

**MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE  
NATIONAL AVIATION UNIVERSITY**

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**REINFORCED CONCRETE  
AND STONE STRUCTURES**

Approved by the Ministry of Education  
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for students of higher educational institutions  
doing the course in «Civil Engineering»

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*Затверджено Міністерством освіти і науки України як підручник  
для студентів вищих навчальних закладів, які навчаються за напрямом  
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The textbook provides main information about construction materials (concrete, reinforced concrete, stone), their physical and mechanical properties, resistance calculation of reinforced concrete and stone structures for strength, rigidity and crack endurance.

**Першаков В. М.**

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У підручнику розглянуто загальні відомості про матеріали (бетон, арматурну сталь, камінь) та їхні фізико-механічні властивості; основи розрахунку залізобетонних та кам'яних конструкцій за міцністю, жорсткістю, тріщиностійкістю і способи їх конструювання.

Для студентів вищих навчальних закладів авіаційного та будівельного профілю, а також спеціалістів, магістрів з проектування, будівництва аеропортів, промислового та цивільного будівництва.

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## FOREWORD

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This textbook complies with the new syllabus of "Reinforced concrete and stone structures" for students of specialities "Industrial and Civil Engineering" (specialization "Construction and Maintenance of Airports") and "Highways and Aerodromes" (specialization "Construction and Maintenance of Aerodromes").

This textbook is based upon many-year teaching experience of the course by the group of lecturers of the department of computer technologies in construction at the Airport faculty of Kyiv national aviation university.

The methods of designing elements of structures are presented in accordance with the current construction standards and regulations.

This textbook helps students acquire knowledge of basics in projection, calculation and design of building constructions during their manufacture, assembly and maintenance.

Data on materials (concrete, reinforcing steel, reinforced concrete, stone) and their physical and mechanical properties are also provided. Classes and brands of concrete, reinforced elements and their joints, shrinkage and creep of reinforced concrete are considered as well.

Besides students get acquainted with the basics of calculating reinforced concrete structures and methods of their calculation for strength, rigidity and crack resistance with the design of multistory (flat monolithic and precast reinforced concrete floors, columns, foundations) and one-storeyed (slabs, beams, trusses, covering, columns, foundations) industrial constructions.

The author thanks teachers of the NAU: S.I. Tkachenko, S.L. Maksimov and V.Yu. Gyrych for their assistance in editing the textbook.

## LIST OF SYMBOLS

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**Forces and moments** due to external loads and prestressing force in a cross-section of a member are designated by:

$M$  = bending moment;  $Q$  = shearing force;  
 $N$  = longitudinal force;  $T$  = torque.

**Characteristics of materials:**

$R_c, R_{c,ser}$  = design axial compression concrete resistance for extreme conditions of groups I & II respectively;

$R_{ct}, R_{ct,ser}$  = design tensile strength of concrete respectively;

$R_s, R_{s,ser}$  = design strength of steel respectively;

$R_{s,w}$  = design strength of transverse steel;

$R_{s,c}$  = design strength of compressed steel;

$E_c$  = tangent modulus of elasticity of concrete in compression and tension;

$E_s$  = modulus of elasticity of steel;  $n = E_s / E_c$ ;

**Characteristics of cross-sections:**

$b$  = width of a rectangular section, width of a rib of a T- or I-section;

$h$  = height of a rectangular, T- or I-section;

$b_f, b_f'$  = width of a flange in the tension and compression zones of a T- or I-section;

$A$  = area of concrete in the member cross-section;

$h_f, h_f'$  = depth of the flange in the compression and tension zones of a T- or I-section;

$A_c$  = cross-sectional area of concrete in compression;

$A_{red}$  = cross-sectional area of a transformed section;

$I_{red}$  = moment of inertia of a transformed section;

$A_s, A_s'$  = cross-sectional area of stressed and unstressed reinforcement of S and S' respectively;

$A_{sp}, A_{sp}'$  = cross sectional areas of stressed part of reinforcement respectively S and S';

$a, a'$  = distances from the compressive resultant in the steel to the nearest side of the member cross-section, respectively;

$h_o, h_o'$  = effective depth of the reinforced concrete cross-section, equal to  $h-a$  or  $h-a'$  respectively;  
 $x$  = depth of the compressed zone of a cross-section;  
 $\xi = x/h_o$  relative depth of the compression zone of a cross-section;  
 $z_1$  = distance from the compressive to tensile resultants in a section (the arm of the internal force couple);  
 $z_c$  = distance from the centroid of concrete in compression to the resultant of the forces in the steel of area  $A_s$  or  $A_{sp}$ ;  
 $e, e'$  = distance from the point of application of the longitudinal force  $N$  to the resultant of the forces in the steel of area  $S$  or  $S'$  respectively;  
 $e_o$  = eccentricity of the longitudinal force  $N$  with respect to the centroid of a transformed section;  
 $e_{op}$  = eccentricity of the prestressing force  $P$  with respect to the centroid of a transformed section;  
 $e_{o,tot}$  = eccentricity of the resultant of the longitudinal force  $N$  and prestressing force  $P$  with respect to the centroid of a transformed section.

# 1. INTRODUCTION INTO THE SUBJECT "REINFORCED CONCRETE AND STONE STRUCTURES"

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## 1.1. General information about the subject

The title of this series of lectures is "Reinforced concrete and stone structures" They are designed for the forth-year students (the eighth terms). All the forth-year students are to take a credit test.

The theoretical course consists of 32 academic hours, the laboratory research – 16 academic hours. You have 40 academic hours for your term thesis. Your independent training course is 60 hours.

"Reinforced concrete and stone structures" is one of the main special subjects. "Reinforced concrete and stone structures" is called the basic subject for civil aviation engineers in the specialty "Industrial and civil construction" with specialization "Airport construction and operation".

This training course includes learning the methods of analysis and design of building structures. The bearing elements of buildings and structures are made of different building materials according to the normative regulations.

This training course includes both scientific and technical information, which is necessary for specialists of the capital construction departments. The maintenance of buildings, the operation of aerodrome facilities, interaction with the repair and building offices are the fields of activity for the employees of the capital construction departments.

This subject is basic for the following training courses: "Aerodromes of civil aviation", "Technical operation of buildings and constructions", "Building organization and planning", "Building economics".

Students prepare their diploma thesis at the department of computer building technologies. The contents of the course "Reinforced concrete and stone structures" determine the appropriate part of diploma thesis.

The *purpose* of the subject is to provide a future specialist with knowledge in the field of bearing building constructions designing for structures of civil aviation.

The *task* of the subject is to study the practical methods for analysis and projecting of bearing elements of buildings and structures.

The knowledge of scientific and technical principles of the subject "Reinforced concrete and stone structures" facilitates the activities of the civil aviation specialists at the following departments: capital construction, ground buildings operation, aerodrome service, repairing and building offices, building and assembly offices and so on.

As a result of studying this subject the student must have all the necessary knowledge and skills for qualification characteristics of aviation specialists.

## **1.2. Fields of Reinforced Concrete Application**

Reinforced concrete structures serve as a basis for modern industrial ground and underground construction. Reinforced concrete members are used in one- and multistoried industrial buildings, thermal power stations, storehouses, civil buildings, including blocks of flats and structures for agricultural purposes.

Reinforced concrete is widely used in large-span thin-walled shells for industrial and civil buildings, bunkers, tanks, and smoke stacks. It has also proved indispensable in subways, highway and railway bridges and tunnels, hydro electrical power plants, nuclear power stations and reactors, irrigation works, mine yards, structures for underground workings, etc.

The amount of steel used in linear reinforced concrete structures is by 2–3 times less than that in all-steel structures. Reinforced concrete floors, pipes, bunkers and the like require one tenth of the amount of steel used in similar sheet-steel structures.

Reasonable combination of reinforced concrete, metal and other structures with the right use of the best properties of each material is of great economic importance.

By the production method there are distinguished prefabricated structures manufactured at special factories and assembled at the construction sites, monolithic ones erected at the construction site and prefabricated monolithic ones which are made of prefabricated concrete elements and monolithic concrete.

Prefabricated reinforced concrete structures meet to the best, the requirements of construction industrialization. The use of prefabricated reinforced concrete improves considerably the structure quality, reduces labour as compared with in-situ structures, cuts down (or, sometimes, eliminates) the need for formwork and false work, and considerably

reduces the time of construction. Reinforced concrete structures can be erected all the year round, including the winter season, without any significant rise in the cost of construction; whereas structures cast in-situ are still mostly erected during the warm season of the year, and, if erected in winter, involve additional expenditures (to heat the concrete while it hardens, and so on).

### **1.3. The use of reinforced concrete structures in modern construction**

Prefabricated and monolithic reinforced concrete with prestressed reinforcement is the basic material of capital construction.

At a concrete product plant the products are wall blocks and panels of both normal-weight concrete and light one, columns, floor and covering slabs; trusses, beams, arches, crane beams, foundation blocks.

For engineering structures – prefabricated elements of bridge spans, battering walls, pedestrian subways, tunnel finish as well as supports of contact system, sleepers, pipes, filler elements and others.

Examples: In St. Petersburg the bus station cover of 144×96 m is performed as prefabricated reinforced concrete vaults. The cover consists of twelve thin shells with the size of 96×12 m each with prestressed tie-beams. The shell represents a "barrel" vault of ribbed prestressed units of 3×12 m.

The air terminal at the airport Borispol is ceiled by sloping double curved shell. The shell size in plan is 48×58 m, with the rise of 8.9 m. The shell is mounted of reinforced concrete ribbed slabs of 2.6×8.0 m and side element – pipes with the diameter of 720 mm.

Enterprise building cover consists of built-up arches with chord 36 and 24 m with prestressed tie-beams and flat ribbed roofing slabs. Middle arches with the chord of 36 m rest on prefabricated reinforced concrete columns 29 m high with the span of 12 m.

Monolithic reinforced concrete is used for construction of power-generating structures, in car roads and airfield pavement, in silo towers, stacks and so on. An example: in monolithic prestressed reinforced concrete tower 540 m high of the Moscow TV Station a reinforced concrete part has the height of  $h = 385\text{m}$  (the author – Nikitin N.V.).

At present in the domestic and foreign practice civil multistoried buildings are erected in movable, rolling and knock-down forms or by the method of concreting (quick hardening) with their further jacking up, with the use of type 3 cements.



In CIS, USA and other countries for bearing elements they usually use unstressed and prestressed structures of light concretes of B30-B40 classes with density 1600–1900 kg/m<sup>3</sup>. The light concrete use for the multistoried building structures, big span covers reduces their empty weight by 25–30 per cent, cuts down reinforcement consumption by 15–10 per cent, reduces their cost by 8–12 per cent.

In the former USSR over 30 prefabricated lightweight aggregate concrete bridges, including the first in Europe Bridge with prestressed reinforcement across the river Akhtuba have been built. Lightweight aggregate concrete is used for piles, trusses, crane beams, floors, airfield pavement and roadway coverings and so on. In Armenia and Georgia pumice concrete is used for bridging and hydraulic engineering works.

Another example: the Administrative Office building of light concrete in the city of Sydney. The main tower is round, on plan with the diameter of 41 m and 184 m high (50 storeys). Basements and columns of ground floors are made of cast-in-place reinforced concrete; all the other floors are made of precast light reinforced concrete with density 1730 kg/m<sup>3</sup> and strength 30 MPa.

Building cost is reduced by 33 per cent as compared to that of heavy concrete. Light concrete with nonprestressed and prestressed reinforcement can be used for the erection of very important buildings and constructions.

For making concrete wall slabs, roof and floor slabs cellular concretes are used; for light covers of big spans thin slab armored cement construction are used. Prestressed reinforced concrete guy shell in Krasnoyarsk consists of stays with a span of 78m every 1,5 m on which precast reinforced concrete slabs 25 mm thick with two side ribs are laid.

Stays of bar reinforcement of A-III<sub>B</sub> class are fixed to reinforced concrete beams rested on columns. The span of columns and prestressed guys is 12 m. The guys are grappled to the ground by means of build-up monolithic disk anchors. Water drainage is conducted due to sag. Shell prestressing is carried out by super load equivalent to roof & snow weight with subsequent solidification formation of joints between plates.

The suspended structure of bus station cover in Kiev of round plan with the diameter 160 m has a hip form with stays fixed on the central strut. Reinforced concrete columns are placed along the perimeter of the building.

The sport complex "Druzba" (Moscow) has a hall cover of precast monolithic of space reinforced concrete element consisting of sloping

central shell 48×48m and side folding shells. The upper reinforced concrete bearing ring and steel tie beam endure oval ring forces and stability of folder shells. On the plan there is an 88×96 m oval.

Steel structures are expedient in buildings and constructions with big loads, spans and heights, at plant shops of technological processes, which are accompanied by the release of large quantities of heat. Reinforced concrete structures can fail due to reinforced concrete drying while steel ones are reliable at the temperature of 100–150<sup>0</sup> C.

#### **ASSIGNMENTS TO CHAPTER 1**

1. What is the strength of building constructions?
2. Give definition of rigidity.
3. What rigidities do you know?
4. What is the building stability?
5. Determine general rules of  $M$  and  $Q$  diagrams control at flat transverse bending of straight bars.
6. What way can be used for quick check of efforts in the chords of beam truss?
7. How can we check the sign correctness in the elements of beam truss grid, which is loaded symmetrically by linearly distributed loading?
8. How do efforts change in these braces during motion from support to the middle of truss?
9. Given four equal size span beam, which is loaded with the constant uniformly distributed load? Temporary load is the same. How should we distribute temporary loading over the spans?

## 2. GENERAL INFORMATION ON CONCRETE AND REINFORCED CONCRETE

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### 2.1. Reinforced Concrete as a Construction Material

Numerous tests have shown that concrete is strong in compression and weak in tension. A simply supported plain concrete beam subjected to transverse bending has to resist tension in the zone lying below the neutral axis (Fig. 2.1, *a*); accordingly, its load-bearing capacity is poor as a result of poor resistance to tension.

The same beam containing reinforcing steel in the tension zone (Fig. 2.1, *b*) has a load-bearing capacity, which is much higher and can be 20 times higher than that of the plain concrete beam.

Reinforced concrete members (such as columns) intended to support compressive loads are also reinforced with steel bars (Fig. 2.1, *c*). Since steel has high compressive and tensile strength, it considerably improves the load-bearing capacity of compressed members.

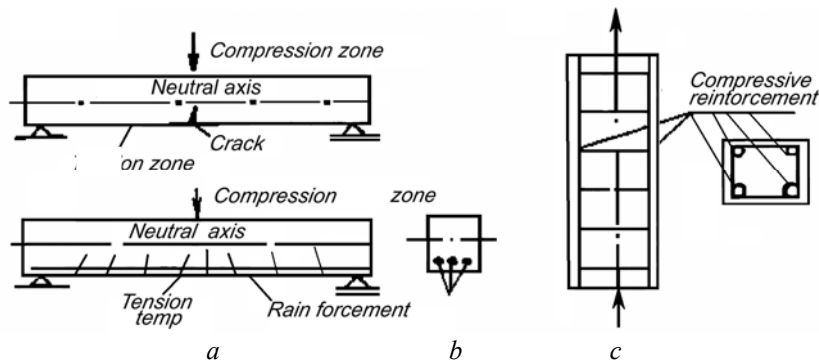


Fig. 2.1. Behaviour of members under load

That joint work of concrete and reinforcing steel results from the beneficial combination of their physical and mechanical properties:

- a) strong bonds forming between the hardened concrete and steel make them deform together under load;
- b) dense concrete (having a high cement content) protects the encased steel against corrosion and direct exposure to fire;

c) the coefficients of linear thermal expansion of steel and concrete are about the same; so, when the ambient temperature varies within  $\pm 100^\circ\text{C}$ , the initial stresses occurring in the materials are insignificant, and the steel does not slip from the concrete.

Reinforced concrete has found extensive application in construction owing to its durability, fire resistance, resistance to environmental effects, high dynamic resistance and low maintenance costs of buildings and structures, etc. Also, due to availability of fine and coarse aggregates in large quantities almost everywhere, reinforced concrete can be used practically all over the country.

Reinforced concrete is more durable than other construction materials. If used properly, reinforced concrete structures may serve for an infinitely long time, with their load-bearing capacity unaffected. This is due to the fact that unlike other materials, the strength of concrete increases with time, and the embedded steel is protected against corrosion. Reinforced concrete owes its good fire resistance to the fact that structures with a proper concrete cover subjected to medium fires for several hours suffer damage starting from their surface, so they lose their load-bearing capacity gradually.

Loaded reinforced concrete structures tend to develop cracks in the concrete of the tension zone. In most structures, the crack width under service loads is small, and cracking does not affect their normal behaviour. In practice (especially with high-strength materials), however, it is often necessary to prevent members from cracking, or limit the crack width. For this purpose, the concrete is prestressed before loading; as a rule, this is done by tensioning the steel. In this case, we speak of prestressed reinforced concrete. A relatively large self-weight of reinforced concrete, although useful under certain conditions, is generally undesirable. The weight of structures is reduced by using thin-walled members and hollow members (with voids) or members made of porous-aggregate concrete.

## **2.2. Off-Site Fabrication**

Reinforced concrete members should be designed so that their overall dimensions, cross-sections and reinforcement would be optimum for manufacture at special factories and erection at the construction sites. Members design solution and off-site fabrication technologies are closely connected. Members, who can be mass-produced by high efficiency equipment without labour-consuming manual operations, are always preferable. The design of a member and the manufacture procedures are closely related. There exist several technologies for precast concrete manufacture.

**Line Production.** With this type of manufacture, concrete members are made in forms installed on cars moving by rail from one work station to another. As a car moves on, workmen successively carry out all the necessary operations, namely erection of the reinforcement, tension of the prestressing steel, installation void formers into hollow-core slabs, placement and compacting of the concrete mix, removal of void thermo-wet treatment of concrete to accelerate its hardening. All cars are advanced at a predetermined speed. This high efficiency type of manufacture is utilized by major plants in the quantity production of members of relatively small mass.

**Stage-by-Stage Production.** With this type of manufacture, all operations are carried out by appropriate plant shops, and a form with a member is transported from station to station by cranes. The technological rate of car movement is not predetermined nor is it forced.

**Casting-Bed Technology.** A distinction of this technology is that members remain at the same place during their manufacture and heat treatment, and the units executing the necessary operation move along the stationary forms. Casting beds are equipped with travelling cranes, concrete placers, and vibrators to compact the concrete mix. Structural members are fabricated in plain or shaped forms (matrices or cassettes). This technology is used to manufacture large and prestressed members for industrial buildings (such as trusses, roof beams, crane beams, columns, and so on).

Panels for floors, ceilings and walls of civil buildings are widely made in cassettes. Members are made in stationary beds consisting of a faggot of vertical metal cassettes, so that several panels may be produced at a time. The assembly and disassembly of cassettes are mechanized. Reinforcing cages are installed inside each compartment of a cassette. Concreting is by a thin-consistency mix pneumatically supplied through pipes. Owing to their vertical position, the surface of such members is even and smooth.

In the vibration-rolling process, thin floor, ceiling and wall panels are manufactured on a continuously moving belt, the smooth or corrugated surface of which serves as a mould. After a reinforcing cage has been installed, the concrete mix is placed on the belt, which is then vibrated and compacted from above by rollers. The members being rolled are covered from above and heated from below. During their movement on the belt (for several hours) they attain the necessary strength and after cooling on racks finished products are transported to a storehouse. Technological operations are carried depending on the speed of the forming belt.

The whole set of precast members required for the construction of a building, cannot be fabricated by any one of the above processes. That

is why precasting plants and yards use several production processes simultaneously. The development of new and more promising designs may in some cases require certain improvements of the production techniques or creation of new technologies, which, in its turn, may necessitate a re-design of the product.

### **2.3. Historical Background of Reinforced Concrete**

Invention and development of building constructions including reinforced concrete ones, is closely related to the society living conditions, productive forces and production relations development. Reinforced concrete was invented in the latter half of the 19th century when rapidly growing industry, trade and transport necessitated the construction of a large number of factories, bridges, ports and other structures. By that time, the cement industry and iron-and-steel manufacture had already been sufficiently developed to provide the necessary basis for reinforced concrete production.

In the period from 1850 to 1885, the first reinforced concrete structures were erected in France (1850, 1854, 1867-1880), Britain (1854), and the USA (1855-1877).

From 1885 to 1917, reinforced concrete began to find still wider application in economically developed countries, such as Britain, France, the USA, Germany and Russia, in the floors of industrial buildings, buried pipes, wells, walls, tanks, bridges, overpasses, trestles, fortification structures and so on.

By the end of the 19th century, the work conducted by researchers and engineers in various countries had given birth to the main principles of elastic theory based on the elastic behaviour of materials under load.

In Russia, reinforced concrete structures developed under the influence of foreign experience and home practice. Of great importance were the experiments on various structures (such as slabs, tanks, vaults, funnels, precast corn-bins, and arch bridges), spiral reinforcement of columns, indirect reinforcement of compression members, cementation and reinforced concrete piles, arrangement of floors and ceilings without beams, precast flat floor slabs (solid and cored), and the like.

The development and application of reinforced concrete in the Soviet Union can be divided into two periods.

During the first period (1918–1945), reinforced concrete was mainly used in industrial and hydro-engineering construction. In the late 20s, large institutions were set up to design industrial structures. At the same

time, various research institutes and laboratories came into being to carry out research work on concrete and reinforced concrete.

To satisfy the demands of large-scale construction and to save metal, reinforced concrete was widely used instead of steel structures. These were the years when reinforced concrete was a dominating construction material. It was used in in-situ multispans, beam floors, multispans and multistage frameworks, arches and similar structures for industrial buildings, hydro electrical power plants, elevators, silos, etc. In 1928, the first precast reinforced concrete was used in some industrial buildings and hydro electrical power plants. At about the same time, thin-walled in-situ roofs were first applied in domes, folds, cylindrical shells, and marquees. Advances in construction were accompanied by the development of design theory for multispans, beams and frames, shells, slabs, plates, and other structural systems.

The early achievements in the field of reinforced concrete were covered in scientific research publications, monographs, and textbooks.

During this period, further steps were taken in preparing, transporting and placing the concrete mix, winter concreting, and standardization of formwork.

With the growing scale of construction, the drawbacks of elastic theory regarding reinforced concrete as an elastic material became more evident. To overcome them, the basic principles of plastic (or collapse) design were formulated at the end of 1931. This theory suggested that plastic strains in the steel and concrete of a bending beam at destruction reached their ultimate values, which determined the critical bending moment.

This approach was backed by a number of experiments and wide theoretical work, which gave birth to a fundamentally new theory of designing and reinforcing concrete structures. Some time later the new method was extended to cover members in eccentric compression and reinforced structures containing stiff reinforcement.

Plastic theory served as the basis for new Soviet standards and specifications requiring that reinforced concrete structures should be designed in terms of the breakdown stage.

In the 30s, the idea of improved prestressed concrete suggested at the end of the past century was put into practice owing to the work done by French and German scientists. In the Soviet Union, the first experiments on prestressed concrete began in 1930.

In the early 40s, Soviet researchers put forward a theory of crack resistance and stiffness of reinforced concrete, which gained general recognition.

The second period in the use of reinforced concrete in the USSR and Ukraine began at the end of World War II (1945) and still continues.

In the meantime, reinforced concrete has become the basic material not only in industrial and hydro engineering structures, but also in civil buildings, thermal power stations, highways, railroads, and agricultural structures. The advent of precast reinforced concrete structures has ushered in a new era in construction. Now, structural members are factory-made and erected at the site. This technology has improved the strength of reinforced concrete members, and has gone a long way towards mechanized erection. Progress has been made in the design of statically indeterminable reinforced concrete structures with allowance for inelastic strains on the basis of the equilibrium between the ultimate external and internal forces. Considerable advances have been made in the field of creep in concrete, and design and arrangement of underground structures (such as subways and various tunnels). In the 50s, the theory of design and arrangement of heat-resistant reinforced concrete structures was developed.

During this period, the design layout of structures has undergone significant change due to the shift to structures made of entirely precast members and application of prestressed members, which are manufactured nowadays by almost every plant. Precast skeleton and bearing-wall multistory structures have been built and a theory of their design has been advanced.

A major step in the development of reinforced concrete in the Soviet Union has been the establishment of a complete range of standard structural members for large-scale manufacture and use.

In 1955, the limit-state design of reinforced concrete members underlying the present-day standards and specifications was developed and put into practice in the USSR.

The latest relevant standards embody the results of the experiments on new types of reinforcement; design of reinforced concrete members; layout of structures; specifications for heavy, heat-resistant, light-weight and cellular concretes, and so on.

