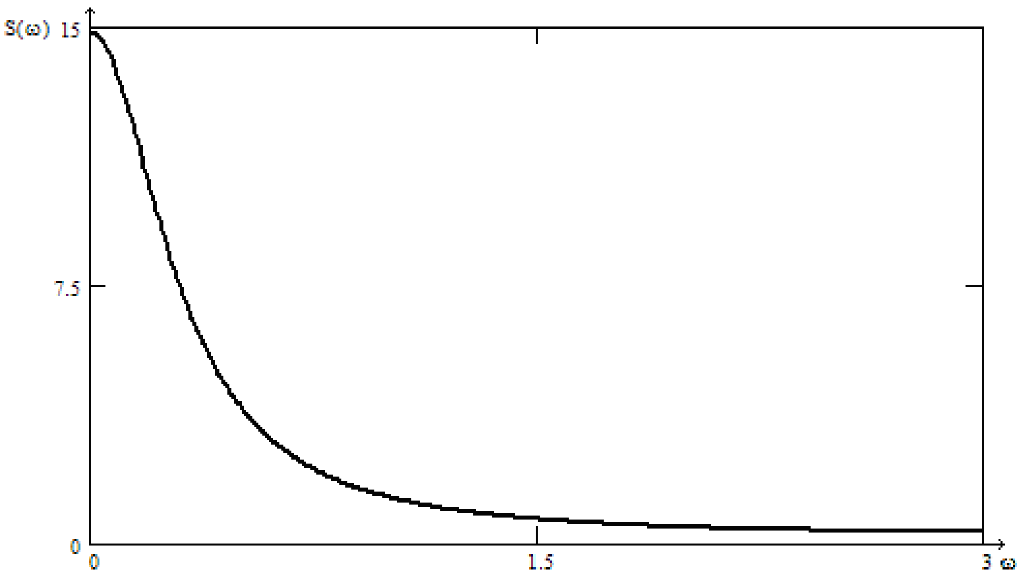
Evaluation of the function at realization is defined as follows:

$$\rho(s) = \frac{1}{(n-1)\varkappa_2}\left[\sum_{i=0}^{n-s}(\xi_{i+s} - \varkappa_1)(\xi_i - \varkappa_1)\right]$$



Pic. 3. The Graph of evaluation of correlation function at realization $\rho(s)$ ***and*** $\rho_1(s)$**.**

Spectral power density $S(\omega)$, received from simulation according to the theoretical formula (5) is represented in picture 4.



Pic. 4. The Graph of spectral power density $S(\omega)$**.**

Conclusions. The research paper considers and illustrates the construction of RC-noise model. Evaluation of assembly average and dispersion has been obtained and the graphs of correlation function and spectral density has been built.

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