A considerable contribution to the modern theory and structural design of reinforced concrete in the Soviet Union has been made by higher educational establishments which have carried out research in the composite action of precast structures in plane and spatial systems; design of civil multistory buildings; economic design of reinforced concrete structures; spatial behaviour of spans in trestle structures; polymerized concretes and structures using polymerized concrete members; limit-state design of domes; creep in concrete



states of stress in some roofs; systems used to strengthen reinforced concrete structures; biaxial bending and eccentric compression bond between steel and concrete, etc.

#### **ASSIGNMENTS TO CHAPTER 2**

1. What main technological requirements should concrete meet as a material for reinforced concrete constructions?
2. How are concretes distinguished by different properties?
3. What main classes of concrete do you know?
4. What is the armature of concrete? How does it influence strength and deformability of concrete?
5. What is shrinkage of concrete? How does it influence stresses in concrete?
6. What main types of samples are used to test concrete for compression, tension?
7. What is the influence of sample sizes on concrete strength at compression?

### 3. MAIN PHYSICAL AND MECHANICAL PROPERTIES OF CONCRETE

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#### 3.1. Concrete for Reinforced Concrete Structures

As a material for reinforced concrete structures, concrete must have certain predetermined physical and mechanical properties. Among other things, it must be sufficiently strong, have a high bond resistance, and be watertight enough to protect reinforcing bars against corrosion.

Also, according to the purpose of a reinforced concrete structure and exposure conditions, concrete must meet some special requirements. It must be frost resistant (it must well withstand freezing and thawing cycles) for example, in exterior wall panels, outdoor structures, etc.; resist long time exposure to high temperatures; withstand attack by corrosive environments and so on.

Concretes may be classified according to:

a) inner structure, into "no-voids" concretes where the space between the aggregate grains is entirely occupied by a hardened binding agent; coarse-grained concretes using no or little sand; induced-porosity concretes using aggregates and binding agents with artificially produced open pores; and cellular concretes with artificially produced closed pores;

b) average density (bulk unit weight), into super heavy concretes with an average density of more than 2500 kg/m<sup>3</sup>; heavy concretes with an average density of more than 2200 to 2500 kg/m<sup>3</sup>; medium-heavy concretes with an average density of more than 1800 to 2200 kg/m<sup>3</sup>; and light-weight concretes with an average density of more than 500 to 1800 kg/m<sup>3</sup>;

c) form of aggregate, into nonporous-aggregate (usually, sand-and-broken stone) concretes; porous-aggregate concretes; concretes using special aggregates providing biological shielding, heat resistance, etc.;

d) aggregate size, into coarse concretes using both coarse and fine aggregates, and fine concretes using fine aggregates;

e) manner of hardening, into naturally hardened concrete; normal-pressure moist-cured concrete; and high-pressure steam-cured (autoclaved) concrete.

For brevity, concretes used in bearing reinforced concrete structures are referred to in Soviet practice as:

– heavy concretes; these are "no-voids", heavy, nonporous coarse-aggregate concretes using cement as the binding agent, hardening under any conditions;

– porous-aggregate concretes; these are dense, medium-heavy or lightweight, coarse porous-aggregate concretes using cement as the binding agent, hardening under any conditions.

Nonporous aggregates for heavy concretes are broken stone crushed from various rocks, such as sandstone, granite and diabase, and natural quartz sand. Porous aggregates are naturally occurring perlite, pumice or shell rock, or manufactured materials, such as ceramsite, slag, etc. Those using porous aggregates are divided into ceramsite concrete, slag concrete, perlite concrete, and so on.

Induced-porosity, cellular, and lightweight-aggregate concretes with an average density of  $1400 \text{ kg/m}^3$  and lower are mostly used for filler walls. Super heavy concretes are used in structures providing biological shielding against radiation. Fine concretes find application mostly for making joints in structures from precast (prefabricated) members.

In order to produce concrete having a predetermined strength and durability, one needs to mix in the correct proportion the necessary ingredients which include various cements, coarse and fine aggregates, admixtures improving the workability or frost resistance of the concrete, and so on.

The strength of concrete depends on the aggregate grading (the aggregate should be graded so that the volume of voids in the mixture would be minimum), aggregate strength and surface condition, cement grade and its amount, amount of water, etc. Uneven and rough aggregate grains provide a better bond in the mix. So, sand-and-broken stone concretes have a greater strength than sand-and-gravel concretes.

In more detail, the proportioning of concrete is discussed in the course on construction materials.

The necessary density of concrete is attained by intelligent aggregate grading, thorough tamping in forms, and use of a sufficient amount of cement (which ranges between 250 and  $500 \text{ kg/m}^3$ ). The strength of the concrete increases with increasing density. In order to reduce the consumption of cement, its strength should exceed the required strength of the concrete.

### **3.2. Inner Structure of Concrete. Its Effect on Strength and Stress-Strain Behaviour**

The strength and stress-strain behaviour of concrete considerably depend on its inner structure. To get a better insight into the

matter, let us see what happens when concrete is produced. When water is added to a mixture of aggregates and cement, the cement combines with the water (hydration is said to take place). Hydration produces what is known as cement gel, a porous mass consisting of cement particles, which have not yet reacted, and an insignificant amount of complex crystal substances suspended in water. As the mix is agitated, the gel envelopes the aggregate grains, the paste becomes gradually stiff, and the crystals intertwine into conglomerates growing with the time. The stiffening gel turns into hardened cement combining the coarse and fine aggregates into concrete.

A very important factor affecting the strength and inner structure of concrete is the amount of water added to the mix; it is expressed in terms of the water-cement ratio which is determined as the weight ratio of mixing water to cement per unit volume of the concrete mix. A water-cement ratio of about 0.2 is usually required for all of the cement to be hydrated. However, the concrete thus proportioned would be so stiff that it would be extremely difficult to place and finish. Consequently, additional water must be added to make the mix workable. For example, fluid concrete mixes, which can be poured into place, have a water-cement ratio of 0.5 to 0.6, and hard mixes, which are vibrated into forms, have a water-cement ratio of 0.3 to 0.4.

As the water-cement ratio decreases, the volume of voids in the hardened cement reduces and the strength of the concrete rises. That is why prefabricated concrete structures are made of hard concrete mixes having the lowest possible cement ratios. Concretes made of hard mixes have a higher strength; require less cement and shorter aging in forms.

Thus, the inner structure of concrete is rather non-uniform. It is formed by a space lattice of hardened cement embedding sand and broken stone grains of various size and shape, and containing a vast number of pores and capillaries filled by free water, vapour and air. Physically, concrete is a capillary and porous material with discontinuities throughout its bulk, in which all the three phases – solid, liquid and gaseous – are present. The hardened cement is also of non-uniform inner structure and consists of an elastic crystal conglomerate and a viscous gel filling it.

The slow processes taking place in such a material, namely the change in water balance, decrease in volume of the stiffening gel, growth of crystal conglomerates, are responsible for the specific elastic-plastic behaviour of concrete under load and exposure to variations in ambient temperature and humidity.

Experience shows that the theories of strength suggested for other materials are inapplicable to concrete. Relations between the composition, inner structure, strength and stress-strain behaviour of concrete have yet to be studied. Present day knowledge about the strength and stress-strain behaviour of concrete is based on a great number of experiments carried out in laboratories and field conditions.

### **3.3. Shrinkage and Initial Stress**

Concrete decreases in volume when it is allowed to harden under normal atmospheric conditions (this is known as shrinkage) and increases in volume when it hardens in water (this is called swelling). Concretes using expansive or shrinkage-compensating cements do not shrink. Experimental data show that shrinkage increases 1) with increasing amount of cement per unit volume of concrete and also with active and alumina cements; 2) with increasing amount of mixing water, that is, a higher water-cement ratio; and 3) with decreasing aggregate size; fine sand and porous broken stone result in a greater shrinkage.

The amount of shrinkage may be reduced by using aggregates with a higher stress-to-strain ratio, that is, a higher modulus of elasticity. The amount of shrinkage may further be reduced by using unevenly graded aggregates as this reduces the volume of voids.

As a rule, the rate of shrinkage is especially high during the initial period of hardening and the first year after manufacture; later, it gradually slows down. The rate of shrinkage and the resultant strain increase with decreasing ambient humidity. Work in sustained compression speeds up, whereas work in sustained tension slows down the shrinkage.

Shrinkage is closely related to the physical and chemical processes related to the hardening and decrease in volume of the cement gel, the loss of excess water by evaporation into atmosphere and by hydration of the cement particles that have not yet reacted. As the cement gel hardens, reduces in volume and forms crystal conglomerates, shrinkage becomes less. Capillary phenomena caused in the hardened cement by the excess water also affect the extent of shrinkage – the surface tension in the menisci exerts pressure on the capillary walls and gives rise to volume strain.

When the concrete hardens, the cement is prevented from shrinkage by the aggregate, which acts as an internal restraint and induces initial tensile stresses in it. A similar restraint is produced by the crystal conglomerates formed in the hardening gel. The concrete dries non-

uniformly throughout its bulk, so it shrinks also non-uniformly, which results in initial shrinkage strains. The exposed surface layers of the concrete, which dry faster, are subjected to tension, whereas the wetter internal layers, in restraining the shrinkage of the surface layers, are subjected to compression. Such initial tensile stresses produce shrinkage cracks in the concrete.

Initial shrinkage stresses do not enter into the strength analysis of reinforced concrete structures explicitly. Instead, they are taken into account by coefficients covering various aspects of concrete strength, as well as by concrete structures reinforcement measures. Initial shrinkage stresses can be reduced by suitable proportioning of the concrete mix, by steam and moist treatment, and also by providing shrinkage joints in structures.

### 3.4. Strength of Concrete

**Basics.** Since concrete is a non-uniform material, it is non-uniformly stressed when exposed to external load. In a compressed test specimen, stresses concentrate on the harder particles having a higher modulus of elasticity. They result in forces, which concentrate on the particles and tend to break the bond. In addition, stresses concentrate near pores and voids. As is known from the theory of elasticity, in a compressed test specimen, compressive and tensile stresses concentrate near holes in the material, the tensile stresses acting in planes parallel to the compressive force (Fig. 3.1). Since concrete is abundant in pores and voids, tensile stresses at one hole or pore overlap with those at an adjacent hole. As a result, an axially compressed specimen is subjected to longitudinal compressive and lateral tensile stresses (this is known as a secondary stress field).

Experimental data show that compression test specimens collapse due to the lateral rupture of the concrete. At first, microscopic bond failure cracks appear throughout the specimen bulk. As the load increases, the bond failure cracks merge into visible cracks running in parallel or at a small angle to the direction of the compressive force (Fig. 3.2). Then, the cracks open up, and this is accompanied by an apparent increase in specimen volume. Finally, the specimen fails completely.

The point at which microscopic cracks begin to form in concrete under load may be determined by ultrasonic tests. The velocity of ultrasonic waves,  $v$ , spreading at right angles to the direction of the compressive stresses falls as the cracks grow. The point on the curve of

Fig. 3.2 where the velocity of ultrasound begins to decrease defines the compressive stress,  $R_{cr}^0$  at which microscopic cracks begin to appear in the concrete. It is by the value of this stress that we describe the strength and stress-strain behavior of concrete.

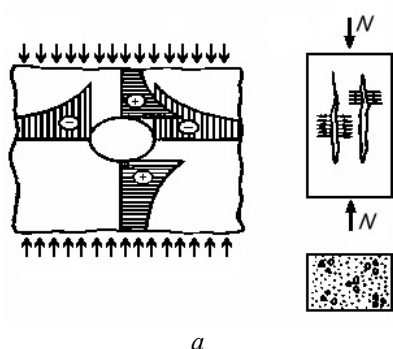


Fig. 3.1. Compressed concrete test specimen:  
*a* — concentration of stress at micropores and microcracking, and cracks;  
*b* — lateral cracking of concrete in axial compression

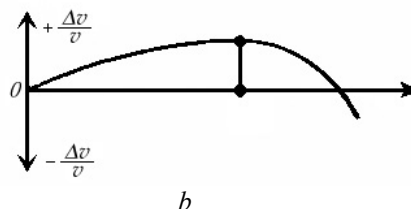


Fig. 3.2. Determination of the compressive stress in concrete at the limit of microcracking,  $R_{cr}^0$ , by the results of ultrasonic test

Since the particles and pores making up concrete are spaced non-uniformly and differ in size, there is a certain range in strength between different test specimens made of the same concrete mix. The strength of concrete depends on many factors, the main of which are:

- type of ingredients and form of mixing;
- age and hardening conditions;
- shape and size of the test specimen;
- type and duration of stress.

The last mentioned point is important because concrete has different ultimate strength in compression, tension or cut.

**Classes and Brands of Concrete.** Concretes are assigned the so-called design classes' brands, or class brand number, depending on the character and purpose of the structure. A design class and brand, or class and brand number are specified by the quality index. Thus, class can be assigned based on axial compressive strength, axial tensile strength, frost resistance, or water resistance average density. Concretes of the desired brand may be obtained by correct proportioning of the concrete mix with subsequent testing for strength. Concrete is widely used as a construction material owing to its ability to withstand high pressure, so the test of compressive strength is the most frequently used one. Also, compressive strength is the

most conveniently determined of all characteristics. This is the reason why the compressive strength of concrete is adopted as its main characteristic.

The relevant standard requires that the compressive strength of concrete should be determined on test specimens in the form of cubes with a 15-cm edge during a 28-day curing period at the temperature of  $20 \pm 2^\circ\text{C}$ . The test result gives the class of the concrete in terms of ultimate compressive strength MPa. Concrete should be treated long enough for it to reach the required strength by the moment when the structure is subjected to a design load. Precast concrete members may be dispatched to a job before they reach the required strength; in this case, their strength is specified according to transportation and erection conditions, time before loading, etc.

According to compressive strength, the following classes and brands are established, namely:

– heavy concretes, B7,5; B10; B12; B15; B25; B30; B35; B40; B45; B50; B55; B60;

– porous-aggregate concretes, B3,5; B5; B7,5; B10; B12,5; B15; B20; B25; B30; B35; B40.

If a concrete structure works mostly in tension, one needs to know the tensile strength of the concrete in addition to its compressive strength. According to their tensile strength, concretes are classed as follows: concretes, Bt0,8; Bt1,2; Bt1,6; Bt2; Bt2,4; Bt2,8; Bt3,2.

Here, the numerals give the ultimate tensile strength in MPa. Brand numbers in terms of frost resistance are assigned to concretes used in structures exposed to freezing and thawing cycles in cold climatic regions, exterior structures in regions with frequent freeze-thaw changes, and to porous-aggregate concrete used in filler walls. They are designated from MPa F15 to MPa F500. The numerals give the number of freeze-thaw cycles that the concrete can withstand under a water-saturated condition.

Brand numbers in terms of water resistance are assigned to concretes used in structures subjected to water pressure, such as tanks and pressure pipes. These brands are designated by the letter "W", they range from W-2 to W-12. Here, the numerals give the limit of water pressure at which the flow of water through channels in the concrete is still prevented.

Brand numbers of average density ( $\text{kg/m}^3$ ) are: heavy concretes from D2200 to D2500, porous aggregate concrete from D800 to D2000, induced-porosity concrete from D800 to D1400.

An optimum design class of concrete is chosen for technical and economic reasons according to the type of reinforced concrete structure, kind of stress, manner of manufacture, exposure conditions, and so on.



It is advisable to use concrete classes of at least B20 for compression members made of heavy or porous-aggregate concrete. Relatively high design concrete classes, such as B30 and B40, have proved to be of great utility in structures subjected to considerable compressive forces (columns, arches, and the like). In prestressed reinforced concrete structures, the best results are achieved with classes B30 through B50, depending on the type of prestressed reinforcement. The optimum class for unprestressed members in bending is B20.

Porous-aggregate cement concretes classed in the same brands in terms of strength, frost and water resistance as heavy concretes are often used in precast or in-situ concrete structures. They are especially effective in many applications, causing mass reduction.

**Cube Crushing Strength of Concrete.** As previously noted, an axially compressed concrete cube fails due to lateral rupture (Fig.3.3). The cracks are inclined because of the friction between the platen and seat of the compression-testing machine and the cube faces. The forces of friction directed inwards prevent the free lateral strain of the cube, thereby causing what we call the grip effect. The restraint decreases with increasing distance from the grips, so, after it has collapsed, the test specimen takes the form of two truncated pyramids with their small bottoms close to one another. If, before testing, we cap the ends of the specimen with neat cement paste or other material, the lateral strain meets no resistance, the rupture cracks become vertical (parallel to the direction of compression), and the ultimate compressive strength of the concrete is nearly halved (Fig. 3.3).

According to the relevant standard, concrete cubes are tested without capping. Experiments show that the strength of concrete taken from the same batch varies with the size of test specimen. For example, if we designate the ultimate compressive strength of a standard cube with a 15-cm edge by  $R$ , this value for a 20-cm edge cube will be about  $0.93R$ , and that for a 10cm edge cube will be about  $1.1R$ .

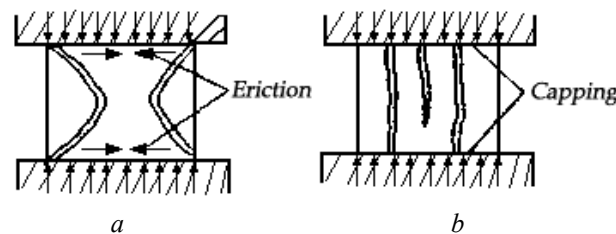


Fig. 3.3. Concrete cube failure:  
*a* — with friction at the cube faces; *b* — without friction

The strength of cube test specimen made of porous-aggregate concrete is less affected by variations in cube size, being  $0.97R$  and  $1.03R$ , for a 20-cm and a 10-cm edge cube, respectively. The variations in strength are explained by the fact that the end grip effect is not the same for different cube sizes. In some countries, for example, in the USA, the test specimens are in the shape of cylinders with the height of 12 inches (30.5cm) and a diameter of 6 inches (15.2 cm). The ultimate compressive strength obtained with cylindrical specimen ranges between 0.7 and 0.75 that of a 15-cm edge cube.

**Prism Crushing Strength of Concrete.** Real reinforced concrete structures differ in shape from cubes. Accordingly, the cube crushing strength of concrete cannot be used directly in the strength analysis of members. Instead, the main characteristic for compression members is the prism crushing strength,  $R_c$  defined as the ultimate axial compressive strength of concrete prisms. Experiments, with concrete prisms with the least square bottom  $a$  and height  $h$  have shown that the prism crushing strength of concrete is less than the cube crushing strength and decreases with increasing  $h/a$  ratio. The curve of Fig.3.4 plotted according to averaged experimental results illustrates the relation between  $R_c/R$  and  $h/a$ .

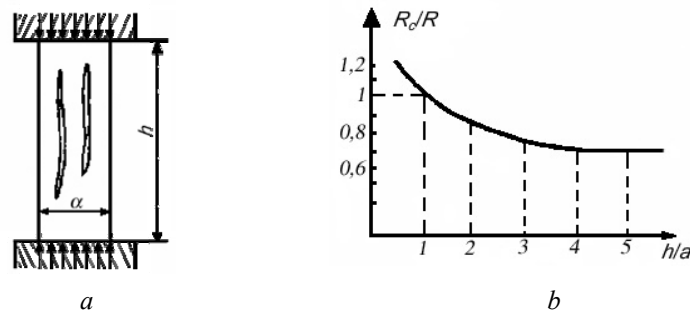


Fig. 3.4. Concrete prism crushing strength ratios

Referring to the figure, the effect of friction at the prism ends decreases with increasing height and, at  $h/a = 4$ ,  $R_c$  becomes almost stable and equal to about  $0,75R$ . The effect of concrete flexibility becomes appreciable only at  $h/a \geq 8$ .

The strength of the compression zones of members in bending is also expressed in terms of  $R_c$ . In this case, the real curved stress diagram for the compression zones in the limit state is replaced by a conventional rectangular stress diagram (Fig. 3.5).

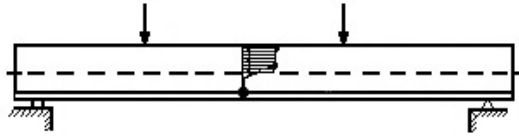


Fig. 3.5. State of stress in the compression zone of a bending reinforced concrete beam

**Tensile Strength of Concrete.** This depends on the strength of the hardened cement and the bonding strength between the cement and aggregate grains. According to experiments, the tensile strength of concrete is 1/10 to 1/20 of its compressive strength, and the relative tensile strength decreases with increasing concrete class number. Here, the spread in strength exceeds that in the case of compression tests. The tensile strength of concrete may be enhanced by raising the cement content, reducing the water-cement ratio, and using broken stone with rough surfaces.

The ultimate axial tensile strength of concrete may be determined by the following empirical formula

$$R_{ct} = 0,233R^{2/3}. \quad (3.1)$$

The results obtained with the help of the above relation are not always correct because concrete is non-uniform in its structure. As a rule,  $R_{ct}$  is determined by a rupture test, splitting test and flexure test (Fig. 3.6), the last mentioned being the most common. Here, a simple beam loaded at three-points is used with a span three times its depth. The moment of rupture of the beam yields the tensile strength of concrete

$$R_{ct} = M/\gamma W = 3,5M/bh^2. \quad (3.2)$$

where  $W = bh^2/6$  is the resisting moment of a rectangular section, and  $\gamma = 1.7$  is the factor taking into account the curved nature of the tension area stress diagram due to inelastic deformation.

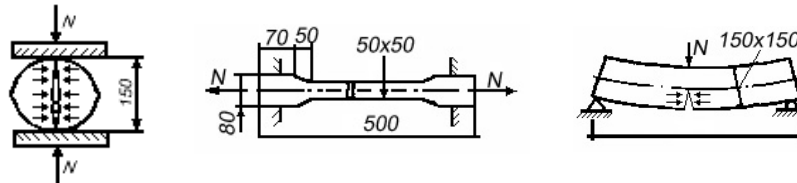


Fig. 3.6. Rupture, splitting and flexure tests for determining the tensile strength of concrete

**Shear Strength and Resistance of Concrete to Spalling.** Ideally, shearing consists in the separation of a member into two parts along the given plane by shearing forces. Grains of the coarse aggregate in the plane of shear act as keys, so their section resistance markedly affects that of the concrete. In shear, stresses are taken to be uniformly distributed over the entire cross-section of the specimen. The ultimate shear strength can be determined by the formula

$$R_{sh} = 2R_{ct}. \quad (3.3)$$

Real reinforced concrete structures are seldom subjected to shearing stresses alone; as a rule, shearing forces act together with longitudinal forces.

Concrete resists spalling when bent until inclined cracks develop. Spalling stresses vary along the depth of a section according to a quadratic parabola. As experimental data show, the ultimate splitting strength of concrete is 1,5 to 2 times greater than  $R_{ct}$ .

**Long Time Strength of Concrete.** Experience shows that concrete subjected to heavy sustained loading fails at stresses below the ultimate axial compressive strength  $R_c$ . This happens because such loads develop considerable inelastic strains and structural changes in concrete. According to experimental data, the ultimate long-time axial compressive strength of concrete, designated  $R_{ct}$  may be as low as  $0.9R_c$  or even lower. If service conditions are favorable for concrete to gain strength, the ratio of the applied stress to the prism strength of the concrete,  $\sigma_c/R_{ct}$  gradually decreases, and the negative effect of sustained loading may not be felt.

**Fatigue Strength of Concrete.** If a concrete structure is subjected to as many as millions repeated compressions, the ultimate compressive strength of the concrete decreases because of microcracking. Experiments show that the fatigue strength of concrete under repeated loading or its fatigue (or endurance) limit,  $R_r$  depends on the number of loading cycles and the ratio of the minimum to the maximum stress in a cycle (or the cycle characteristic),  $p = \sigma_{\min}/\sigma_{\max}$ . Fig. 3.7, *a* shows the endurance curve of concrete. Here, the number of cycles,  $n$ , is marked on as abscissa, and the periodically changing fatigue limit, laid off as ordinate. As the number of cycles rises,  $R_r/R_c$  falls. The value on the horizontal portion of the curve at  $n \rightarrow \infty$  is called the absolute fatigue limit. Practically,  $R_r$  at  $n = 2 \times 10^6$  changes almost linearly with the cycle characteristic  $p$ , the minimum value being  $R_r = 0,5R_c$  (Fig. 3.7, *b*).

As has been found experimentally, the lowest value of the fatigue limit corresponds to the stress at which micro cracking develops in concrete, so  $R_r \geq 0,5R_{cr}^0$ . This relation makes it possible to find the fatigue limit with a single loading of a test specimen by determining the start of micro cracking with the help of ultrasonic testing.

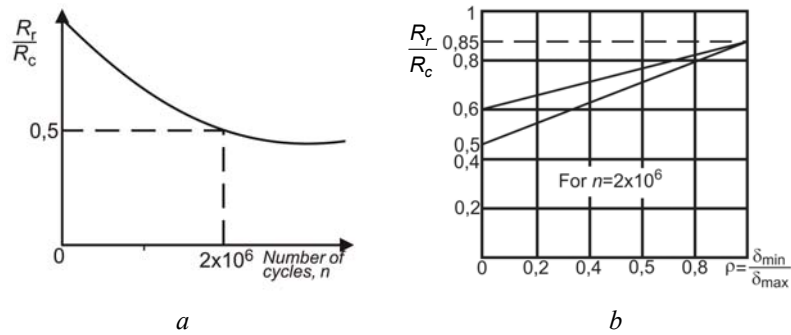


Fig. 3.7. Fatigue limit of concrete as a function of loading cycles (a); and as a function of the cycle characteristic  $p$  determined at  $n = 2 \times 10^6$  (b)

The fatigue limit is necessary for the fatigue analysis of reinforced concrete structures subjected to dynamic loads, such as crane beams, floors in some industrial buildings, and so on.

### 3.5. Stress-Strain Behaviour of Concrete

**Types of Strain.** As a rule, two main types of strain develop in concrete. These are volumetric strain produced by shrinkage and temperature variations, which develop in all directions, and strain produced by applied stresses, usually developing in the same direction as the applied stress. Under the action of longitudinal (axial) stress, concrete deforms laterally. The ratio of lateral to longitudinal deformation (Poisson's ratio) for concrete, designated by  $\mu$  is equal to 0.2. Concrete is a material having both elastic and plastic properties. When concrete is subjected to a load, no matter how small, elastic deformation in it is accompanied by inelastic deformation. In view of this, the strain produced by applied stresses is classified according to the nature and duration of loading into three types, namely strain developed by short-duration single loading, strain caused by a sustained load, and strain produced by repeated loading.

**Volumetric Strain.** Shrinkage strain in concrete varies over a rather wide range. According to experimental data,  $\epsilon_{s1} \approx 3 \times 10^{-4}$  or more for

heavy concretes, and  $\varepsilon_{sl} \approx 4.5 \times 10^{-4}$  for porous-aggregate concretes. Strain caused by swelling ranges between one half and one fifth of that developed by shrinkage.

Temperature strain of concrete depends on the coefficient of thermal expansion of concrete, designated by  $\alpha_{c,t}$ . The thermal coefficient of expansion for heavy concrete and porous-aggregate concrete using quartz sand is  $1 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$  over the ambient temperature range from  $-50$  to  $+50^\circ \text{C}$ . The value of this coefficient varies with the character and amount of cement, aggregate, and moisture state of concrete within  $\pm 30 \%$ .

**Strain Developed by Short-Duration Single Loading.** When concrete is subjected to a short-duration single loading, it undergoes the following strain

$$\varepsilon_c = \varepsilon_e + \varepsilon_{pl} \quad (3.4)$$

composed of elastic and plastic strain (Fig. 3.8). A small proportion of plastic strain (about 10%) recovers after the load has been removed.

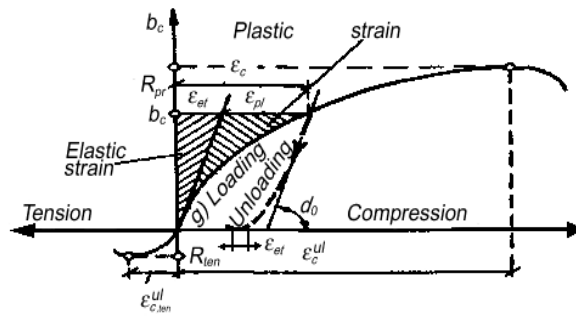


Fig. 3.8. General stress-strain curve of concrete

In Soviet practice, this share of strain is called the elastic recovery strain. If a test specimen is loaded step by step and the amount of strain is  $r$  measured twice each time (that is, just as it is loaded and some time after), we shall obtain a stepped stress-strain curve such as shown in Fig. 3.9. The strain measured immediately after loading is elastic in nature and varies linearly with the applied stress, whereas the strain measured some time after loading is non-elastic, it increases with rising stress and is represented by horizontal portions of the stress-strain diagram. With a sufficient number of loading steps, the relation between stresses and strains may be graphically shown as a curve. A similar curve can be obtained if the load is removed step by step. Again, we measure the strain twice later at each step (first, just as the load has been

reduced and some other time, that is under a new load). With a sufficient number of steps, the stepped line thus produced may be replaced by a smooth curve which in this case is concave (see Fig. 3.8).

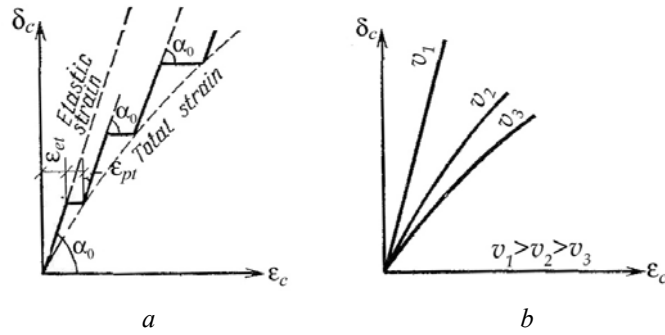


Fig. 3.9. Stress-strain curve of concrete in compression plotted: *a* — for several loading steps; *b* — plotted for different loading rates

As is seen, elastic strain occurs in concrete only if it is loaded instantly, whereas non-elastic strain develops with time and depends on the loading rate,  $v$ . As the loading rate increases at the same stress,  $\sigma_c - \epsilon_c$  and non-elastic strain decreases. Stress-strain curves,  $\sigma_c - \epsilon_c$  plotted for different loading rates,  $V_1 > V_2 > V_3$  are shown in Fig. 3.9.

When concrete works in tension, the resulting strain likewise consists of elastic and plastic components

$$\epsilon_{c,t} = \epsilon_{et} + \epsilon_{pl,t} \quad (3.5)$$

where  $\epsilon_{et}$  is the elastic strain and  $\epsilon_{pl,t}$  is the plastic strain.

**Strain Caused by a Sustained Load.** When a concrete structure is subjected to a sustained load, plastic strain develops in the concrete with time. It has been shown that non-elastic strain is most significant during the first three or four months after loading and may increase during several years. In the stress-strain curve of Fig. 3.10 portion 0-1 represents the strain, which occurs immediately when concrete is loaded, the curvature of this portion depends on the rate of loading. Portion 1-2 shows the growth in plastic strain caused by a constant applied stress.

An increase in plastic strain under a sustained stress is called creep. Creep deformation may be 3 or 4 times as great as elastic deformation.

If creep is allowed to increase freely under a constant sustained load, stresses in concrete remain unchanged. If restraints in concrete, say, reinforcing steel, do not allow creep to develop freely, we speak of

restrained creep; under the circumstances, stresses in the concrete are no longer constant.

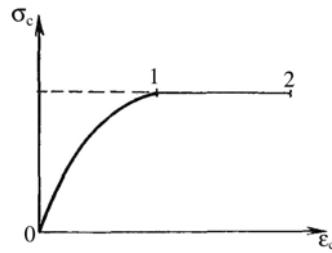


Fig. 3.10. Stress-strain curve for a test specimen subjected to a sustained load

If a concrete test specimen is subjected to an initial stress, developing an initial strain, and restrained from further deformation, there is a progressive decrease in stress with the time.

This is known as relaxation. Creep and relaxation of concrete are similar in nature and markedly affect the work of reinforced concrete and structures under load.

Tests on concrete prisms show that whatever is the loading rate producing the stress, the final creep strain corresponding to this stress is always the same (Fig. 3.11, a).

Creep increases with increasing stress; a plot of strain as a function of time for various stresses,  $\sigma_{c1} < \sigma_{c2} < \sigma_{c3}$  is shown in Fig. 3.11.

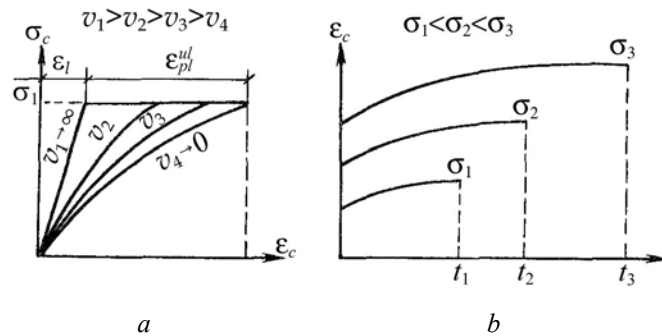


Fig. 3.11. Creep strain:  
 a – creep strain as a function of initial loading rate and;  
 b – as a function of time under load,  $t$ , and stress,  $\epsilon_c$

Concrete loaded at an early age shows a greater amount of creep than the aged one. Creep in concrete cured in a dry atmosphere is more evident



than that in moist-cured concrete. The amount of creep also increases with increasing water-cement ratio and amount of cement per unit volume of the concrete mix. Concretes with stronger aggregate grains and a higher compressive strength show a lower creep. Also, porous-aggregate concretes display a somewhat greater creep than heavy concretes.

The manner in which creep develops in concrete is explained by its inner structure, long crystallization period and decrease in gel volume with the hardening of the cement. When concrete is subjected to a load, the stress is redistributed from the yielding viscous gel to the crystal conglomerates and aggregate grains. At the same time, a proportion of creep is contributed to by capillary phenomena related to the flow of excess water in micro pores and capillaries under the applied load. With the time, the redistribution of stresses slows down and the deformation ceases completely.

Creep is customarily divided into linear creep when the stress-strain behavior of concrete is approximately described by a straight line, and nonlinear creep. When the applied stress exceeds the limit,  $R_{cr}$  at which micro cracking occurs in concrete, strain begins to develop rapidly and non-linear creep takes place. This division is arbitrary because in some tests the stress-strain diagram may be non-linear already at relatively small stresses. It is worth mentioning that nonlinear creep is important in designing prestressed members in bending and also eccentrically compressed and some other concrete members.

Creep and shrinkage occur simultaneously. So the total deformation in concrete is the sum of elastic strain,  $\epsilon_e$ , creep,  $\epsilon_{pl}$ , and shrinkage,  $\epsilon_{sl}$ . Shrinkage, however, is volumetric in nature, whereas creep develops mainly in the direction of the applied stress.

**Strain Caused by Repeated Loading.** Repeated load-unload cycles of a concrete prism result in the gradual accumulation of non-elastic strains. After a sufficient number of such cycles, the rate at which the non-elastic strain corresponding to a given stress accumulates gradually slows down. Creep reaches its maximum and the concrete behaves elastically. Referring to Fig. 3.12, non-elastic strains accumulate with each cycle, and the stress-strain curve gradually becomes a straight-line characteristic of elastic behaviour. This happens only if the applied stresses do not exceed the fatigue limit, that is,  $\sigma_c \leq R_r$ . Otherwise, non-elastic strains begin to grow without bound, and the test specimen collapses. In this case, the stress-strain curve becomes concave and the angle that it makes with the stress axis progressively decreases.

If concrete is subjected to vibration (200 to 600 load-unload cycles per minute), creep develops at a high rate. This is known as vibration or dynamic creep.

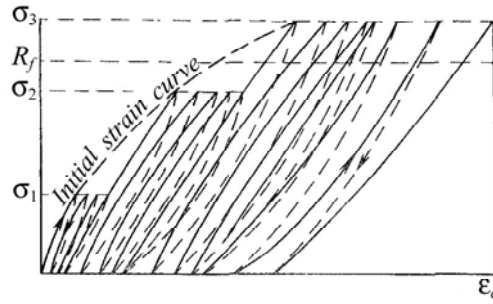


Fig. 3.12. Stress-strain curve for a test specimen due to repeated loading

### 3.6. Modulus of Elasticity and Specific Creep

The initial tangent modulus of elasticity of concrete in compression  $E_c$  corresponds to the only elastic strain produced by momentary working loads. Geometrically, it is determined as the tangent of the angle of the linear part of the stress-strain curve corresponding to elastic strain (Fig. 3.13).

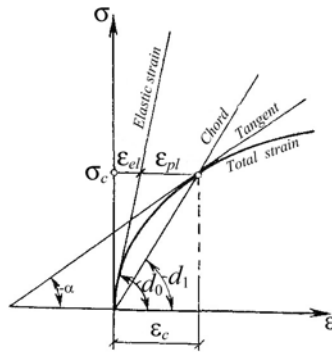


Fig. 3.13. Determination of the concrete elasticity modulus

$$E_c = \rho \operatorname{tg} \alpha_0 \quad (3.6)$$

The tangent modulus of concrete elasticity in compression designated  $E'_c$  corresponds to the total strain in concrete (including creep) and is a variable value. On the diagram, it is defined as the tangent of the angle and the curve at any given point

$$E'_c = d\sigma_c / d\varepsilon_c = \rho \operatorname{tg} \alpha \quad (3.7)$$

Strain in concrete might be determined analytically in terms of the variable modulus of elasticity by integrating the following function

$$\varepsilon_c = \int (1 / E'_c) d\sigma_c \quad (3.8)$$

This way, however, is rather difficult because variations in  $E'_c$  cannot be described analytically. So, in design calculations of reinforced concrete structures, use is made of an average modulus known as the secant modulus of elasticity. This is defined as the slope of the chord drawn through a point on the stress-strain curve corresponding to a given stress

$$E'_c = \rho \operatorname{tg} \alpha_1 \quad (3.9)$$

Since  $\alpha_1$  varies according to the applied stress, the secant modulus of elasticity is also a variable quantity which is smaller than the initial tangent modulus of elasticity.

### ASSIGNMENTS TO CHAPTER 3

1. What classes of concrete by strength are established by norms?
2. What grades of concrete are established by norms?
3. Explain the meaning of "class of concrete strength for compression".
4. What is the value of guaranteed concrete strength for the given concrete class? What supplies is it given?
5. What technical and economical recommendations are given for choosing the concrete class?
6. How do time and hardening conditions influence concrete strength?
7. What basic advantages have constructions made of simple and reinforced concrete?
8. What basic faults have constructions from simple and reinforced concrete?
9. What are the basic requirements to building constructions according to complex criterion of efficiency?
10. What are the rational regions of reinforced concrete constructions application?
11. What are the most perspective directions of various types of reinforced concrete constructions development in Ukraine?
12. What is the basic idea of reinforced concrete constructions creation?
13. What is the basis of compatible work of concrete and armature of reinforced concrete constructions?
14. What factors influence on mechanical properties of concrete?
15. What classes of concrete for durability are established by norms?
16. What brands of concrete are set by norms?

## 4. MAIN PHYSICAL AND MECHANICAL PROPERTIES OF REINFORCING STEEL

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### 4.1. Purpose and Types

Reinforcing steel is primarily the tensile component of reinforced concrete. Also, it is used to reinforce compression areas in members. The necessary amount of reinforcement is determined by appropriate design calculations.

In practice, reinforcing steel is classed into load-bearing reinforcement the steel ratio of which is found by calculations, and erection reinforcement which is used for constructional and other reasons (such as the ease of assembly and erection). Erection reinforcement serves to hold the load-bearing reinforcing bars in place and distribute stresses uniformly between the bars of load-bearing reinforcement. In addition, erection reinforcement may carry some loads, such as shrinkage and thermal variation loads, which generally are not taken into account in design.

Load-bearing and erection reinforcement are combined into welded or tied wire fabric or bar mats, which are placed in reinforced concrete members according to their behaviour under load (Fig. 4.1). Reinforcing steel may come in the form of *hot-rolled bars* and *cold-drawn wires*.

The core bar includes reinforcing bars of any diameter irrespective of how they are delivered to the job—in bars ( $d > 12$  mm, up to 13 m long) or hanks ( $d < 10$  mm, up to 1300 kg in weight).

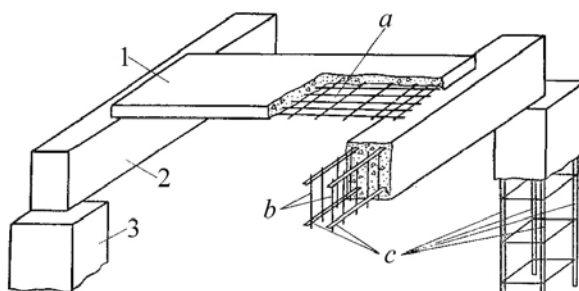


Fig. 4.1. Reinforced concrete members and their reinforcement:  
*a* — wire fabric; *b* — bar mats; *c* — reinforcing cage;  
*1* — slab; *2* — beam; *3* — column

After manufacture, hot-rolled reinforcing steel may be strengthened by heat treatment or cold drawing.

The principal forms that standard concrete reinforcement takes are *plain bars* and *deformed bars*. Standard reinforcing bars are rolled with protruding lugs or deformations which, as well as deformations on cold-drawn wire, serve to increase the bond between the reinforcement and the concrete (Fig. 4.2).

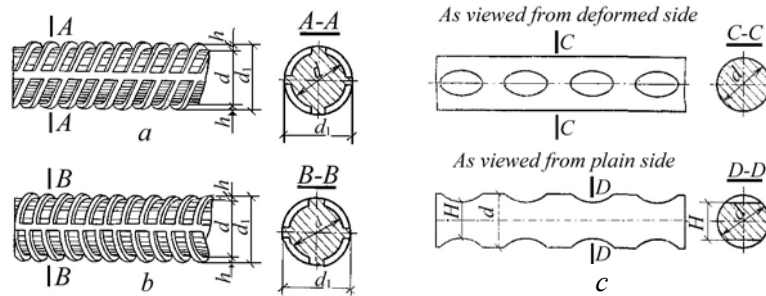


Fig. 4.2. Deformed reinforcing bars:  
*a* — class A-II reinforcing bars; *b* — class A-III  
 and A-IV reinforcing bars; *c* — high-strength wire

In a structural capacity, steel reinforcing bars and wire are utilized in concrete either as *prestressed steel* or as *non-prestressed steel* that is not prestressed before service loadings are applied.

Sometimes, use is made of rolled I-beams, channels or angles, which, before the concrete has hardened, work as a metal structure carrying the dead load including the weight of the false work and fresh concrete. Such reinforcement may prove efficient in in-situ large span floors and ceilings, heavily loaded columns in the bottom floors of multistory buildings, etc.

#### 4.2. Mechanical Properties of Reinforcing Steel

**Strength and Strain Characteristics.** These are determined from stress-strain diagrams,  $\sigma_s - \epsilon_s$ , which are plotted on the basis of tension tests (Fig. 4.3). Hot-rolled reinforcing steel with a definite yield (soft steel) has a considerable elongation (up to 25 %) after rupture (Fig. 4.3, *a*). The first unit stress at which strain increases without a considerable increase in load is called the yield point,  $\sigma_y$ , and the stress (or load) at or just before rupture is known as the ultimate tensile stress,  $\sigma_{ul}$ .

To increase its strength and reduce its elongation at rupture, hot-rolled reinforcing steel is doped with carbon or some other alloying

elements, such as manganese, silicon, chromium, and so on. The carbon contents, however, should not exceed 0,3 to 0,5 %, because otherwise it would reduce the ductility and weld ability of the steel. Manganese raises the strength of the steel without considerably reducing its ductility. Silicon contributes to the steel strength but reduces its weld ability. The percentage of the alloying elements is low, usually ranging between 0.6 and 2 %.

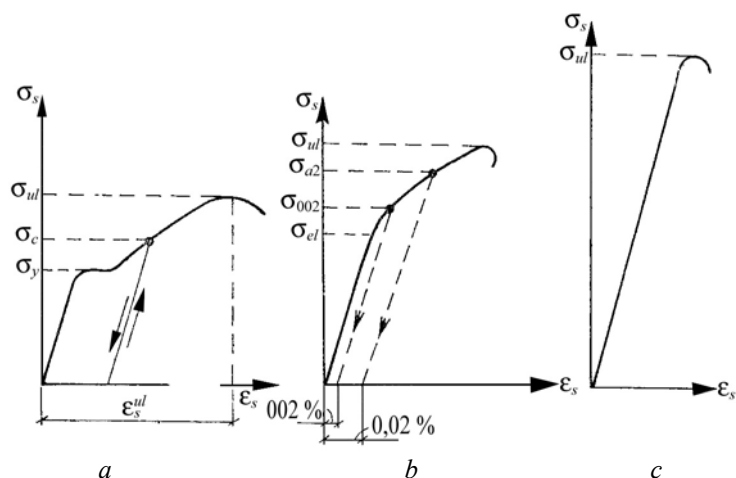


Fig. 4.3. Stress-strain curves for reinforcing steel in tension steel:  
*a* — with definite yield (soft steel); *b* — steel without definite yield;  
*c* — hard steel

Heat treatment and hardening by drawing raise the strength of hot-rolled reinforcing steel several times. Heat treatment includes hardening (heating to 800 or 900° C and rapid cooling) followed by partial tempering (heating to 300 or 400° C and slow cooling). Thermally hardened steel moves into the plastic region gradually and exhibits no yield region on its stress-strain diagram (Fig. 4.3, *b*).

The behaviour of reinforcing steel having no definite yield point is characterized in terms of the proof yield strength designated  $\sigma_{0,2}$ , that is, the stress at which the permanent set is 0,2 %, and the proof elastic limit,  $\sigma_{0,02}$  at which permanent set is 0,02 %.

In hardening by drawing, hot-rolled reinforcing steel is cold-drawn until the stress exceeds the yield point,  $\sigma_k > \sigma_y$ . Cold drawing (or cold working) changes the crystal lattice so that the reinforcing steel hardens. As a result of a second drawing pass,  $\sigma_k$ , becomes a new artificially raised yield point because all of the plastic strain has already been taken up (see Fig. 4.3, *a*). After several drawing passes through a succession of dies with

decreasing diameters, the wire acquires the properties of hard steel, which means that its stress-strain diagram is linear almost till the break. As a result, the ultimate strength of the wire markedly increases, and its elongation at rupture decreases to 4 or 6 % (Fig. 4.3, *c*).

**Ductility.** This property is of great importance for the behaviour of reinforced concrete structures under load, mechanized installation of reinforcement, prestressing, etc. Although reinforcing steel is inherently very ductile, an excessive reduction in its ductility may lead to brittle rupture under load, brittle fracture in prestressed steel at points where it is sharply bent or near anchorage and so on. The ductility of reinforcing steel is expressed in terms of the percentage of elongation determined by a tensile test on standard bars having a length equal to their five diameters (or 100mm) or by bending cold-state standard bars around a mandrel of 3 to 5 bar diameters thick.

The total percentage of elongation, designated  $b$ , is found by dividing the increase in the bar length measured across the fracture by the original bar length. The uniform percentage of elongation  $\delta_{un}$ , is determined by dividing the increase in the bar length without the neck region by the original bar length. The minimum allowable percentage elongation and requirements placed upon the cold-bending test are set up by relevant standards and specifications.

**Weld ability.** The ability of reinforcing steel to be welded into reliable joints having no cracks or other imperfections in and near the weld is expressed in terms of weld ability. This quality is important for shop fabrication of wire fabric and bar mats, splicing reinforcing bars, anchors, various embedded attachment fittings and so on. Hot-rolled low carbon and low-alloy reinforcing steels have good weld ability. Reinforcing steel hardened by heat treatment or drawing may not be welded because thermally hardened steel would be tempered and cold drawn wire would be annealed, so their-hardening would be wasted.

**Cold Brittleness.** Deformed reinforcing bars made of semi-killed open-hearth and converter steel fracture at a temperature below  $-30^{\circ}\text{C}$ ; high-strength wire and thermally hardened bars fracture at a lower temperature.

**Creep and Relaxation.** Creep of reinforcing steel increases with increasing applied stress and temperature. Relaxation, or reduction in stress, occurs in reinforcing bars at a constant length that is when no deformation takes place. Relaxation depends on the mechanical properties and chemical composition of reinforcing steel, manner of its manufacture and exposure conditions. Wire hardened by cold-drawing, thermally hardened reinforcement, and highly alloyed reinforcing bars

show considerable relaxation. Relaxation of hot-rolled low-alloyed reinforcing steels is insignificant. Experimental data show that relaxation is most pronounced during the first several hours after manufacture, but it may continue for a long time. Relaxation of reinforcement markedly affects the behaviour of prestressed structures because it partly reduces the amount of prestressing.

**Fatigue Failure.** This occurs when a structure is subjected to repeated loading; it is brittle by nature. The fatigue limit of reinforcement depends on the number of loading cycles, the ratio of  $\sigma_{\min}$  to  $\sigma_{\max}$  bonding strength, and cracking in the tension area. As the number of loading cycles increases, the fatigue limit of reinforcing steel decreases. Thermally hardened reinforcing steels have a reduced fatigue limit.

**Dynamic Hardening.** When a structure is subjected to high-rate short-time loads, the reinforcing steel hardens. With a high strain rate, reinforcing steels are elastic even at stresses exceeding the yield point. Plastic deformation sets in with a delay. Due to this delay the dynamic yield point,  $\sigma_{\text{dyn}}$  exceeds the static yield point,  $\sigma_y$ . Dynamic hardening has a smaller effect on the proof yield strength of alloy and thermally hardened steels (having no definite yield point), and practically no effect on the ultimate tensile strength,  $\sigma_{\text{ul}}$ , of all types of reinforcing steel including high-strength wire.

**High-Temperature Heating.** When reinforcing steel is heated to a high temperature, its inner structure changes and strength decreases. For example, the yield point of Soviet-made hot-rolled class A-III reinforcing steel heated to 400° C decreases by 30%, those of class A-II and A-I reinforcing steel fall by 40%, and the module of elasticity of all grades decreases by 15%. At a temperature above 350° C, there is significant creep in the reinforcing steel of loaded structures. When heated to a high temperature, wire hardened by cold drawing is tempered and loses its hardness. So, the ultimate strength of high-strength wire decreases quicker than that of hot-rolled reinforcing bars. After heating and subsequent cooling, hot-rolled reinforcing steel recovers its strength completely, and high strength wire only partly.

### 4.3. Classification of Reinforcing Steel

According to their main mechanical characteristics, hot-rolled reinforcing bars are divided into six classes, designated A-I, A-II, A-III, A-IV, A-V and A-VI (Table 4.1). Thermally hardened bars are divided into three classes: A<sub>T</sub>-IV, A<sub>T</sub>-V and A<sub>T</sub>-VI. The letter "T" in the designation stands for thermal hardening.



Steel grades for each class of reinforcing bars have the same mechanical characteristics, but differ in composition. The grade designation shows how much carbon and alloying elements given steel carries. For example, in the designation "25Г2С", the first two-digit number indicates the carbon content in points (hundredths of a per cent), the Russian letter "Г" indicates that the steel is alloyed with manganese, the numeral 2 indicates that the manganese content may reach 2 %, and the Russian letter "С" stands for silicon. In the designations "20ХГ2Ц" and "23Х2Г2Т". "Х" stands for chromium, "Т" for titanium and "Ц" for zirconium (Russian letters throughout).

Table 4.1

**Classification and Mechanical Characteristics  
of Reinforcing Steel**

Type and class of reinforcing steel	Steel grade (Russian)	Diameter, mm	Yield point, MPa	Ultimate strength, MPa	Percentage of elongation
Hot-rolled bars: plain, class A-I	Ст3, ВСт3	6-40	230	380	25
deformed: class A-II	ВСт5	10-40			
	10ГТ	10-32	300	500	19
	18Г2С	40-80			
class A-III	25Г2С	6-40			
	35ГС	6-40	400	600	14
	32Г2Р	6-22			
class A-IV	20ХГ2С	10-22			
	80С	10-18	600	900	8
class A-V	23Х2Г2Т	10-22	800	1050	7
class A-VI					
Thermally hardened bars:					
class A <sub>r</sub> -IIIС	ВСт5СII	10-38	400	600	—
class A <sub>r</sub> -IVС	25Г2С	10-28	600	900	8
class A <sub>r</sub> -V	20ГС	10-22	800	1050	7
class A <sub>r</sub> -VI	20ГС	10-22	1000	1200	6
Ordinary wire: plain, class B <sub>p</sub> -I	—	3-5	—	550-525	—
Ordinary wire: plain, class B-II	—	3-8	—	1900-1400	4-6
deformed, class B <sub>p</sub> -II	—	3-8	—	1800-1300	4-6
Seven-wire strands, class K-7,K-19	—	6-15,14	—	1850-1650	—

Class A-I reinforcing steel is plain, bars of all other classes are deformed.

The yield point of class A-I, A-II, and A-III reinforcing steel ranges between 230 and 400 MPa, the proof yield strength of class A-IV, A-V

and A-VI highly-alloyed reinforcing steel is from 600 to 800 MPa, and that of thermally hardened steel ranges between 600 and 1000 MPa.

The percentage of elongation depends on the class of reinforcing steel. Class A-I has a considerable percentage of elongation ( $\delta = 25\%$ ), classes A-II and A-III have a lower percentage of elongation ( $\delta = 14$  to  $19\%$ ), and classes A-IV, A-V, A-VI and thermally hardened steel of all classes have a relatively small percentage of elongation ( $\delta = 6$  to  $8\%$ ).

The modulus of elasticity of reinforcing bars,  $E_s$ , somewhat decreases with increasing strength. It is equal to  $2,1 \times 10^5$  MPa for classes A-I and A-II,  $2 \times 10^5$  MPa for classes A-III and  $1,9 \times 10^5$  MPa for class A-IV, A-V, A-VI and thermally hardened reinforcing steel.

Wire from 3 to 8mm in diameter is divided into two classes, namely Bp-I and Bp-II. The former is ordinary (cold-drawn, low-carbon) wire mainly used in welded-wire fabric, the latter is high-strength (multi-pass drawn, carbon) wire used as prestressed reinforcement in prestressed members. Deformed reinforcing wire has an additional Russian letter "p" in its designation: Bp-I, Bp-II.

The main mechanical characteristic of reinforcing wire is the ultimate tensile strength, which increases with decreasing wire diameter. The ultimate tensile strength of ordinary wire is equal to 550 MPa, that of high-strength reinforcing wire ranges between 1300 and 1900 MPa. The percentage of elongation of reinforcing wire is relatively small, ranging between 4 and 6%. Rupture of high-strength wire is brittle by nature. The modulus of elasticity of reinforcing wire is equal to  $2 \times 10^5$  MPa for classes B-II and Bp-II;  $1,7 \times 10^5$  MPa for class Bp-I, and  $1,8 \times 10^5$  for class K-7 strands.

In the former Soviet Union, reinforcing steel was classed into standard or gauge sizes according to the nominal diameters. For deformed bars, these are the diameters of plain bars having the same cross-sectional area; for deformed ordinary and high-strength reinforcing wire, these are the wire diameters prior to deformation.

#### **4.4. Application of Reinforcing Steel**

Non-prestressed structures use class A-III hot-rolled reinforcing bars and ordinary wire available in fabric and mats, which have a relatively high strength. It is also possible to use class A-II reinforcing steel; if not all of the strength of class A-III reinforcement is utilized due to excessive strain or crack width. Class A-I reinforcing steel may be used as erection reinforcement hooks in tied mats or transverse reinforcing bars in welded mats.

For prestressed structures, it is recommended to use class A<sub>T-VI</sub>, A<sub>T-V</sub> and A<sub>T-IVC</sub> thermally hardened bars and class A-IV, A-V and A-VI hot-rolled reinforcing steel. It is advisable to reinforce members over 12m long with reinforcing strands and high-strength wire.

The more recent additions are class A<sub>TII</sub>-V reinforcing bars having an increased corrosion resistance; class K-19 strands and some other new types of reinforcing steel.

Cold-brittle steels are not used to reinforce structures intended for service at sub zero temperatures (out-of-doors and in unheated locations). Below -30° C, use is made of class A-II grade BCT5πc2 and class A-IV grade 80C reinforcing steel; below -40° C, class A-III grade 35ГC reinforcing steel.

One of important factors determining the applicability of reinforcing steel is its weld ability. Class A-I, A-II, A-III, A-I, A-V and A-VI hot-rolled reinforcing bars and ordinary wire in fabric form are good for resistance welding. Thermally hardened bars of all classes and high-strength wire cannot be welded because they would lose their hardness. Also, arc welding is inapplicable to class A<sub>T-V</sub> and A<sub>T-VI</sub> reinforcing bars.

#### 4.5. Wire Fabric and Bar Mats

As a rule, reinforcing steel for non-prestressed structures comes in the form of welded wire fabric and bar mats. Longitudinal and transverse wires in fabric and mats (which are usually at the right angles to one another) are joined by resistance spot welding. Manufacture of fabric and mats by welding has industrialized reinforcement work and reduced labour requirements and the cost of reinforcement.

**Welded-Wire Fabric.** According to the appropriate Soviet standard, welded wire fabric is made of ordinary reinforcing wire of 3 to 5 mm in diameter and class A-III reinforcing steel of 6 to 10 mm in diameter. Welded wire fabric comes in rolls and mats (Fig. 4.4).

The maximum diameter of longitudinal wires in rolls is 7mm. Both longitudinal and transverse wires may serve as the load-bearing reinforcement. Wires placed at the right angles to load bearing wires work as erection reinforcement serving to distribute stress uniformly among the load-bearing wires. Fabric in which the transverse wire provides the minimum steel area necessary for fabricating and handling is termed "one-way". Where significant reinforcement is provided in both transverse and longitudinal directions, the fabric is called "two-way". The maximum width,  $B$ , of welded-wire fabric in rolls is 3.8 m.

The length of this type of fabric is limited by the weight of rolls, which may range from 900 to 1300 kg. The maximum width of fabric in mats is 2,5 m, and the maximum length,  $L$ , is 9m. The values  $B$  and  $L$  are taken between the centres of edge wires. Type designations of welded-wire fabric contain the main characteristics of the type. The numerator of a designation is the type of fabric stated in terms of  $t/t_1/d/d_1$  (Fig. 4.4), and the denominator gives the width and length of the fabric,  $BL$ . For example, a welded-wire fabric mat 2300 mm wide and 5900 mm long ( $L$ ) has longitudinal wires 4mm in diameter ( $d$ ) spaced 250 mm apart ( $t$ ) and transverse wires 8 mm in diameter ( $d_1$ ) spaced 200 mm apart ( $t_1$ ). Type designations for welded-wire fabric in rolls give only the width  $B$  in the denominator.

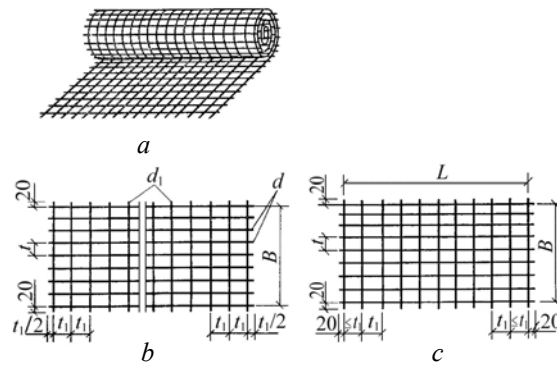


Fig. 4.4. Welded-wire fabric:  
*a* — in rolls; *b* — unrolled fabric; *c* — in mats

**Welded-Bar Mats.** These are fabricated from longitudinal load-bearing and erection bars and transverse bars (Fig. 4.5). In the plane of a mat, the longitudinal bars may be placed in one row (Fig. 4.5, *a* through *c*) and in two rows (Fig. 4.5, *d* and *e*).

In addition, they may be located on one side (Fig. 4.5, *c* and *d*) or either side of the transverse bars (Fig. 4.5, *a*, *b* and *e*). The longitudinal bars placed on one side of the transverse bars facilitate resistance spot welding and contribute to the bond between the mat and the concrete. Sometimes, use is made of double bar mats in which single bar mats are joined by arc welding (Fig. 4.5, *g*), or bar mats in which the longitudinal load-bearing bar is joined by arc welding to an additional load-bearing bar (Fig. 4.5, *h*).

Concrete structural members may be reinforced by reinforcing cages which may be one-piece or built up from sections. Reinforcing cages (Fig. 4.5) may be wholly fabricated by welding in the shop or assembled

at the job from bar mats joined by arc-welded transverse bars. In welded-bar mats and reinforcing cages, it is advisable to space transverse apart bars at equal distances, along the full or some part of mat length so that welding machines need not be readjusted.

The quality of spot welding on welded-wire fabric and bar mats depends on the diameter ratio of the longitudinal and transverse bars, which should be at least 0,3. For column reinforcement and deformed wire fabric, the ratio may be reduced to 0,25. The minimum bar spacing also depends on bar diameters. The necessary data for the design of welded wire fabric and bar mats.

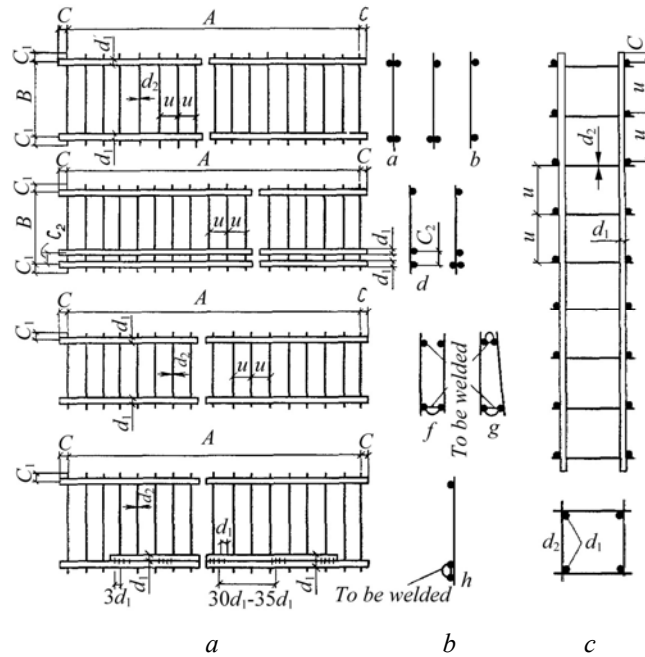


Fig. 4.5. Reinforced cages:  
*a* — through; *b* — welded-bar mats; *c* — reinforcing cage

#### 4.6. Prestressed Concrete Reinforcing

Prestressed-concrete structures are reinforced by wires combined into strands and cables. The time and labour required to place and tension many small prestressing wires can be extensive. Besides, the need to maintain the necessary spacing between the wires leads to an excessive cross-sectional area of a prestressed-concrete member.

To reduce the placing and tensioning expense, strands composed of several prestressing wires twisted together can be used (Fig. 4.6). They consist of several wires having the same diameter helically twisted around a central straight wire so that they cannot untwist. During the manufacture of a strand, the wires are deformed and fit closely to one another. At present, use is most commonly made of class K-7 seven-wire strands made of wires 1.5 to 5mm in diameter. The diameter of a class K-7 strand is equal to three diameters of its wires. The irregular surface of strands ensures a good bond between the concrete and strands. In addition, very long strands are available, so that they can be used in continuous joint-free structures.

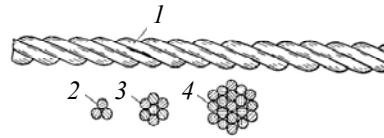


Fig. 4.6. Wire strands:  
1 — side view; 2-4 — cross-sections of 3, 7 and 19 wire strands

Prestressing cables consist of high-strength wires running in parallel to one another (Fig. 4.7). One-layer cables are made of 14, 18 or 24 wires placed around a circumference with spacing to let grout inside the cable, and wrapped by soft wire. Stronger cables are composed of parallel strands instead of separate wires. In multilayer cables, as many as 100 wires 4 to 5 mm in diameter may be used. Prestressing cables are not supplied by reinforcing steel manufacturers, so they are fabricated directly at the job or erecting shops.

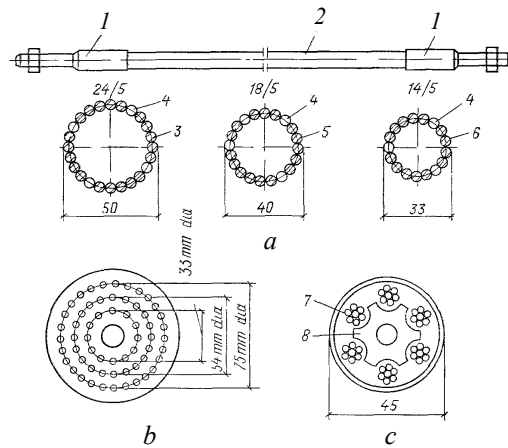


Fig. 4.7. Prestressing cables:  
*a* — one-layer cables; *b* — multilayer cables; *c* — cable using seven-wire strands;  
1 — anchorage; 2 — side view; 3, 5 and 6 — cross sections of 14-, 18- and 24-wire cables; 4 — short wires; 7 — wire strand; 8 — spacer

## 4.7. Splicing Reinforcing Steel

**Types of Welded Joints.** The preferred reinforcing bar splice welds are the butt-welded joints, which can be made by various techniques in the shop or directly at the job.

In the shop, class A-I, A-II, A-III, A-IV, A-V and A-VI reinforcing bars (for example, reinforcing-bar blanks and large diameter splice bars) are joined by resistance welding (Fig. 4.8, *a*). The diameter ratio of the bars being joined,  $d_1/d_2$ , should not be less than 0.85, the minimum bar diameter being  $d_1 > 10$  mm. If a special welding technique is used, the diameter ratio may be reduced to 0,5.

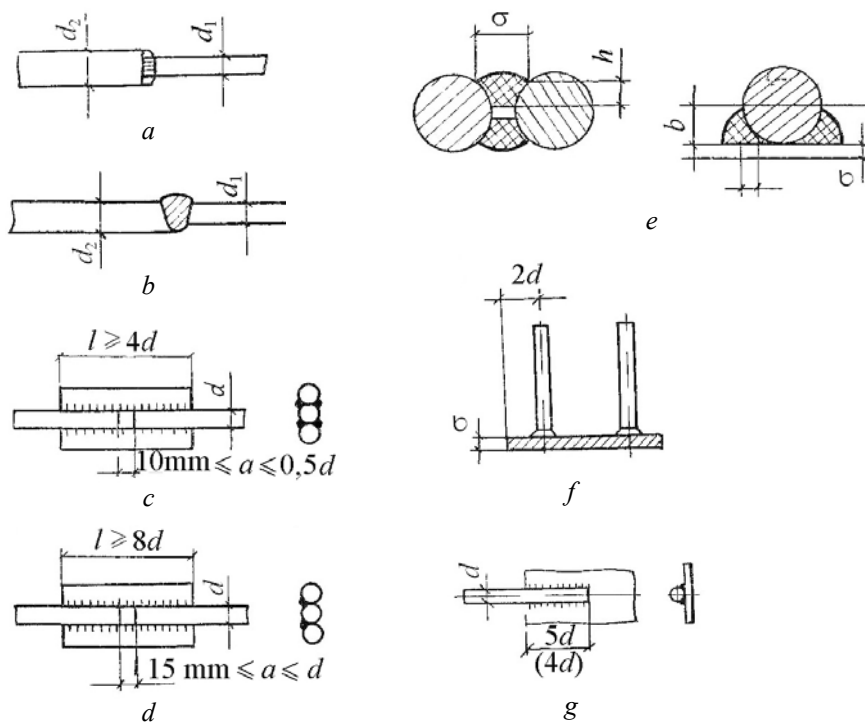


Fig. 4.8. Welded joints:

*a* — butt resistance welding; *b* — arc welding in a pool of molten metal contained in reusable mould; *c* — arc welding with splice bars, four side fillet welds; *d* — arc welding with splice bars, two side fillet welds; *e* — legs of a weld; *f* — T-joint of reinforcing bars and a plate; *g* — lap splice of a reinforcing bar and a plate

In the field, class A-I, A-II, A-III and A<sub>T</sub>-III reinforcing bars (for example, stick-outs in precast members) is arc welded in a pool of molten metal contained within a permanent (reusable) mould (Fig. 4.8). If the bars to be welded are less than 20mm in diameter, splices are made by double-lap joints. To this end, reinforcing bars are arc welded to splice bars by four (Fig. 4.8, *c*) or two longitudinal side fillet welds (Fig. 4.8, *d*). In the former case, the weld length is  $l = 4d$ ; in the latter, use is made of longer splice bars, and the weld length is  $l = 8d$ . The throat of the weld,  $h$ , should be equal to one fourth of the bar diameter, but it must be not less than 4mm; the width of the weld,  $b$ , should be one half of the bar diameter but not less than 10mm (Fig. 4.8, *e*).

T-joints of reinforcing bars 10 to 16mm in diameter and plates  $0,75d$  thick (made of steel sheets or strips) are made by submerged arc welding (Fig. 4.8, *f*). Lap splices of reinforcing bars 8 to 40mm in diameter with plates or flat rolled elements may be made by longitudinal fillet arc welding (Fig. 4.8, *g*).

**Non-welded Lap Splicing.** Class A-I, A-II and A-III reinforced bars may be lap-spliced where not all of the reinforcement strength is used. The lap of a splice should be from 20 to 50 bar diameters. This, however, is the least desirable type of splicing because it requires additional steel and is imperfect in construction.

Wire fabric may be lap-spliced in the load bearing direction (Fig. 4.9). The load-bearing bars of the fabric pieces to be joined may be located in one or different planes. In the tension zone, each piece should contain at least two transverse bars welded to all the longitudinal bars of the fabric. If the load bearing bars are deformed, one or both fabric pieces may have no welded transverse bars within the lap.

The necessary lap of a splice is determined according to the necessary anchorage by calculation.

Lap splices on bar mats may be made when the longitudinal bars are located on one side of the transverse bars. Like welded wire fabric, bar mats are spliced in the load bearing direction. Here, additional stirrups or transverse bars with a spacing of not more than 5 longitudinal bar diameters are placed within the lap.

The joints in welded wire fabric and bar mats in a structure must be staggered.

In the non-load-bearing direction (for erection wires), welded wire fabric may also be lap-spliced (Fig. 4.10). The lap of a splice is taken to be 50mm for erection wires less than 4mm in diameter, and 100 mm for erection wires more than 4mm in diameter.



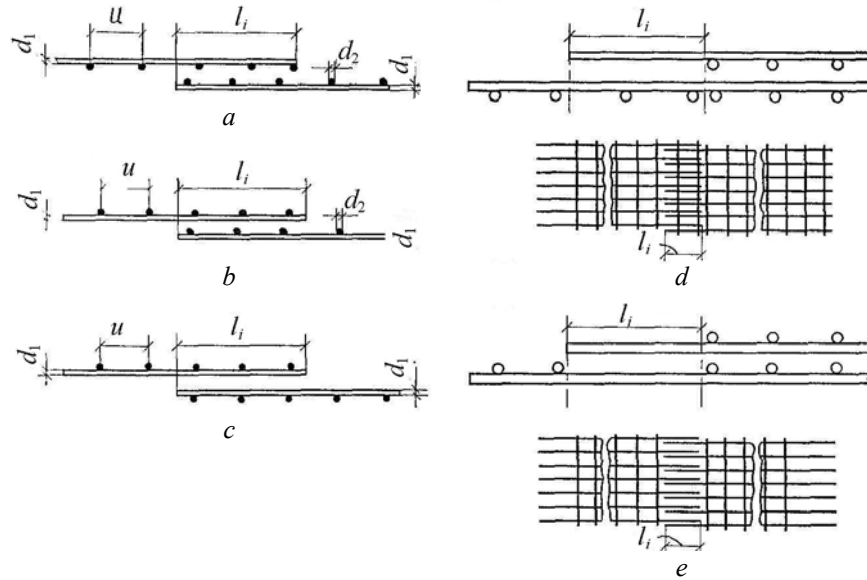


Fig. 4.9. Splicing of welded-wire fabric in the load bearing direction: *a* — plain wires, transverse wires in the same plane; *b, c* — plain wires, transverse wires in different planes; *d* — deformed wires, no transverse wires within the lap in one piece to be spliced; *e* — deformed wires, no transverse wires within the lap in both pieces to be spliced

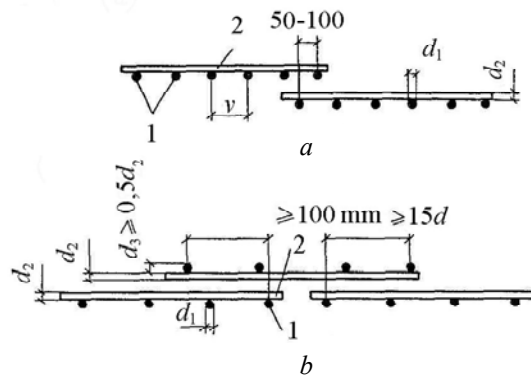


Fig. 4.10. Splicing of welded-wire fabric in the direction of the erection bars: *a* — lap splicing; *b* — with additional splice fabric; *1* — load bearing wires; *2* — erection wires

When the load bearing bars are 16mm in diameter or more, additional splicing fabric is used, with an overhang of 15 diameters, but not less than 100 mm on either side.

#### **ASSIGNMENTS TO CHAPTER 4**

1. Name the signs of steel armature classification.
2. Name mechanical properties of steel armature.
3. Name the classes of armature steel and wire.
4. What does the brand of armature steel depend on?
5. How does armature influence on shrinkage and creep of concrete?
6. Name all types of metals corrosion of reinforced concrete constructions.
7. Specify the size of minimum thickness of protective layer of concrete for the longitudinal working armature of reinforced concrete beams and columns.
8. Specify accepted in the normative documents modern method of calculation of reinforced concrete constructions.
9. Name the stages of the stressed-deformed condition of reinforced concrete constructions at bending.
10. What is the critical condition of constructions?

## **5. MAIN PHYSICAL AND MECHANICAL PROPERTIES OF REINFORCED CONCRETE**

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### **5.1. Prestressed Concrete**

Prestressed members are those in which considerable compressive stresses are artificially induced by tensioning the reinforcing steel before they are loaded. Initial compressive stresses are applied to those concrete areas which, when loaded, work in tension. Prestressing imparts greater crack resistance to the member and enables the designer to take advantage of high-strength steel to cut down steel consumption and the cost of a structure.

It pays to use high-strength steel instead of hot-rolled reinforcing bars. In non-prestressed structures, however, high-strength steel may not be used because the high tensile stresses in it and the respective tensile strain in the concrete result in wide cracks, which cause the loss of necessary qualities of the structure.

The major advantages of prestressed concrete are its cost reduced by the use of high-strength steel and high crack resistance, which, in turn, contributes to its stiffness, dynamic load strength, corrosion resistance, and longer service life.

In a loaded prestressed beam, the concrete begins to work in tension only after all of the compressive prestressing has been counteracted. Here, the load, which causes cracking or limited opening of cracks, exceeds the service load. As the load increases to the ultimate breaking value, the stresses in the reinforcement and concrete reach their ultimate values. In a similar non-prestressed beam, the load is less than service one but breakage load is about the same for either beam because the ultimate stresses in the reinforcement and concrete are equal.

As is seen, loaded prestressed structures work with no or narrow cracks, whereas non-prestressed structures develop cracks under load. This is the main difference between prestressed and non-prestressed members, affecting their analysis, design and manufacture.

## 5.2. Methods of Prestressing

There are two basic methods of prestressing concrete, pretensioning and post tensioning. In the former, the tendons (single wires or wire strands) are erected in a form where they are anchored to an abutment at one end and tensioned at the other by jacks or other devices until the required stress is attained (Fig. 5.1, *a*). After the concrete has attained the necessary cube crushing strength before the compression,  $R_{cp}$ , the wires or strands are released from the abutments. Being bonded to the concrete, the wires or wire strands are unable to return to their original length and transfer the prestressing force to the concrete by bond resistance together with radial compression (Fig. 5.1, *b*). In the so-called continuous reinforcement, the form is placed on a bed equipped with studs, which carry sleeves. A special machine winds the wires on the sleeves at the specified stress, after which the ends of the wires are clamped by a die clip (Fig. 5.1, *c*). After the concrete has attained the necessary strength, the member with embedded sleeves is removed from the bed studs, and the released wires compress the concrete.

Bar tendons may be pretensioned by the electrothermal process. Bars with upset ends are heated by an electric current to 300–350°C, placed in a bed and anchored to abutments. As they cool, the bars tend to restore their initial length, and are thus pretensioned.

In post-tensioning, the tendons (steel wires, strands, or bars) are tensioned against and anchored to the concrete after it has developed adequate strength,  $R_{cp}$  (Fig. 5.1, *d*). Here, the tendons are inserted in ducts, tensioned, anchored, and grouted (Fig. 5.1, *e*).

The stress in the tendons is checked after all of the compressive force has been transferred to the concrete. The ducts or raceways in the concrete, exceeding the tendon diameter by 5 to 15 mm, are produced by installing, before the concrete is poured in the forms; withdraw able steel spins, rubber hose or permanent corrugated steel tubing. After the concrete has been compressed, the steel is bonded to it by injecting cement paste or grout under pressure into the raceways. This is done through pipes placed during the manufacture of the tendon.

If prestressing steel is located outside the concrete (for example, in pipes, tanks, and so on), it is wound by special wrappers, and the compressive stress is simultaneously transferred to the concrete. Here, after the steel has been tensioned, the protective cover is applied by shotcreting (concreting under pressure).

Pretensioning can be mechanized more easily, so it is mostly used in the off-site manufacture of concrete members. Post-tensioning is

mainly used in the production of large-size structures and for their joining at the job.

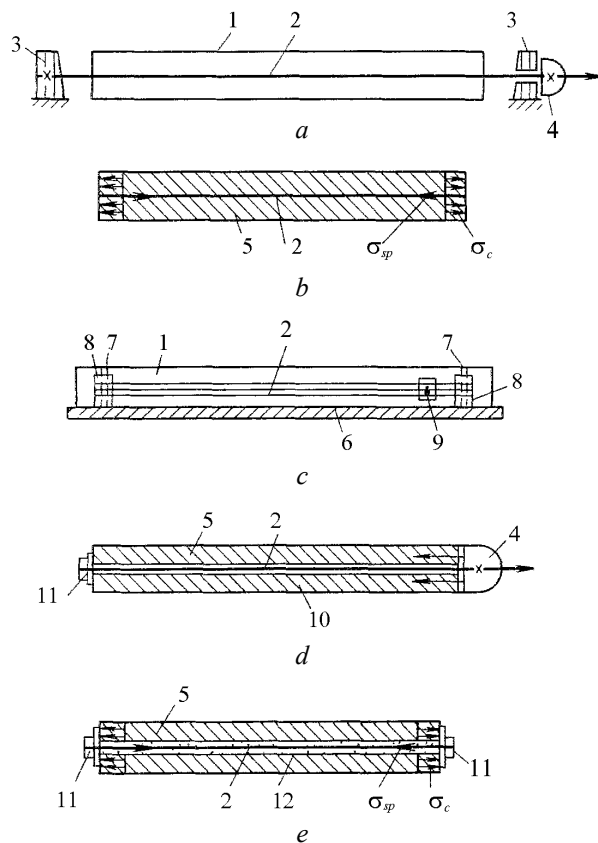


Fig. 5.1. Methods of prestressing:

*a* — pretensioning (schematic diagram); *b* — finished member; *c* — pretensioning in continuous reinforcement; *d* — post-tensioning (schematic diagram); *e* — finished member; 1 — mould; 2 — reinforcement; 3 — abutment; 4 — jack; 5 — hardened concrete; 6 — casting bed; 7 — studs in the casting bed; 8 — sleeves; 9 — clip; 10 — raceway; 11 — anchorage; 12 — grouted raceway member

### 5.3. Bond between Reinforcing Steel and Concrete

Owing to the bond between the materials, the reinforcing steel does not slip in the concrete under load. Bonding strength is tested by the "pull-out" and "press-in" tests (Fig. 5.2, *a*). Experiments show that

the reliance for the bond strength is on (1) the bearing of lugs and the strength of concrete between lugs (Fig. 5.2, *b*); (2) friction between the steel and the concrete resulting from shrinkage; and (3) adhesion between the cement paste and the steel surface, attributed to the adhesive properties of the cement gel.

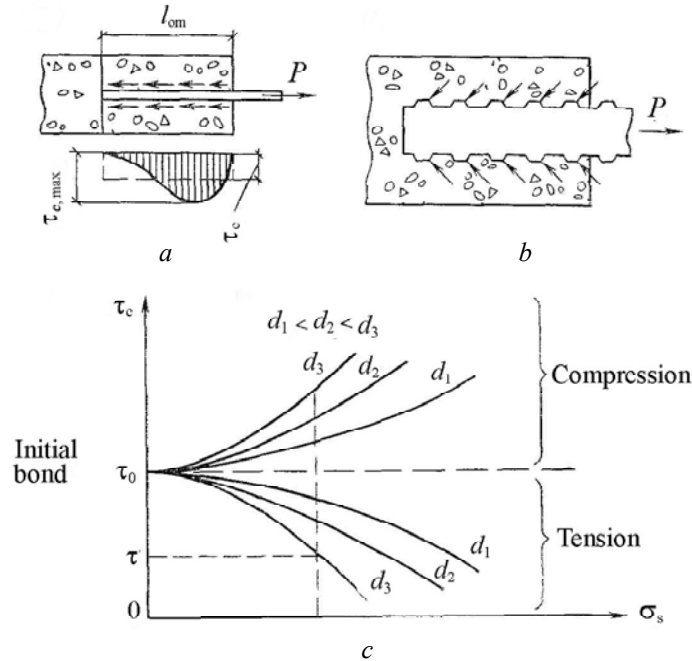


Fig. 5.2. Bond between reinforcing steel and concrete

The first factor accounts for about 75 % of the total resistance to slip. With plain reinforcing steel, the resistance to slip is one half or one third of that with deformed bars. According to experimental data, the bearing stress is not uniformly distributed along the embedded length of a bar, and the maximum bearing stress,  $\tau_{c, max}$ , does not depend on the embedded length,  $l_{em}$ . The average bearing stress is defined as the ratio of the force in a bar,  $N$ , to the surface of the embedded part of the bar.

$$\tau_{b, av} = N / l_{em}u \quad (5.1)$$

where  $u$  is the bar circumference. For plain bars and medium concrete classes, the average bearing stress ranges between 25 and 40 kg/cm<sup>2</sup>.

The bond strength increases with increasing concrete brand number, decreasing water-cement ratio, and age. When the embedded length of a

bar is an insufficient, additional short bar or washers are welded on (class A-I plain bars are hooked at the ends). When a bar is pressed in concrete, the bond strength is greater than in pulling out because the concrete resists the lateral expansion of the compressed bar. As the bar diameter and the stress,  $O_s$ , increase, the bond strength in compression rises, and that in tension falls (Fig. 5.2, *c*). So, for better bond strength the diameter of bars in tension should not be less than a certain minimum value.

#### 5.4. Anchorage

The anchorage of reinforcing steel in concrete structures is carried out by embedding the steel in the concrete past the section in question for a length sufficient to transfer stresses from the steel to the concrete (due to the bond between the steel and the concrete), and also by anchoring devices.

**Anchorage of Non-prestressed Steel.** Class A-I plain bars have hooks at their ends, with a diameter of  $2.5d$  in nonporous-aggregate concrete, and  $5d$  in porous-aggregate concrete (Fig. 5.3, *a*). In welded wire fabric and mats made of plain bars, the anchors are transverse bars, so they are used without hooks at the bar ends. Deformed bars form a much better bond with concrete, so no bent bar anchorage is used in this case, either.

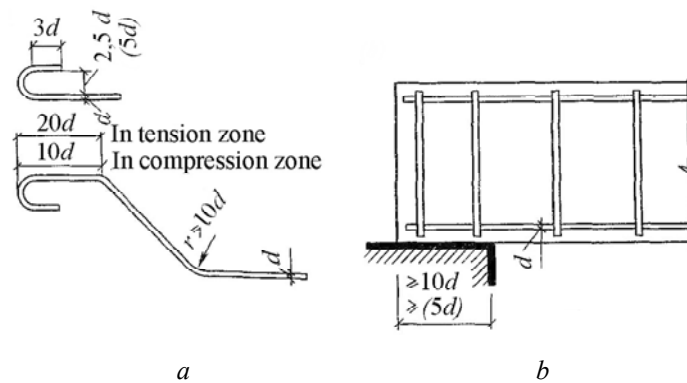


Fig. 5.3. Anchorage of non-prestressed reinforcing steel:  
*a* — plain bars; *b* — deformed bars at a free support

Non-prestressed deformed bars are embedded past the section normal to the longitudinal axis of a member, within which the reinforcing steel is assumed to develop the full design strength for the entire anchorage length

$$l_{an} = (\omega_{an}R_s/R_c + \Delta\lambda_{an})d \quad (5.2)$$

but not less than  $l_{an} = \lambda_{an}d$  where  $\omega_{an}$ ,  $\Delta\lambda_{an}$ ,  $\lambda_{an}$  and also the minimum safe value of  $l_{an}$  are taken from Table 5.1;  $R_s$  is the design strength of the reinforcing steel;  $R_c$  is the design axial compressive strength of the concrete; and  $d$  is the bar diameter.

If reinforcing bars are embedded past the section normal to the longitudinal axis of a member, within which not all of their design strength is used,  $R_s$  should be multiplied by the ratio of the cross-sectional area of the steel required when the total design strength is utilized, to the cross-sectional area of the actual reinforcing steel.

At the outer free supports of bent members, the tension longitudinal bars are carried past the internal face of each support for not less than  $10d$ ; if there are no inclined cracks in the tension area, the bars are carried past the internal face for not less than  $5d$  (Fig. 5.3).

Table 5.1

**Determination of the Anchor Length  
of Non-prestressed Deformed Bars**

State of stress and anchorage conditions	$\omega_{an}$	$\Delta\lambda_{an}$	$\lambda_{an}$	$L_{an}^{mm}$ in
Tensile steel in tension concrete	0,7	11	20	250
Tensile or compressive steel in compression concrete	0,5	8	12	200

**Anchorage of Prestressed Steel.** In pretensioning deformed bars or wire strands in structures made of sufficiently strong concrete, no special anchoring devices are needed. In post-tensioning wire cables or pretensioning plain high-strength wire having poor bond resistance, anchors are always used. The anchorage length of prestressed steel used without anchoring devices is taken to be equal to the length of the area where the stress is transferred from the reinforcing steel to the concrete, which is determined as

$$L_p = [\omega_p(\sigma_{sp}/R_c) + \lambda_p]d \quad (5.3)$$

where  $\omega_p$  and  $\lambda_p$  are determined from Table 5.2;  $R_c$  is the transfer strength of the concrete (the cube crushing strength of the concrete attained by the moment of the stress transfer);  $\sigma_{sp}$  is the preliminary stress in the steel with allowance for losses (accepted to be equal to  $R_s$  or  $\sigma_{sp}$  whichever is the greater).



For members made of porous-aggregate concrete, the value calculated by Eq (5.3) should be multiplied by 1.2. For all types of deformed bars,  $l_p$  should be at least  $15d$ . When the stress is instantaneously transferred from deformed bars up to 18mm in diameter (released from abutments by cutting),  $l_p$  is multiplied by 1.25. With structural members used at a design temperature below  $-40^\circ\text{C}$ , the values of  $\lambda_{an}$  are doubled.

The stress in the steel is considered to be linearly varying from zero at the edge of a member to its maximum value at the section lying within  $l_p$  of the member edge.

Table 5.2

**Determination of the Stress Transfer Length  
for Stressed Reinforcement without Anchorage**

Type and class of reinforcing steel	$\omega_p$	$\lambda_p$
Deformed bars of any class and diameter	0,25	10
Class Bp-II high-strength wire:		
5 mm in diameter	1,4	40
4 mm in diameter	1,4	50
3 mm in diameter	1,4	60
Class K-7 wire strands:		
15 mm in diameter	1,0	25
12 mm in diameter	1,1	25
9 mm in diameter	1,25	30
Class K-19 wire strands	1,0	25

In order to prevent the concrete from spalling when transferring the stress, the member ends are reinforced by embedded items with anchorage bars, stirrups, mesh, and so on.

Wire strands and deformed bars are gripped, tensioned and fastened to buttresses by grip tensioning units (Fig. 5.4, *a*); in addition, use is also made of welded short bars or washers (Fig. 5.4, *b*), threaded attachments which do not weaken the cross section of the tendons (Fig. 5.4, *c*), button or rivet heads (Fig. 5.4, *d*), and upset bar ends with bushes (Fig. 5.4, *e*).

In post-tensioning, anchorage should ensure good transfer of the prestress from the tendons to the concrete. At the member ends where anchors are placed, the concrete is reinforced by additional stirrups, welded-wire mesh, and spirals. For uniform transfer of the prestress, anchors are placed on steel bearing plates.

Prestressing cables are anchored by factory-made socket-type anchorages which consist of a threaded stud inserted into a cable, and a mild steel fitting put on the cable (Fig. 5.5, *a*). When the anchor is drawn

through an upsetting ring, the metal of the fitting yields and compresses the cable wires (Fig. 5.5, *b*).

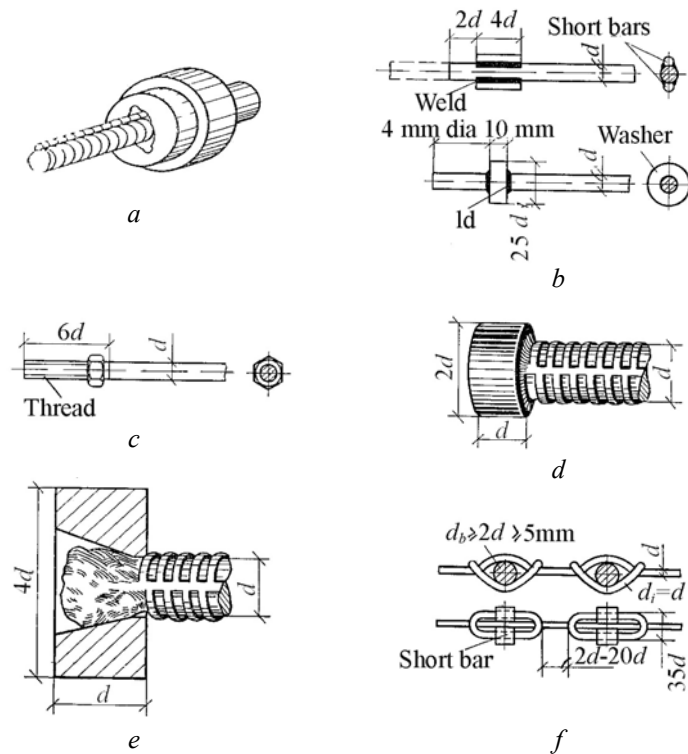


Fig. 5.4. Anchorage of prestressed reinforcing steel:  
*a* — grip tensing unit; *b* — short bars and washers welded to reinforcing bars; *c* — nut at the end of a threaded bar; *d* — button or rivet head;  
*e* — upset bar end with a bush; *f* — loops and short bars for the anchorage of plain high-strength wire

After the cable has been post-tensioned by a jack, it is anchored by turning the nut on the stud as far as it will go.

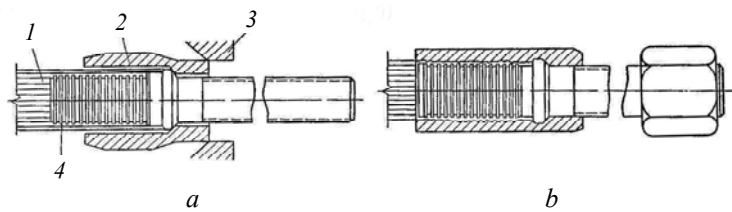


Fig. 5.5. Socket-type anchorage:  
*a* — before pressing; *b* — after pressing; 1 — cable;  
 2 — socket; 3 — upsetting ring; 4 — threaded stud

Fig. 5.6 shows the Freyssinet system using a wedging principle. A double-acting hydraulic jack is placed in position and gripped to the wires by wedges. The main piston is pumped to the required pressure and, while maintaining that pressure, the inner piston is pumped to drive the plug home.

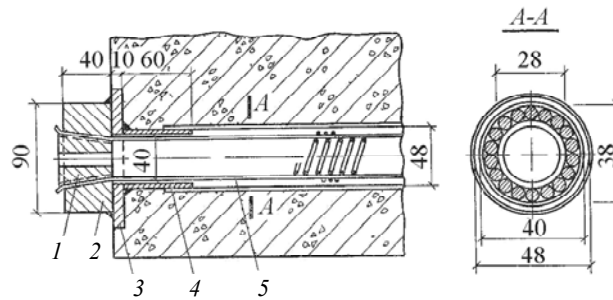


Fig. 5.6. Freyssinet anchorage:  
 1 — cone plug; 2 — female anchor cone; 3 — steel bearing plate;  
 4 — sleeve; 5 — reinforcing cable

Multilayer cables are anchored by basket-type anchorages (Fig. 5.7). The cable is post-tensioned by a jack to the required stress. Then, the gap between the anchorage and the member end is filled up by notched shims, which maintain the tension.

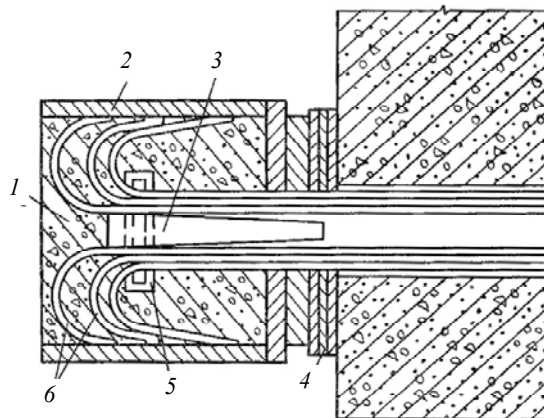


Fig. 5.7. Basket-type anchorage for post-tensioned multilayer cables:  
 1 — concrete pressed into the basket to hold the cable wires;  
 2 — steel basket with welded bottom

## **ASSIGNMENTS TO CHAPTER 5**

1. What is the main idea of reinforced concrete manufacture?
2. What is the basis of joint work of concrete and steel in reinforced concrete structures?
3. Name positive and negative properties of reinforced concrete.
4. Characterize composition and types of reinforced concrete.
5. What factors influence the strength property of reinforced concrete?
6. Name classes and brands of reinforced concrete.
7. How is reinforced concrete deformed due to short-time, long-time and many-time repeated loading?
8. Characterize mechanical properties and types of steel reinforcement.
9. Name classes and brands of steel reinforcement.
10. Characterize the effect of steel reinforcement on shrinkage and creep of reinforced concrete.

## 6. EXPERIMENTAL BASIS OF THE REINFORCED CONCRETE STRENGTH THEORY. METHODS OF REINFORCED CONCRETE STRUCTURAL DESIGN

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### 6.1. The Importance of Experiments

Experimental studies into the composite action of concrete and reinforcing steel differing in physical and mechanical properties have been carried out since the invention of reinforced concrete. It has been experimentally proved that nonlinear strain in concrete and cracks in tension zones have a significant effect on the stress-strain behavior of reinforced concrete members. Indeed, the assumption that stress-strain relations are linear, and the associated strength equations for elastic materials are often invalid for reinforced concrete.

The strength theory of reinforced concrete is based on experimental data and laws of mechanics, and proceeds from the real stress-strain state of members at various stages of external loading. As more experimental data are accumulated, the methods for the design of reinforced concrete structures are improved.

### 6.2. Three Stages in the Stress-Strain State

Experiments on various reinforced concrete members, including those in bending, eccentrically tension, and eccentrically compression with negative and positive stress diagrams, have shown that as the external load is gradually raised; three characteristic stages can be traced in the stress-strain behaviour of the member. *Stage I* continues until cracking begins in the tension zone of the concrete; during this stage, the stress in the concrete is below the ultimate tensile strength, and both the concrete and the steel are subjected to the tensile stress. *Stage II* begins after cracking has occurred in the tension zone; during this stage, the tensile stress at the cracks is received by the steel and the concrete areas above the cracks, and both the concrete and the steel are subjected to the tensile stress between the cracks. *Stage III* is the breakdown stage taking place during a relatively short period of time; during this stage, the stress in the tension reinforcing bars reaches its physical or proof

yield point, that in high-strength wire reaches the ultimate tensile strength, and that in the compression zone of the concrete reaches the ultimate compressive strength. Depending on the amount of reinforcement, the tension zone may break down ahead of the compression zone, or vice versa.

Let us consider the three stages for a reinforced concrete member in pure bending (Fig. 6.1).

**Stage I.** When the load is small, the stress in the concrete and steel is low, the strain is mostly elastic in nature, relation between the stress and the strain is linear, and the diagrams of normal stresses in the tension and compression concrete zones are triangular. As the load increases, no elastic strain develops in the tension zone, the stress diagram becomes curved, and the stress approaches the ultimate tensile strength. This is the end of Stage I. Any further increase in the load results in cracks in the tension zone, which signals the beginning of Stage II.

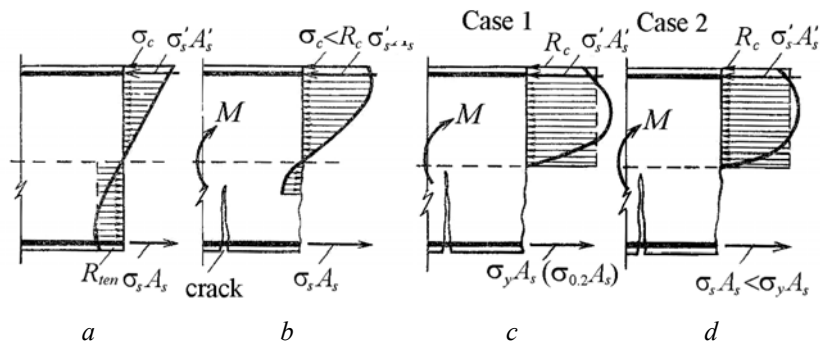


Fig. 6.1. Stress-strain stages at normal sections of a nonprestressed member in bending:  
*a* — Stage I; *b* — Stage II; *c* — Stage III

**Stage II.** As already noted, at the cracks appearing in the tension zone the tensile stress is carried by the steel and the tension concrete areas above the cracks. Between the cracks, the steel retains its bond to the concrete, and the tensile stress in the concrete increases and that in the steel decreases on moving away from the crack edges. As the load increases, no elastic strain develops in the compression concrete zone, the normal stress diagram becomes curved, and the stress peak moves from the edge towards the centre of the section. Stage II ends when appreciable plastic strain develops in the reinforcing steel Stage III. As the load increases further, the stress in the reinforcing bars reaches the physical or proof yield point; under the action of the increasing sag and

decreasing depth of the compression zone, the stress in the compression zone of the concrete reaches the ultimate compressive strength. As this takes place, the reinforced concrete member collapses; the steel in the tension zone is the first and the concrete in the compression zone is the last to break down. This failure is plastic in nature; we shall call it Case 1. If in the tension zone the member is reinforced by a high-strength wire having a low percentage elongation at rupture (about 4 %), the wire breaks down simultaneously with the concrete of the compression zone. The failure is brittle by nature, and it also falls under Case 1.

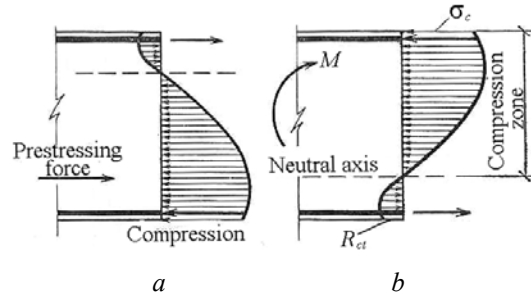


Fig. 6.2. Stress distribution at normal sections of a prestressed member in bending:  
*a* — during the transfer of the prestress;  
*b* — after the external load has been applied. Stage I

If a member is over reinforced, the failure is caused by the breakdown of the concrete in the compression zone. Stage II changes to Stage III suddenly. The failure of over reinforced members is always brittle by nature, with the strength of tensile reinforcing steel being only used in part; we shall call it Case 2.

**In Stage III**, the non-prestressed steel in the compression zone is subjected to a compressive stress, the value of which depends on the ultimate strain of the concrete  $\sigma'_s \leq \epsilon_{uc} E_s$ .

A particular stage of the stress-strain state is not the same for different sections along a reinforced concrete member. For example, Stage I takes place in the areas with low bending moments, Stage II – in the areas with higher bending moments, and Stage III in the areas of a maximum bending moment. Similarly, different stress-strain stages may occur during manufacture, prestressing, transportation, erection, and exposure to various service loads.

When the prestress is transferred from the steel to the concrete, a rather high stress may develop in a prestressed member. Owing to non-

elastic strain, the compressive stress diagram becomes curved. As the member is gradually loaded, the compressive prestress is balanced out and the appearing tensile stress approaches the ultimate tensile strength of the concrete (Fig. 6.2). As the strain in the concrete,  $\sigma_c$ , gradually increases and  $E'_c$  decreases from the axis to the outer fibre of the section, the maximum stress,  $\sigma_c = \varepsilon_c E'_c$  moves closer to the centre of the section. The difference between ordinary and prestressed reinforced concrete members is most pronounced in Stage I of the stress-strain state. In prestressed members, the external load causing cracking is several times that of ordinary members, the stress in the compression zone and the depth of this zone also considerably increase. The interval between Stage I and Stage III decreases. After cracking has occurred in Stages II and III, the stress-strain states of prestressed and noprestressed members are similar.

### 6.3. Crack Development in Tension Zone

Cracks in reinforced concrete members may be caused by hardening and shrinkage conditions, compressive prestress during manufacture, and overstressing of the materials in service, that is, overloading, settlement of supports, temperature variations, and the like. Overstressing is likely to cause cracking in tension zones rather than in compression zones. Invisible cracks appear even in the tension zones of perfectly designed reinforced concrete structures. This is a result of low extensibility of concrete, which is unable to follow the considerable steel elongation at high working stresses. In prestressed members, cracking occurs at a relatively high load. Experience shows that as long as the width of these cracks lies within certain limits, they are not dangerous and do not affect the integrity of reinforced concrete.

Reinforcing steel in the tension zone of a member somewhat reduces the negative effect of the non-uniform inner structure and discontinuity of concrete, but with the usual amount of reinforcement, the tensile strength of reinforced concrete only slightly exceeds that of plain concrete. Cracks in compression zones usually are an indication that the section is inadequate to carry the compressive stress. Such cracks constitute a threat to the strength of the structure.

Cracking in tension zones may be divided into three stages, namely 1 — incipient cracking when cracks may still be invisible, 2 — visible cracking, and 3 — crack opening until a maximum width is attained. In members with a normal amount of reinforcement, the first two stages merge together, so we may consider only two stages, namely visible cracking and crack opening.



#### 6.4. Elastic design

This method of design of members in bending was historically the first to develop. It uses Stage II of the stress-strain state and the following assumptions as its basis: 1 — concrete does not resist tensile stresses; all of the tensile stress is applied to the reinforcing steel; 2 — the concrete in compression zones is elastic, the stress-strain relation is linear and obeys Hook's law; and 3 — right sections which were plane before bending are plane after bending (that is, Bernoulli's assumption remains true).

As a consequence of these assumptions, the stress diagram in the compression zone is taken to be triangular and the ratio of the elastic moduli of steel and concrete,  $\alpha = E_s/E_c$ , is constant (Fig. 6.3).

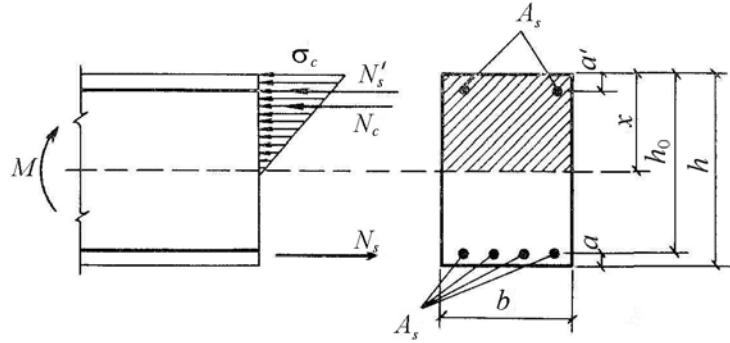


Fig. 6.3. Elastic design of a rectangular beam

Here, we consider a uniform section in which the steel area,  $A_s$ , is replaced by its transformed "concrete area" equal to  $\alpha A_s$ . Since the strains of the concrete and steel are equal, the stress in the outer fibre is determined as to the transformed uniform section. Based on two materials deformation equation

$$\varepsilon_s = \sigma_s/E_s = \varepsilon_c = \sigma_c/E_c \quad (6.1)$$

with the help of  $\alpha$  we can obtain dependence of reinforced concrete and concrete stresses

$$\sigma_s = \alpha \sigma_c \quad (6.2)$$

The outer fibre stress is determined as to the transformed uniform section

$$\sigma_c = Mx/I_{\text{red}} \quad (6.3)$$

To stresses in the tensile and compressive steel can be determined by the following formulas

$$\sigma_s = \alpha M (h_0 - x) / I_{red} \quad (6.4)$$

$$\sigma'_s = M(x - a') / I_{red} \quad (6.5)$$

where  $h_0 = h - a$  is the effective depth of the section;  $h$  – is the overall depth of the section;  $a$  – is the distance from the tensile face to the centroid of the tensile steel;  $a'$  – is the distance from the compressive face to the centroid of the compressive steel; and  $x$  – is the depth of concrete in compression.

The value of  $x$  is found from the condition that the static moment of the transformed section about the neutral axis is zero

$$S_{red} = bx^2/2 + \alpha A'_s(x - a') - \alpha A_s(h_0 - x) = 0 \quad (6.6)$$

The moment of inertia of the transformed section is

$$I_{red} = bx^3/3 + \alpha A_s (h_0 - x)^2 + \alpha A'_s (x - a')^2 \quad (6.7)$$

In elastic theory, the permissible stress for concrete is defined as a certain fraction of the ultimate compressive strength of concrete,  $\sigma_c = 0,45R$  (where  $R$  is the concrete brand number equal to the cube crushing strength of concrete). The permissible stress for steel is defined as a fraction of its yield point,  $\sigma_s = 0.5\sigma_y$ .

The main drawback of elastic design is that concrete is regarded as an elastic material. The actual stress distribution in a concrete section in Stage II cannot be described by a triangular stress diagram. Also,  $\alpha$  is not constant, being dependent on the stress in concrete, the duration of this stress and some other factors. Adjustment of  $\alpha$  to suit a given concrete brand number has also proved ineffective. Furthermore, it has been determined that actual stresses in reinforcing steel are below those calculated. With elastic theory, it is impossible not only to design structures with a predetermined safety factor, but also to determine real stresses in the materials. In some cases, this leads to an excessive material consumption or the use of reinforcement in the compression zones of the concrete.

The disadvantages of elastic design became especially obvious with the advent of new types of concrete (such as high-strength heavy concretes and lightweight porous-aggregate concretes) and stronger reinforcing steel.

## 6.5. Plastic or collapse design

The drawbacks of elastic theory necessitated the development of a new design method which would better conform to the elastic and plastic properties of concrete. In Soviet practice, new standards and specifications based on plastic or collapse design were put in effect in 1938.

Basically plastic design uses Stage III of the stress-strain state. Again, tensile stresses in the concrete are neglected as an element of strength. Permissible (working) stresses in the design formulas are replaced by the ultimate compressive strength of concrete and the yield point of steel. With these values, the modular ratio need not be known. In the early days of plastic analysis, the stress diagram in the compression zone was taken to be curved; nowadays, it has been replaced by a rectangular diagram. The maximum safe force which may be applied to a structure in service is determined by dividing the limit or collapse force by the total safety or load factor,  $k$  (whence another name: the load factor method). Accordingly, for members in bending

$$M = M_p / k \quad (6.8)$$

and for members in compression

$$N = N_p / k. \quad (6.9)$$

When determining the collapse load for members working in Case 1 (breaking down in the tension zone), Bernoulli's assumption is replaced by the plastic collapse principle according to which the stress reaches its ultimate value simultaneously both in the concrete and in the steel. This principle (first formulated by the Soviet scientist A.F. Loley) yields the design formulas defining the collapse load for bending and axially loaded members.

For a bending member of any symmetrical cross section (Fig. 6.4), the depth of the compression zone is determined from the equilibrium of internal forces in the breakdown stage

$$R_u A_c + R_s A'_s = R_s A_s \quad (6.10)$$

where  $R_u$  is the ultimate compressive strength of the concrete in bending equal to  $1.25 R_c$ ;  $R_s$  is the yield point of the steel; and  $A_c$  is the concrete area in compression.

The collapse moment is determined as the moment of internal forces about the centroid of the tensile steel

$$M_p = R_u S_c + R_s A'_s (h_0 - a) \quad (6.11)$$

where  $S_c = A_c z_c$  is the static moment of the compression zone about the centroid of the tensile steel; and  $z_c$  is the distance from the centroid of the tensile steel to the centroid of the compression zone.

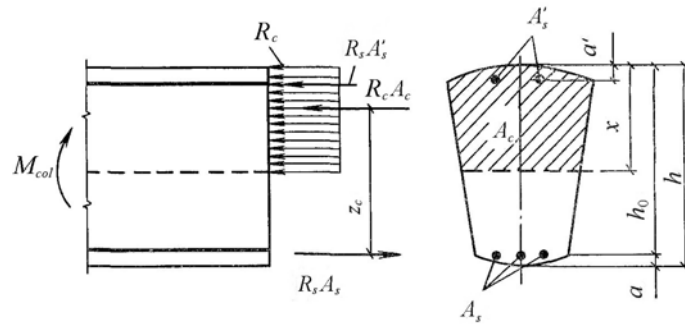


Fig. 6.4. Plastic design of a beam symmetrical in cross-section

The demarcation line between Case 1 and Case 2 is drawn according to experimental data: at  $S_c/S_0 \leq 0.8$ , concretes of B40 class and below fall under Case 1. Here,  $S_0$  is the static moment of the entire effective area of concrete about the centroid of the tensile steel. For rectangular and T-beams with compressed flanges, the depth of the concrete in compression should not exceed  $0.55h_0$ .

The safety (load) factor used in the design equation by this method is the same for the entire member. It is established according to the stress distribution at failure, load combination and the ratio of the live load,  $T_v$ , to the dead load,  $T_g$ . If live load prevails, a structure is more likely to be overloaded, so the safety factor must be greater. For example, for slabs and beams subjected to the basic combination of loads and at  $T_v/T_g < 2$ ,  $k = 1.8$ , at  $T_v/T_g > 2$ ,  $k > 2$ , and so on. For off-site precast members, the safety factor decreases by 0.2, but its specified value should not be less than 1.5.

In plastic design, the internal forces  $M$ ,  $Q$  and  $N$  due to loading are also determined in the breakdown stage, that is, with allowance for the formation of plastic hinges. For some structures (such as slabs, continuous beams and frames), this design method results in significant economy.

**Advantages and Disadvantages of Plastic Design.** Plastic design, which takes into consideration the elastic and plastic properties of concrete, describes the actual behaviour of members under load more correctly and has been an important advance in the strength theory of reinforced concrete.

A major advantage of this method over elastic design is that the total safety factor of a structure thus found is closer to its actual value.

In addition, structures designed by the load factor method require less reinforcing steel than those designed by the elastic method. For example, plastic design calculations usually show that members in

bending do not require compressive reinforcing steel. A disadvantage of this method is that the likely deviations of actual loads and strength of the structural materials from their design values cannot be taken care of by the common factor of safety.

#### **ASSIGNMENTS TO CHAPTER 6**

1. Speak on the importance of experimental research for the theory of RCC resistance.
2. How does the process of cracks development in stretched zones of RCC proceed?
3. Describe stressed deformed condition of RCC under loading and possible changes at stages.
4. Explain what the physical essence of case 1 and case 2 is in the third stage of the stressed-deformed condition of the SDS of an element.
5. What are the features of three stages of SDS under loading of pre-stressed elements?
6. What are the basic provisions of the method of calculation of cuts durability in the elastic circuit according to allowed pressure? Name disadvantages of the method.
7. What are the main provisions of the method of calculation of durability of sections by destroying effort with uniform factor of stock? What are the advantages and drawbacks of the method?

## **7. LIMIT-STATE DESIGN OF REINFORCED CONCRETE**

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### **7.1. General**

Limit-state design has originated from plastic design. The main difference between the two methods is that limit-state design clearly establishes the limit states of structures and sets up a system of design coefficients, which guarantee that a structure will not attain such states under the worst load combinations and at the minimum strength of the materials. This method is likewise based on the breakdown stage, but the safety of a structure under load is expressed in terms of several design coefficients rather than a total safety factor. Structures designed by this method are more economical.

### **7.2. Two Groups of Limit States**

Limit states are those in which structures no longer meet the service requirements, that is, they cease to resist external loads and other factors or are displaced or locally damaged more than it is allowed.

Limit-state design involves two groups of limit states. One refers to the necessary load bearing capacity (this is the first group of limit states) and the other to fitness for a particular normal service (the second group of limit states).

The design in terms of the first group of limit states is carried out to prevent a structure from:

- brittle, viscous or any other failure (for this purpose we find the strength of a structure with allowance, where necessary, for the sag before failure);
- loss of shape (stability analysis of thin-walled structures) and position (overturning and sliding analysis of retaining walls and eccentrically loaded high foundations; analysis of buried and underground tanks for tendency to float, etc.);
- fatigue failure (endurance analysis of structures subjected to repeated live or pulsating load, such as crane beams, sleepers, frame foundations and floors for unbalanced machines, and so on);

– failure under the combined action of load and unfavorable exposure conditions (periodic or permanent exposure to corrosive environments, freeze-thaw cycles, and the like).

The design in terms of the second group of limit states is done to prevent a structure from:

– excessive or long-term crack opening (if it is not allowed by the service conditions);

– excessive displacement (sagging, rotation angles, warping angles and vibration).

The limit-state design of an entire structure and of its members or parts is carried out for each stage, namely manufacture, transportation, erection and service. For each stage, an appropriate loading and scheme should be used, compatible with the type of structure involved.

### **7.3. Design Factors**

Design factors which include various loads and the mechanical characteristics of concrete and reinforcing steel (such as the ultimate strength and the yield point) are statistical values, that is, vary from case to case. For example, loads and other factors may exceed, and the mechanical characteristics of the materials may be below the average values as found by probability laws. Limit stage design takes into account the statistical variability in load and mechanical characteristics of the materials, non-statistical factors, and various favorable and unfavorable exposures, manufacture and service conditions to which concrete and reinforcing steel are subjected in buildings and other structures.

All loads, mechanical characteristics of materials and design coefficients are subject to relevant standards and specifications.

### **7.4. Classification of Loads. Basic and Design Loads**

**Classification of Loads.** According to their duration, loads are divided into dead and live. In turn, live loads may be subdivided into long time, short time and special loads.

**Dead loads** are loads which are related to the self-weight of the load bearing and filler members of the structure, the mass and pressure of the soil, and the prestress in prestressed concrete structures.

**Long time** loads include the weight of stationary equipment on floors (machines, apparatus, motors, tanks, etc.); the pressure of gases, liquids, and loose materials in tanks; loads in storehouses, refrigerators,

archives, libraries and the like; the specified part of the live load in civil buildings; long-time exposure to heat from stationary equipment; the load due to one hoist or traveling crane multiplied by 0.6 for medium-duty cranes and by 0,8 for heavy-duty cranes; and the snow load for some climatic regions reduced by  $70 \text{ kg/m}^2$ . The above values of crane, snow and some live loads constitute only a part of the total value; they are used when it is necessary to evaluate their long time effect on displacement, deformation and cracking. The total values apply to short time loads.

**Short time** loads include the loads constituted by people, spare parts and materials in attendance and maintenance areas, aisles and other locations free from equipment; floor loads in residential and other civil buildings; loads arising during the manufacture, transportation and erection of structural members; loads presented by hoists and overhead cranes used in the erection and service of buildings and other structures; snow and wind loads; and climatic temperature factors.

**Special loads** are related to earthquake and explosion loads; loads caused by failed equipment and sudden changes in temperature; non-uniform strain in foundations accompanied by a radical change in the soil structure (for example, strains caused by the soaking of settling soils or thawing of permafrost soils); and so on.

**Basic Loads.** These are established by relevant standards according to the predetermined probability of exceeding average values or are set according to rated values. For dead loads, the basic values are deduced from the design geometrical and structural variables and average densities. For live erection and service loads, they are deduced from the maximum safe values ensuring normal service; for snow and wind loads, they are deduced from the worst values averaged over a year or some other repetition period.

**Design Loads.** For the strength and stability analysis of structures, design loads are found by multiplying appropriate basic loads by the respective safety factor of load, which is usually greater than unity, for example,  $q = q_n \gamma_f$ . The weight safety factor of load for plain and reinforced concrete structures is 1.1. For lightweight concrete structures (with an average density of  $1600 \text{ kg/m}^3$  and lower) and various strainers, fillings, heat insulation and similar materials, the weight overload factor is 1,2 when factory-made and 1.3 when cast in-situ. For various live loads it may vary according to their values from 1,2 to 1,4. In the design of structures intended to resist floating, overturning and sliding, and also in some other cases when a reduction in weight might affect the behaviour of a structure, the safety factor of load is taken equal to 0.9.



To determine the loads existing during the erection stage, the design short-time loads must be multiplied by 0,8. For the purpose of strain and displacement analysis (the second group of limit states) design safety factor of load is taken equal to the respective basic values, so  $\gamma_f = 1$ .

**Load Combinations.** Any structure should be designed for different load combinations or, in the case of plastic design, for the respective forces. According to the loads involved, there are basic load combinations consisting of dead, long time and short time live loads or the appropriate forces, and special combinations which are composed of dead, long time, likely short time and one of special live loads or the respective forces.

Basic load combinations are subdivided into two groups. The first group includes dead, long time and one short time live load, the second contains dead, long time and two (or more) short time live loads. Here, the values of short time loads or the respective forces should be multiplied by the combination factor equal to 0,9.

When structures are designed to resist special load combinations, short time loads or the respective forces should be multiplied by the combination factor of 0,8, except the cases covered by specifications for structures in earthquake regions. The special load proper should be taken into account without any reduction.

**Load Reduction.** When designing columns, walls and foundations for industrial buildings, the vertical forces induced by short time loads (such as people, spare parts, repair materials, and so on) acting in areas free from equipment, transportation facilities and materials stored for a long time and occupying the whole floor area may be reduced by multiplying by coefficients).

Relevant standards and specifications also permit reduction in short time loads for beams and girders, in proportion to the floor area being actually loaded.

## 7.5. Basic and Design Concrete Strength

Basic concrete strength values include basic prism axial crushing strength  $R_{cn}$  and basic axial tensile strength  $R_{ctn}$ , being determined depending on the concrete strength class (at 0,95).

The basic prism crushing strength is determined by the following empirical formula

$$R_{cn} = B(0,77 - 0,00125B) \quad (7.1)$$

The basic axial tensile strength is determined by Eq. (7.2)

$$R_{ct} = 0,233R^{2/3}. \quad (7.2)$$

For a check of the axial tensile strength of concrete, it may alternatively be taken equal to its design axial tensile strength where  $B_{ctn}$  is the ultimate tensile strength of the concrete as given by its respective class number.

**Design Strength of Concrete.** For the first group of limit states, it is found by dividing the basic strength by the respective safety factors for concrete: in compression,  $\gamma_{cc} = 1.3$  and in tension,  $\gamma_{ct} = 1.5$  (or  $\gamma_{ct} = 1.3$  as a check for the tensile strength). Thus, the design axial compressive strength is

$$R_c = R_{cn} / \gamma_{cc} \quad (7.3)$$

and the design axial tensile strength of concrete is

$$R_{ct} = R_{ctn} / \gamma_{ct} \quad (7.4)$$

For design brands B50, B55 and B60 of heavy concrete, the design strength should be multiplied by the coefficient taking into account the mechanical properties of high strength concrete (reduced creep strain). This coefficient is 0.95, 0.925 and 0.9 respectively.

In the design of structural members, the design concrete strengths  $R_c$  and  $R_{ct}$  are reduced or, sometimes, increased by multiplying them by an appropriate service factor,  $\gamma_{ci}$ . These factors take care of the duration and cycling of loading; conditions, nature and stage in the behavior of a structure; manner of manufacture, cross-sectional area and so on.

For the second group of limit states, the design strength of concrete is determined assuming that the safety factor for concrete is  $\gamma_c = 1$ ; in other words, use is made of the basic values

$$R_{c,ser} = R_{cn}; R_{ct,ser} = R_{ctn}. \quad (7.5)$$

In calculations, these values are multiplied by the service factor  $\gamma_{ci} = 1$ , except when reinforced concrete members are designed to resist cracking under the action of repeated loading. In such cases, use is made of the service factor  $\gamma_{ci}$ .

## 7.6. Basic and Design Strength of Reinforcing Steel

**Basic Strength.** This value, designated  $R_{sn}$ , is established with allowance for the statistical variability of strength and is taken as the least observable value of the physical yield point  $\sigma_y$  or that of the proof yield point  $\sigma_{0.2}$  for reinforcing bars, and as the least observable ultimate strength  $\sigma_{0.2} = 0.8\sigma_u$  for reinforcing wire. The minimum confidential probability for the basic strength of reinforcing steel required by relevant specifications is 0.95.

**Design Strength.** For the first group of limit states, it is found by dividing the design strength by the respective safety factors for reinforcing steel

$$R_s = R_{sn}/\gamma_s \quad (7.6)$$

The safety factors for reinforcing steel are given in Table 7.1.

Table 7.1

**Safety Factors for Reinforcing Steel**

Type of steel	Safety factor
Bars:	
class A-I and A-II	1.05
class A-III $\phi$ 6...8 mm	1.1
class A-III and A-IIIC	1.07
class A-IV and AT-IVC, A-V and AT-V	1.15
class A-VI, AT-V and AT-VI	1.2
Wire:	
class Bp-I	1.1
class B-II and Bp-II	1.2
class K-7 and K-19	1.2

The design compressive strength of steel  $R_s$ , used in the design on the basis of the first group of limit states on the assumption that the steel remains bonded to the concrete, is taken equal to the respective tensile strength of the steel  $R_s$  but not more than 400 MPa (according to the compressive strain capacity of the concrete,  $\epsilon_{uc}$ ). For structures made of heavy or porous-aggregate concrete, the design strength is chosen for long time loading with the service factor  $\gamma_{c2} > 1$  applied, so  $R_{sc}$  may be taken equal to 450 MPa for class A-IV and AT-IVC reinforcement and to 500 MPa for class A-V, AT-V, AT-VI, B-II, Bp-II and K-7, K-19 reinforcement. This is justified because long time loading leads to an increase in the compressive strain capacity. At the same time, the transverse reinforcement should be positioned so as to prevent the longitudinal compressive steel from buckling; the spacing should not exceed 500 mm or twice the width of a given member face. When there is no bond between the concrete and the steel,  $R_{sc} = 0$ .

**7.7. Requirement for Crack Resistance of Reinforced Concrete Structures**

The crack resistance of a reinforced concrete structure is its ability to resist cracking in Stage I and crack opening in Stage II of the stress-strain state.

The requirements for the crack resistance of a reinforced concrete structure or its parts vary according to the exposure conditions and type of reinforcing steel. They are related to normal and inclined cracks and may be divided into three categories:

- category one, no cracks are allowed;
- category two, short time opening of limited width cracks is allowed, provided that they close tightly after the load has been removed;
- category three, short- and long-time opening of limited-width cracks is allowed.

"Short time" refers to the opening of cracks under the action of dead, long and short time live loads; "long time" refers to the opening of cracks under the action of dead and long time live loads only. The limiting crack width (designated  $a_{cr1}$  for short time opening and  $a_{cr2}$  for long time opening) at which structures continue to behave normally, reinforcing steel remains protected against corrosion and the service life of the structure is unaffected, ranges from 0,05 to 0,4 mm according to the category of crack resistance requirements.

Table 7.2

**Loads for Crack Resistance Design**

Category of requirements	Loads for			
	cracking analysis	Crack width analysis		Crack closure analysis
		Short time	Long time	
One	All loads (except special) act together, $\gamma_f > 1$ (as in strength analysis)	—	—	—
Two	All loads (except special) act together, $\gamma_f > 1$ (analysis is carried out to make sure whether checks on short-time crack width and crack closure are necessary or not)	All loads (except special) act together, $\gamma_f = 1$	—	Dead and long-time live loads act together, $\gamma_f > 1$
Three	All loads (except special) act together $\gamma_f = 1$ (analysis is carried out to make sure whether crack-width check is necessary or not)	Same	All loads (except special) act together, $\gamma_f > 1$	—

The requirements of the first category apply to prestressed concrete members subjected to the pressure of liquids or gases (such as tanks or pipes), members reinforced by bars or wire used below the ground water level and with their sections entirely in tension, and members reinforced by wire 3mm in diameter or smaller with their section partly in compression.

The requirements of the third categories hold for other classes of prestressed members, depending on service condition and type of reinforcement. The requirements of the third category apply to non-prestressed structures reinforced by class A-I, A-II and A-III, A-IIIC bars.

These requirements also hold in the calculation of forces arising during transportation and erection.

In crack resistance analysis, the loads taken into consideration are treated according to the applicable category of crack resistance requirements.

Within the transmission length  $l_p$  at the ends of prestressed members, it is required that the combined action of loads (except special) multiplied by the overload factor  $\gamma_f \leq 1$  should cause no cracking. This is because the premature cracking of the concrete at the ends of members might force the reinforcement to slip under load and the structure to fail suddenly.

Cracks that might form during manufacture, transportation or erection in a zone which will be in compression when loaded, would reduce the force required to cause cracking in the tension zone and increase crack width and sagging. The effect of such cracks is taken care of in the structural design. In members subjected to repeated loads, and designed for endurance such cracking is not allowed.

## 7.8. Main Points of Design

**The First Group of Limit States.** As already noted, strength analysis is based on Stage III of the stress-strain state. A section is said to be sufficiently strong if the forces induced by the design load do not exceed the design strength of the materials, taken with an appropriate service factor applied. A force due to the design loads,  $T$  (for example, a bending moment or a longitudinal force), is a function of basic loads, overload factors and some other factors designated  $C$  (loading system, dynamic factor, and so on). The force resisted by a section,  $T_{per}$ , is, in turn, a function of the shape and sizes of the section,  $S$ , material strength,  $R_{cn}$  and  $R_{sn}$ , safety factors,  $\gamma_{ci}$  and  $\gamma_{si}$ , and service factors,  $\gamma_{ci}$  and  $\gamma_{si}$ .

The condition for strength is described by the following inequality

$$T(g_n, v_n, \gamma_f, \gamma_n, C) \leq T_{per}(S, R_{cn}, \gamma_c, \gamma_{ci}, R_{sn}, \gamma_s, \gamma_{si}). \quad (7.7)$$

Since  $g_n \gamma_f = g$ ;  $v_n \gamma_f = v$  and  $R_{cn} \gamma_c = R_c$ ;  $R_{sn} \gamma_s = R_s$ , may be rewritten as

$$T(g, v, \gamma_n, C) \leq T_{per}(S, R_c, \gamma_{bi}, R_s, \gamma_{si}). \quad (7.8)$$

**The Second Group of Limit States.** Analysis for the formation of normal and inclined cracks is carried out to check the crack resistance of

members designed to meet the requirements of the first category, and also to determine whether cracks may occur in members designed to meet the requirements of the second and third categories. It is assumed that no normal cracks will occur in the concrete, if the force  $T$  (a bending moment or a longitudinal force) induced by the external load does not exceed the force  $T_{cr}$  which may be resisted by the section

$$T \leq T_{cr} \quad (7.9)$$

The load combination and overload factors necessary to determine the force,  $T$ , is given in Table 7.3.

It is also assumed that no inclined cracks will occur if the principal tensile stresses in the concrete do not exceed the design values.

Crack opening analysis consists in determining the width of normal and inclined cracks at the level of tensile reinforcing steel and comparing it with the limiting crack width.

It is required that

$$a_{cr} \leq a_{cr,u} \quad (7.10)$$

Table 7.3

**Limits of Sag Reinforced Concrete Members**

Members	Limit of sag as a fraction of span	Loads with $\gamma_t = 1$
Electric crane beams	Service requirements 1/600	For one crane
Beams, trusses, ceiling and floor plates with span: l = 6m l = 12m l = 24m	Esthetic and Psychological requirements 1/200 1/250 1/250	Dead and long-time live loads
Filler wall panels (in plane design) with spans: l = 6m 6m ≤ l ≤ 7.5m l > 7.5m	Structural requirements 1/200 3cm 1/250	Dead, long and short time live loads

The displacement analysis of a structure consists in finding the sag of a member caused by the external load and adjusted for the duration of loading, and comparing it with the sag limit. It is required that

$$f \leq f_{lim} \text{ with } \gamma_f = 1 \quad (7.11)$$

The limit of sag is set according to service and structural requirements. The service requirements provide for the normal operation of cranes,

machines and other process equipment. The structural requirements arise from the effect of adjacent members limiting the strain, the necessity to maintain specified slopes, and also the external appearance of the structure.

The limit of sag for prestressed members may be increased by the amount of hogging, if this does not run counter to service and structural requirements.

When a structure is to meet certain service or structural requirements, sagging is determined for dead, long and short time live loads; if sagging is limited by considerations of the external appearance, it is determined for dead and long time live loads.

The limits of sag prescribed for reinforced concrete members by appropriate specifications are given in Table 7.3. The limits of sag related to the cantilever overhang are doubled.

Reinforced concrete members such as slabs in floors, flights of stairs and landings, not connected to adjacent members should additionally be analyzed for fixity: the additional sag produced by a short time concentrated load of 1000 H at the most unfavorable disposition of loads should not exceed 0,7 mm.

#### **ASSIGNMENTS TO CHAPTER 7**

1. What are the main provisions for calculation of constructions by the method of limiting condition? Name two groups of limiting conditions.
2. What should calculation according to limiting condition of the first group provide?
3. What should calculation for a limiting condition of the second group provide?
4. What classification of loadings has been accepted? What is the purpose of reliability factor introduction?
5. How is the normative resistance of concrete established? What allowance does it provide?
6. How is calculation resistance for concrete of 1-st and 2-nd groups of limiting conditions determined? What purpose are the factor of reliability and factor of operating conditions introduced for?
7. How is the normative resistance for various classes of steels established?
8. What calculated resistance of armature, factor of reliability and operating condition of armatures are used?
9. How are conditions of elements calculation as to limiting conditions of groups 1-st and 2-nd written down?
10. What are the requirements to crack stability of RCC? Are they divided into categories?

## 8. PRESTRESS IN STEEL AND CONCRETE

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### 8.1. The Value of Prestress

Prestress in reinforcing steel and concrete is very important for the subsequent behavior of members under load. When the prestress in the steel and concrete is low, the prestressing effect, will be gone in some time because of the relaxation of stress in the reinforcement, shrinkage and creep of the concrete, and some other service and structural factors. If the stress in the steel is close to its ultimate strength, wire reinforcement may break down in tension and hot-rolled reinforcement may suffer considerable permanent set. Present evidence shows that the prestress in the steel placed in the tension and compression zones, designated  $\sigma_{sp}$  and  $\sigma'_{sp}$  respectively, should be specified according to the following conditions

– for reinforcing and wire reinforcement bars

$$\sigma_{sp} + p \leq R_{sn} \text{ and } \sigma_{sp} - p \geq 0,3R_{sn} \quad (8.1)$$

where  $p = 0,05\sigma_{sp}$  MPa for mechanical tensioning;  $p = (30 + 360/l)$  MPa for electro thermal and electro thermal mechanical tensioning (that is, tensioning by electric heating);  $l$  is the length of the bar being tensioned, in meters (the length out to out of the bearing plates).

In electro thermal tensioning, the temperature should not exceed 300 or 350° C to prevent the steel strength loss.

In pretensioning, the initial jacking stress in the steel with allowance for anchorage strain losses,  $\sigma_3$ , and friction losses in templates,  $\sigma_4$ , is equal to

$$\sigma_{con} = \sigma_{sp} - \sigma_3 - \sigma_4 ; \quad \sigma'_{con} = \sigma'_{sp} - \sigma_3' - \sigma_4' \quad (8.2)$$

The initial jacking stress in post-tensioning (with a part of the prestress transferred to the concrete) is equal to

$$\sigma_{con} = \sigma_{sp} - \alpha\sigma_{cp}; \quad \sigma'_{con} = \sigma'_{sp} - \alpha\sigma_{cp}' \quad (8.3)$$

where  $\sigma_{cp}$ ,  $\sigma_{cp}'$  are the prestress (with regard to the early losses).

Any possible deviation from the specified prestress in manufacture is taken care of by the tension accuracy factor

$$\gamma_{sp} = 1 \pm \Delta\gamma_{sp} \quad (8.4)$$



$$\gamma_{sp} = 0,5(p/\sigma_{sp})/(1+1/\sqrt{n}) \geq 0,1 \quad (8.5)$$

where  $\Delta\gamma_{sp}$  is the extreme deviation in the prestress in the steel. The plus sign is taken when the prestress has a negative effect on the member behavior (for example, in the strength analysis of reinforcing steel placed in the zone compressed under load, also in the calculations for the manufacture and erection stages), the minus sign is taken when the effect of the prestress is positive.

For the prestress loss, crack width and displacement calculations,  $p$  may be taken as zero.

The transfer or cube strength of concrete by the instant when the prestress is transferred from the steel to the concrete, designated  $R_{bp}$  should be chosen so that the ratio  $\sigma_{cp}/R_{cp}$  would not be too high, because otherwise there would be considerable creep strain in the concrete and the prestress in the steel would be gone. The recommended value is  $R_{cp}$  equal to 11 MPa by calculation and not less than 15,5 MPa for class AT-VI bars and wire strands.

The stress transferred to the concrete during compression,  $\sigma_{cp}$  is limited for the same reason; it should not exceed the limiting values of Table 8.1, given as fractions of the transfer strength,  $R_{cp}$ .

Table 8.1

**Limiting Prestress in Concrete  
(for a Design Winter Temperature)**

State of stress in section	Method of tensioning	Prestress in concrete, fractions of $R_{cp \max}$	
		Axial prestressing	Eccentric prestressing
Prestress decreases under external load	Pretensioning	0,65	0,75
	Pretensioning	0,55	0,65
Prestress increases under external load	Pretensioning	0,5	0,55
	Pretensioning	0,45	0,5

If  $\sigma_{cp}$  is reduced by external loading (which happens most often) its value for the eccentrically transfer of compression and pretensioning should not exceed  $\sigma_{cp} \leq 0,75 R_{cp}$ .

According to the type and class of prestressed steel, its diameter and anchorage, the concrete for prestressed members ranges in brand number (Table 8.2). As the diameter and strength of the reinforcing steel increase, the brand number is also increased.

Table 8.2

### Design Concrete Classes for Prestressed Members

Prestressed steel	Design concrete class
Wire: class B-II with anchorage class Bp-II without anchorage, max 5mm in diameter the same, 6mm and more in diameter class K-7, K-19 strands	B20 B20 B30 B30
Deformed bars without anchorage, 10 to 18mm inclusive in diameter: classes A-IV and AT-IV classes A-V and AT-V class A-VI and AT-VI	B15 B20 B30
The same, 20mm and more in diameter: classes A-IV and AT-IV classes A-V and AT-V class A-VI and AT-VI	B20 B25 B30

### 8.2. Prestress Losses in Reinforcing Steel

The initial prestress in the steel decreases with time. Losses in prestress are customarily divided into the early losses taking place during the manufacture and transfer to concrete, and the late losses occurring after the prestress has been transferred to the concrete.

#### Early Losses

1. Losses due to relaxation of stress in the steel in pretensioning and according to the type of tensioning and reinforcing steel:

*Mechanical tensioning, MPa:*

– high-strength wire and wire strands

$$\sigma_1 = (0,22\sigma_{sp}/R_{sn} - 0,1)\sigma_{sp} \quad (8.6)$$

– bars

$$\sigma_1 = 0,1\sigma_{sp} - 20 \quad (8.7)$$

*Electro thermal and electrotermomechanical tensioning:*

– high-strength wire and wire strands

$$\sigma_1 = 0,05\sigma_{sp} \quad (8.8)$$

– bars

$$\sigma_1 = 0,03\sigma_{sp} \quad (8.9)$$

Here  $\sigma_{sp}$  is taken without losses.

2. Losses due to the difference in temperature between the steel and devices resisting the prestress during the moist or heat curing of the concrete

$$\sigma_2 = 1,25\Delta t, \quad (8.10)$$

where  $\Delta t$  is the difference in temperature between the steel and abutments resisting the prestress in °C; if no data are available,  $\Delta t$  is taken as 65°C.

3. Losses due to the deformation of anchorages located near tensioning devices, caused by upset shims, crushed button or rivet heads, and displaced bars in grips in mechanical pretensioning

$$\sigma_3 = (\lambda/l)E_3, \quad (8.11)$$

where  $\lambda = 2\text{mm}$  with upset shims or crushed button or rivet heads;

$$\lambda = 1.25 + 0,15d, \quad (8.12)$$

with displaced bars in reusable grips; here,  $d$  is the bar diameter in millimeters, and  $l$  is the length of the bar being tensioned in millimeters (the distance out to out of the bearing plates).

In electro thermal tensioning,  $\sigma_3 = 0$ .

In post-tensioning,

$$\sigma_3 = (\lambda_1 + \lambda_2)E_s/l, \quad (8.13)$$

where  $\lambda_1$  is the amount of upsetting in shims placed between the anchorages and the concrete, equal to 1mm;  $\lambda_2$  is the displacement of basket-type anchorages, Freyssinet anchorages, anchoring nuts and grips, taken as 1mm; and  $l$  is the length of the bar being tensioned (the length of the member).

4. Losses due to steel friction:

a) against raceway walls or structure surface in post-tensioning (Fig. 8.1).

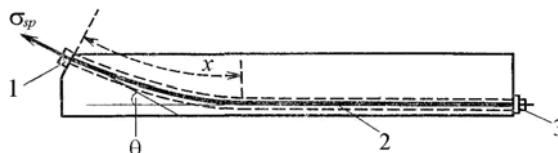


Fig. 8.1. Determination of frictional losses in raceways:  
1 — jacking end; 2 — tendon in a raceway; 3 — anchorage

$$\sigma_4 = \sigma_{sp}(1 - e^{-\omega t - \delta\theta}) \quad (8.14)$$

where  $x$  is the length of the tendon portion;  $\theta$  is the total angular change of the tendon along the curved portion, in radians;  $\delta$  is the coefficient of

friction between the tendon and surrounding material and  $\omega$  is the wobble coefficient. The values of  $\delta$  and  $\omega$  are given in Table 8.3.

Table 8.3

**Friction and Wobble Coefficient**

Duct	Values		
	$\omega$	cables and strands	deformed bars
With metal sheathing	0.003	0.35	0.4
With concrete surface:			
made by rigid duct-former	0	0.55	0.65
made by flexible duct-former	0.0015	0.55	0.65

b) against templates in pretensioning

$$\sigma_4 = \sigma_{sp}(1 - e^{-0.25\theta}) \quad (8.15)$$

where  $\theta$  is the total angular change of the tendon, in radians.

5. Losses due to the deformation of steel forms in tensioning by jacks

$$\sigma_5 = [(n - 1) / 2n](\Delta l / l)E_s \quad (8.16)$$

where  $\Delta l$  is the yield of the abutments along the resultant prestressing force, determined by the mould design;  $l$  is the distance out to out of the bearing plates; and  $n$  is the number of tendon groups tensioned simultaneously.

If no data about the mould design are available,  $\sigma_5$  is taken as 25 MPa. In pretension by a wrapping machine,  $\sigma_5$  is halved; in tensioning by electric heating,  $\sigma_5 = 0$ .

6. Losses due to instantaneous creep depend on the hardening conditions, stress-strain ratio and design concrete class numbers. The loss of prestress occurs during the transfer of the prestress from the steel to the concrete (and also during the first 2 or 3 hours after the transfer). When a member is allowed to harden fully in the air, the loss of prestress is

$$\sigma_6 = \begin{cases} 40\sigma_{cp}/R_{cp} & \text{at } \sigma_{cp}/R_{cp} \leq a \\ 40a + 80\beta(\sigma_{cp}/R_{cp} - a) & \text{at } \sigma_{cp}/R_{cp} > a \end{cases} \quad (8.17)$$

where  $a = 0,25 + 0,025R_{cp} \leq 0,8$  and  $\beta = 5,25 - 0,185R_{cp} \leq 2,5$ ,  $\beta \geq 1,1$ .  $\sigma_{cp}$  is the prestress in the concrete at the centroid of the prestressed  $A_s$  and  $A_s'$  developed by the prestressing force  $P$  taken with allowance for the losses  $\sigma_1$  through  $\sigma_5$ . In heat and moist curing at atmospheric pressure, the losses are multiplied by 0,85.

**Late Losses.** 7. Losses due to relaxation in post-tensioned high strength wire and bars, are taken equal to the losses in pretension, that is,  $\sigma_7 = \sigma_1$ .

8. Losses caused by shrinkage and shortening of a member depend on the type of concrete, method of tensioning, and hardening condition. The values of  $\sigma_8$  will be found in Table 8.4.

Table 8.4

**Loss of Prestress due to Shrinkage (in MPa)**

Concrete	Pretensioning		Post-tensioning
	in air entire time	heat cured at atmospheric pressure	Irrespective of hardening conditions
Heavy concrete:			
– B35 and lower	40	35	30
– B40	50	40	35
– B45 and higher	60	50	40
Porous-aggregate concrete:			
– dense fine aggregate	50	45	40
– porous fine aggregate, except expanded perlite sand	70	60	50

9. Losses due to creep of concrete (caused by the respective shortening of a member), depending on the type of concrete, hardening conditions, and stress-strength ratio.

For heavy concrete of classes B35, B40, B45 and higher light concrete with dense fine aggregate

$$\begin{aligned} \sigma_9 &= 150\alpha\sigma_{cp} \quad \text{at } \sigma_{cp}/R_{cp} \leq 0,75 \\ \sigma_9 &= 300\alpha(\sigma_{cp}/R_{cp} - 0,375) \quad \text{at } \sigma_{cp}/R_{cp} > 0,75 \end{aligned} \quad (8.18)$$

where  $\sigma_{cp}$  is determined as with the loss due to instantaneous creep taking into account;  $\alpha = 1$  when the concrete is allowed to harden in the air entire time; and  $\alpha = 0,85$  when the concrete is cured by heat at atmospheric pressure.

10. Losses caused by crushing of the concrete under the turns of spiral or ring reinforcement (at a pipe and tank diameter of up to 3m), defined as

$$\sigma_{10} = 70 - 22d \quad (8.19)$$

where  $d$  is the external diameter of a structure, m.

11. Losses due to upset joints between precast blocks, are determined as

$$\sigma_{11} = (n\lambda/l)E_s \quad (8.20)$$

where  $\lambda$  is the amount of upsetting equal to 0,3 mm for concreted points and 0.5mm for dry joints;  $n$  is the number of joints along the prestressed steel; and  $l$  is the length of the prestressed steel, in millimeters.

The shrinkage and creep losses,  $\sigma_8$  and  $\sigma_9$ , considerably depend on the time of curing and ambient humidity. If we know the time of loading in advance, the loss is multiplied by the following coefficient

$$\psi_t = 4t/(100 + 3t) \leq 1 \quad (8.21)$$

where  $t$  is the time reckoned from the instant when concreting is completed (for  $\sigma_8$ ) or from the day when the prestress is applied to the concrete (for  $\sigma_9$ ), in days.

For structures exposed to an ambient humidity of less than 40 %, the shrinkage and creep losses increase by 25 %. For structures intended for service in dry and hot climatic regions the losses due to shrinkage and creep should be increased by 50 %.

With pretension, the early losses include relaxation in the steel, thermal losses, anchorage deformation, frictional losses in templates, steel mould deformation, and instantaneous creep

$$\sigma_{\text{los},1} = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \sigma_5 + \sigma_6, \quad (8.22)$$

and the late losses include losses due to shrinkage and creep

$$\sigma_{\text{los},2} = \sigma_8 + \sigma_9. \quad (8.23)$$

With post-tensioning, the early losses are composed of anchorage deformation and losses due to friction between the steel and duct walls (or structure concrete surface)

$$\sigma_{\text{los},1} = \sigma_3 + \sigma_4 \quad (8.24)$$

and the late losses include relaxation in the steel, shrinkage and creep in the concrete, crushing of the concrete under steel turns, and deformation of joints between blocks (for structures made of pre-cast members)

$$\sigma_{\text{los},2} = \sigma_7 + \sigma_8 + \sigma_9 + \sigma_{10} + \sigma_{11}. \quad (8.25)$$

The total losses for any method of tensioning are determined as

$$\sigma_{\text{los}} = \sigma_{\text{los},1} + \sigma_{\text{los},2}. \quad (8.26)$$

They may reach 30 % of the initial prestress. In design calculations, the total losses should be taken as not less than 100 MPa.

### **ASSIGNMENTS TO CHAPTER 8**

1. How is initial previous stress in armature established?
2. What is the stress in supervised armature equals to?
3. What is the accuracy factor of tightening armature equal to and what is this factor introduced for?
4. What is the transfer strength of concrete? How is its value determined?
5. What does the physical essence of the previous stress losses in armatures consist in?
6. What are 1-st and 2-nd losses of the previous stress in armature made up of at armature tightening on supports and on concrete?
7. What is a resulted concrete section?
8. What is the sequence of change of stressed condition in a prestressed central element?
9. What is the sequence of change of stressed condition in a prestressed bending element?

## 9. MEMBERS IN BENDING

### 9.1. Construction Features

The most common reinforced concrete members in bending are slabs and beams. Slabs are defined as flat members whose thickness,  $h$  is much smaller than the length,  $l$ , and width,  $b$ . Beams are defined as structural members whose length  $l$  considerably exceeds their depth,  $h$ , and width,  $b$ . These members are used in many reinforced concrete structures, most often in precast, in-situ and precast, in-situ beam and slab floors and roofs (Fig. 9.1). There exist single- and multi-span slabs and beams.

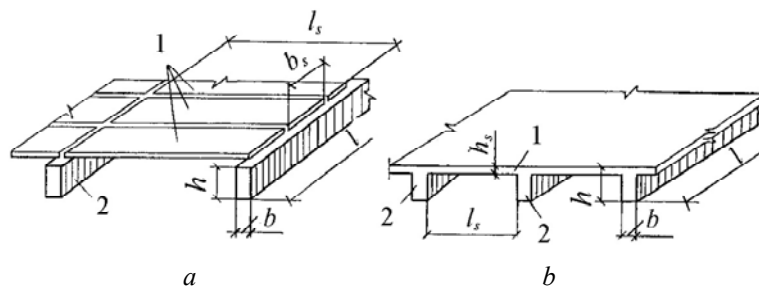


Fig. 9.1. Reinforced concrete floors:  
 $a$  — precast floor;  $b$  — in-situ floor;  $l$  — slabs;  $2$  — beams

**Slabs.** The thickness of slabs cast in-situ ranges between 50 and 100mm, whereas precast slabs should be as thin as possible.

Fig. 9.2,  $a$  shows a one-way single-span slab carried by two opposite side supports; Fig. 9.2,  $b$  shows an in-situ one-way multi-span slab carried by a number of parallel supports. Such slabs deform, as beams under various loads provided the loads do not change in the direction normal to the span. Slab reinforcement consists of welded-wire fabric. The fabric is placed with its load-bearing wires along the span to resist tensile forces induced by bending according to the bending moment diagrams (Fig. 9.2). So, within the span, the fabric is placed in the lower part of the slab; in multi-span slabs, the fabric is additionally placed in the upper part above



the intermediate supports. The fabric reinforcement used in multi-span slabs may be continuous (Fig. 9.2, *b*) and separate (Fig. 9.2, *c*).

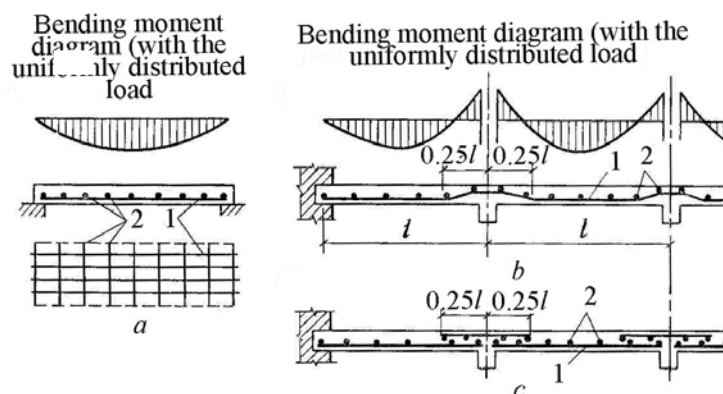


Fig. 9.2. Slab reinforcement:  
*a* — single-span slab; *b* — multi-span slab with continuous reinforcement;  
*c* — multi-span slab with separate reinforcement;  
*l* — load-bearing bars; 2 — erection bars

Load-bearing wires are from 3 to 10mm in diameter, with a spacing of 100 to 200 mm.

The minimum concrete cover for load-bearing reinforcement is 10mm; in super thick slabs (thicker than 100mm), it is at least 15 mm.

Transverse wires (distribution reinforcement) are placed to hold load-bearing wires in the design position, reduce shrinkage and thermal strain in structures, and distribute the stresses due to concentrated loads over a larger area. They are chosen to be smaller in diameter than load-bearing wires, but the total transverse steel area should be not less than 10% of the load-bearing steel area at places where the bending moment is maximum. Transverse wires are spaced from 250 to 300 mm apart, but not wider than 350 mm. In some cases (in slabs of a complex configuration or having a large number of voids), where standard welded-wire fabric cannot be used, slabs are reinforced using separate wires held together in manually applied tie wire.

**Reinforced Concrete Beams.** There exist rectangular beams, T-beams, I-beams and trapezoidal beams (Fig. 9.3).

The beam depth, *h*, varies over a wide range; according to the load and type of structure, it may be from 1/10 to 1/20 of the span. For unification, the beam depth is taken as a multiple of 50mm if it does not exceed 600 mm, or a multiple of 100 mm if it is greater.

The width of rectangular beams,  $b$ , is taken as from  $0,3h$  to  $0,5h$  namely 100, 120, 150, 200, 220, or 250 mm; the greater sizes being taken as a multiple of 50 mm.

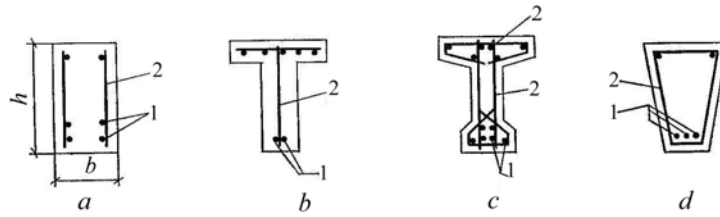


Fig. 9.3. Beam cross sections and types of reinforcement:  
 $a$  — rectangular beam;  $b$  — T-beam;  $c$  — I-beam;  $d$  — trapezoidal beam;  
 $1$  — longitudinal bars;  $2$  — transverse reinforcement

To reduce the concrete consumption, the beam width should be taken as small as possible. Over the cross section, the load-bearing reinforcement is positioned in the tension zone in one or two rows with spacing sufficient for concreting without voids and cavities.

The necessary spacing and covers are shown in Fig. 9.4. The clear space between longitudinal non-prestressed or pretensioned bars should be not less than the maximum bar diameter; for bottom horizontal bars (so positioned during concreting), it should be at least 25 mm, and for top bars, 30 mm. If there are more than two bottom bar rows, the horizontal separation between the bars in the third and higher rows (as reckoned from the bottom) should be at least 50 mm.

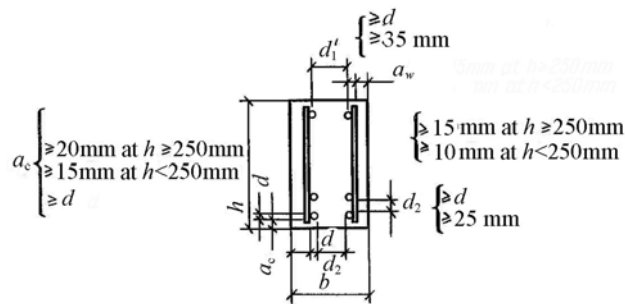


Fig. 9.4. Distribution of reinforcement over a beam cross-section:  
 $a_c$  — concrete cover for load-bearing reinforcement;  
 $a_c'$  — concrete cover for erection reinforcement;  $d$  — largest diameter of load-bearing bars;  $e_b$  — clear space between bottom bars (when placing concrete);  $e_t$  — clear space between top bars (when placing concrete)

If little space is available, bars may be placed in pairs without separation.

The clear space between deformed reinforcing bars is taken according to their nominal diameters without allowance for lugs.

As in slabs, longitudinal load-bearing reinforcement in beams is placed according to the bending moment diagrams in the tension zones where it is to resist longitudinal tensile forces induced by bending loads.

In members under bending, the calculated area of the longitudinal load-bearing reinforcement,  $A_s$ , should be at least 0,05 % of the effective area of the section. In rectangular beams, this area is equal to the product of the section width  $b$  and the effective depth  $h_0 = h - a$  (where  $h$  is the overall depth of the section, and  $a$  is the distance from the resultant force in the bars to the section face near which the reinforcement is located).

As a rule, longitudinal reinforcement consists of deformed bars 12 to 32 mm in diameter; plain bars are used more rarely.

Beams with a width of 150 mm and more are reinforced with at least two longitudinal bars which extend as far as the beam supports. If a beam is less than 150 mm wide it may be reinforced with one bar (or a bar mat).

In addition to bending moments, reinforced concrete beams are subject to shearing forces. To carry these forces, beams require transverse (or shear) reinforcement. The area of transverse reinforcement is determined by an appropriate analysis and from design considerations.

Longitudinal and transverse reinforcement is welded into bar mats; if no welding machines are available, they are held together with tie wire. Tied reinforcement is rather labour-consuming, and its use is warranted only when it is impossible to manufacture welded bar mats.

Welded bar mats are combined into reinforcing cages, using horizontal transverse bars spaced 1 to 1,5 m apart.

The reinforcement of a single-span beam using welded bar mats is shown in Fig. 9.5, *a*. When use is made of tied bar mats (Fig. 9.5, *b*), stirrups in rectangular beams are of closed design, whereas in T-beams where the rib is inseparable from the solid flange on either side, the stirrups may be open at the top (they are known as U stirrups). Beams wider than 35 cm use multiple stirrups. In tied bar mats, the stirrups should be at least 6mm in diameter for beams up to 800 mm deep, and not less than 8mm in diameter for deeper beams.

To meet design and erection requirements, in members without bent bars, the spacing between the transverse bars (or stirrups) in the longitudinal direction should be not more than  $h/2$ , or 150 mm for beams up to 400 mm in depth, and not more than  $h/3$ , or 500 mm for beams deeper than 400 mm, whichever is the smaller. In beam-subjected to a uniformly distributed load, this requirement holds for  $1/4$  of the span

starting from the support; in beams subjected to concentrated loads, the portion in question extends farther out to the nearest load. In the remaining part of the span, the space between transverse bars (or stirrups) may be greater, but not more than  $3/4h$  or 500 mm whichever is the smaller.

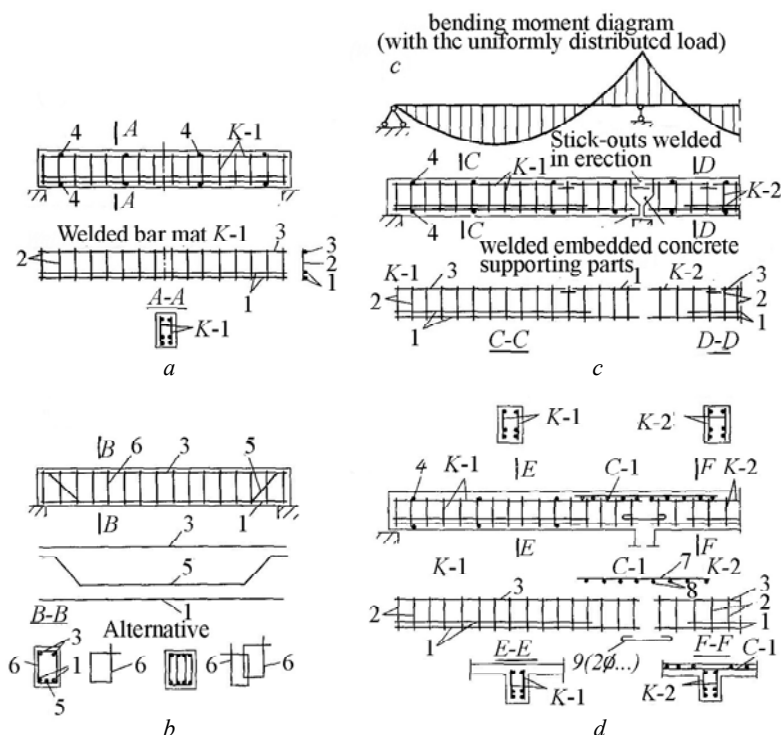


Fig. 9.5. Types of beam reinforcement:  
*a* — single-span beam with welded bar mats; *b* — single-span beam with tied reinforcement;  
*c* — precast multi-span rectangular beam;  
*d* — in-situ multi-span T-beam;  
 1 — longitudinal load-bearing bars; 2 — transverse bars in mats;  
 3 — longitudinal erection bars; 4 — transverse erection bars;  
 5 — bent load-bearing bars; 6 — stirrups in tied mats; 7 — load-bearing bars in support fabric; 8 — spacer bars in support fabric; 9 — erection bars (two bars with a diameter of not less than 10mm, not less than one half of the bottom erection bar diameter)

Transverse bars (stirrups) in beams (and ribs) more than 150 mm in depth are placed even if they are not required by the analysis; if the depth is less than 150 mm, transverse reinforcement may not be used.

In beams more than 700 mm in depth, the side faces are additionally reinforced by longitudinal bars spaced not more than 400 mm apart in

the vertical direction. The cross-sectional area of each bar should be at least 0,1 % of the area that they are intended to reinforce (the depth of this area is equal to a half-sum of the distances to the nearest bars, and the width is equal to a half of the member width, but maximum to 200 mm). These bars together with transverse reinforcement prevent inclined cracks on the side faces from opening.

The top beam faces are reinforced by longitudinal distribution bars 10 to 12 mm in diameter supplied during erection; they are necessary to combine all reinforcing members into a cage that would be stable during concreting, and to anchor the ends of the transverse bars. In precast beams, the distribution bars may serve as load-bearing (or main) bars during transportation and erection.

Instead of, or in addition to, transverse bars, use may be made of inclined bars. They are more effective than the former because they better coincide with the direction of the principal tensile stresses in a beam. Transverse bars, however, are preferable because they are more convenient to install.

Inclined bars are usually placed at  $45^\circ$  to longitudinal bars. In beams deeper than 800 mm, they may be placed at up to  $60^\circ$ , whereas in beams with a smaller depth and beams subjected to concentrated loads, the inclined bars are positioned at  $30^\circ$ .

When beams are reinforced with tied cages (made of class A-I and A-II steel), it is good practice to bend up some longitudinal load-bearing bars to reduce the steel consumption and improve the construction of the cage (Fig. 9.5, *b*). The minimum bend radius is  $10d$ . Bends should end in straight portions at least  $0.8l_{an}$  long and not less than  $20d$  in tension zones or  $10d$ , whichever is the greater, in compression zones. The straight portions of bends in plain bars should have hooks at their ends.

Precast multi-span beams are fabricated from separate single-span members containing welded reinforcing cages (Fig. 9.5, *c*). The position and length of load-bearing reinforcement in a cage are determined from the bending moment diagram plotted as for a multi-span beam. At the joints above the intermediate supports, the stick-outs of the top load-bearing bars are field-welded together by means of erection plates in a pool of molten metal contained within a reusable mould; the bottom bars are welded to support pads with the help of special support arrangements embedded into the precast members. After the welding, the joints are concreted.

In-situ-cast multi-span T-beams reinforced by welded bar mats in the spans (see Fig. 9.5, *d*), have welded fabric placed above the intermediate supports. The load-bearing bars of the fabric are placed along the span; they are intended to carry the forces appearing in the tension zone above the supports.

In prestressed members subjected to bending, the reinforcement is placed according to the diagrams of bending moments and shearing forces induced by the load. The reinforcement using curved prestressed tendons (Fig. 9.6, *a*) is most efficient because it follows the path of the principal tensile stresses best of all, but it is more complex than the one using straight tendons (Fig. 9.6, *b*). In addition to the steel of area  $A_{sp}$  in the tension zone, beams reinforced with straight prestressed tendons also use the steel of area  $A_{sp}'$  at the opposite face, whose area ranges between  $0,15 A_{sp}$  and  $0,25 A_{sp}$ . This reinforcement is useful in deep beams where the prestressing force applied outside the kern causes at the top face tension which may result in cracking during manufacture.

Beams with a small depth may do without prestressed steel at the top face. Here, cracking can be cancelled by non-prestressed erection (distribution) steel.

The best choices for beams in bending are I-beams (see Fig. 9.3, *c*) and T-beams with sufficiently wide ribs (see Fig. 9.3, *b*). The flanges in compression are proportioned to suit the compressive resultant of the internal couple induced by the bending moment under load. The tension zone should be sufficiently wide to accommodate the necessary reinforcement and resist the prestress in prestressed members.

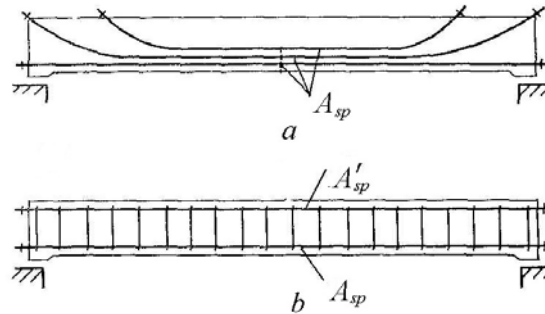


Fig. 9.6. Types of prestressed beam reinforcement:  
*a* — curved prestressed tendons; *b* — straight prestressed tendons

Prestressed steel is placed in tension zones as shown in Fig. 9.7. Here, the concrete cover and the spacing between the pretensioned bars and strands are in accordance with Fig. 9.4. In post tensioning, the concrete cover should not be less than 40mm, or less than the duct width. For the side faces, the cover should be not less than half the duct height. Prestressed steel placed in slots or outside the faces of a member should additionally be covered by at least 20 mm of concrete. The clear space between the ducts for post tensioned tendons should be not less than the duct diameter, or less than 50 mm.

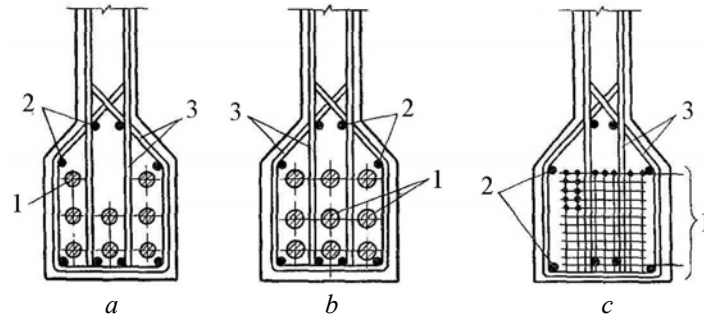


Fig. 9.7. Distribution of reinforcement in the tension zone of prestressed members:

- a* — deformed bars; *b* — strands or cables in ducts; *c* — high-strength wire;  
 1 — prestressed steel; 2 — longitudinal non-prestressed steel;  
 3 — transverse reinforcement

The maximum total angular change of curved post tensioned tendons between two points is taken as  $30^\circ$ , and the minimum radius of curvature is taken (to prevent large losses of prestress) as follows:

- wire 5 mm (and less) in diameter and strands 6 to 9 mm in diameter at least — 4 mm;
- wire 6 to 8 mm (and less) in diameter and strands 12 to 15 mm in diameter — 6 mm;
- bars up to 25 mm in diameter — 15 mm;
- bars 28 to 40 mm in diameter — 20 mm.

For prestressed members, special emphasis should be placed on the design of the reinforcement of beam-ends near supports. Here, considerable prestressing forces are transferred from the steel to tin-concrete via end anchorages (in post tensioning) or grip length of steel (in the absence of anchorages). Also, the eccentric action of the-prestressed steel results in local overstressing at the end of the member, which may cause cracking on its end and top surface. So, tin-ends of prestressed members should be reinforced additionally.

Prestressed members can locally be reinforced at anchorages and jacks by embedded parts attached to the prestressed tendons, by additional transverse bars, and also by enlarging the section area within the transmission length at the ends of a prestressed member. The concrete cover should be increased to at least  $2d$  for bars of class A-IV (A $\tau$ -IV) or lower and strands, and to at least  $3d$  for bars of class A-V (A $\tau$ -V) and higher (where  $d$  is the bar or strand diameter). Here, the concrete cover should be at least 40 mm for bars of all classes and 20 mm for strands. For the ends of a prestressed member with steel

supporting parts reliably anchored in the concrete, or additionally reinforced by transverse steel or reinforcement-acting fixtures embracing all longitudinal prestressed bars, the cover may be the same as in the rest of the beam.

If at the end of a member the prestressed steel is concentrated at the bottom and top faces, the end should be additionally reinforced by prestressed or non-prestressed transverse steel. The transverse steel should be tensioned prior to the longitudinal reinforcement, with the prestressing force being at least 15 % of that in the longitudinal steel in the tension zone at the support. The ends of non-prestressed transverse bars should be welded to embedded parts. The cross sectional area of these bars should be chosen to suit the acting force equal to at least 20% of the force in the longitudinal prestressed reinforcement (in the bottom zone at the support) determined by strength analysis.

The concrete at the ends of prestressed members containing steel with and without anchorages is reinforced by additional fabric or stirrups around all of the longitudinal bars (Fig. 9.8). The length of the zone in question,  $l$ , is taken equal to twice the anchorage length, or, in the absence of anchorages, it should be not less than  $0,6l_p$ , for less than 20 cm.

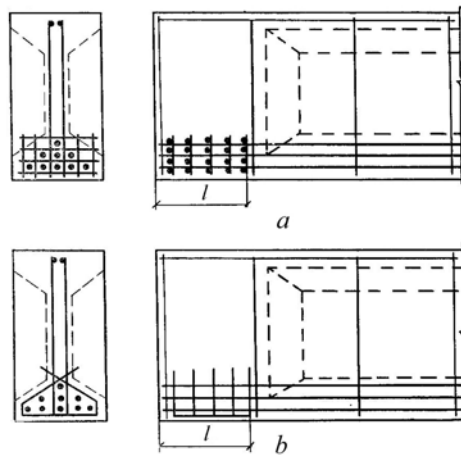


Fig. 9.8. Local reinforcement of prestressed beam ends:  
*a* — transverse welded fabric; *b* — stirrups or embracing welded fabric

At the ends of prestressed members reinforced without anchorages, cracking is not allowed under any load combinations (except special). During the transfer of the prestress from the steel to the concrete, such cracking is allowed in the prestress-tensioned zone of the section if it



contains no prestressed steel without anchors, the transmission length does not exceed  $2h_0$  (where  $h_0$  is the effective depth of the beam), and the web is additionally reinforced in the prestress-tensioned zone near the support by non-prestressed longitudinal bars placed over a length equal to at least  $2h_0$  starting from the beginning of the transmission zone; the cross-sectional area of this steel should be not less than 0.2% of that of the member at the support.

To ensure the necessary grip length for the longitudinal steel at the free (unfixed) ends of beams and slabs in bending, the bars should extend at least  $5d$  beyond the internal face of the support if the calculations show that no cracking is likely to occur at the support; otherwise, the bars should extend at least  $10d$ .

The necessary grip length at simply supported end is determined either by calculation or from the table. If the grip length is less than  $10d$ , it may be taken as given in the table, but not less than  $5d$ . In this case and also when the bar ends are welded to reliably anchored, embedded steel parts. The design strength of the steel at the support is not reduced.

## 9.2. Normal-section Strength Analysis of Rectangular and T-beams

Rectangular Beams with Tensile Steel Only. A non-prestressed rectangular beam with tensile steel only has the following geometry (Fig. 9.9)

$$A_{sc} = bx \text{ and } z_b = h_0 - 0,5x \quad (9.1)$$

where  $h_0$  and  $b$  are the depth and width of the cross section, respectively.

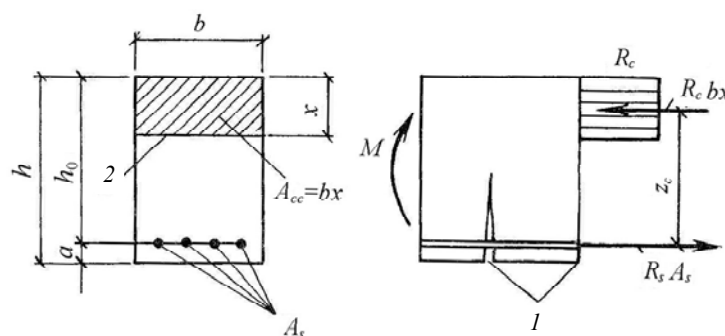


Fig. 9.9. Rectangular beam with tensile steel only and the loading system for the normal-section strength analysis:

1 — normal cracks; 2 — neutral axis

The depth of the concrete in compression,  $x$ , is determined on the basis of Eq. from the following expression

$$bxR_c = R_s A_s. \quad (9.2)$$

The condition for strength is as follows:

$$M \leq R_c bx(h_0 - 0,5x). \quad (9.3)$$

This may also be conveniently expressed in terms of the moments about the centroid of the compression zone

$$M \leq R_s A_s (h_0 - 0.5x). \quad (9.4)$$

Expressions (9.2) and (9.3) or (9.4) are used in combination. They are valid at  $x < \xi_R h_0$ . The reinforcement ratio

$$\mu = A_s / bh_0 \quad (9.5)$$

and the percentage of reinforcement,  $\mu \times 100$ , may, in view of Eq. (9.2) and  $\xi = x / h_0$ , be expressed as

$$\mu = \xi R_c / R_s; 100\mu = 100\xi R_c / R_s \quad (9.6)$$

Hence, we may find the maximum allowable steel contents at the rectangular section from the ultimate values of  $\xi_R$ . If  $x > \xi_R h_0$ , the bending moment is found by Eq. (9.3) or (9.4) at  $x = \xi_R h_0$ .

From the analysis of Eqs. (9.3) and (9.4) it follows that a member may be sufficiently strong with various combinations of sectional dimensions and steel area. Under real conditions, the cost of reinforced concrete members is close to optimal at

$$\begin{aligned} \mu &= 1 \text{ to } 2 \% \text{ and } \xi = 0.3 \text{ to } 0,4 \text{ for beams;} \\ \mu &= 0,3 \text{ to } 0,6 \% \text{ and } \xi = 0,1 \text{ to } 0,15 \text{ for slabs.} \end{aligned}$$

A section with specified  $b$ ,  $x$  and  $A_s$  (the materials and bending moment,  $M$ , are assumed to be known) is tested for strength in the following sequence: first, we find the depth of the compression zone,  $x$ , from Eq. (9.2), then, we test it to see if it satisfies condition  $x \leq \xi_R h_0$ , and finally use Eq. (9.3) or (9.4)

A section is considered to be chosen correctly if its load-bearing capacity expressed in terms of the moment is not more than 3 to 5 % above the specified design moment.

Sections are chosen according to the specified moment with the help of Eqs. (9.2) and (9.3) or (9.4) with their left-hand sides set equal to the right-hand sides.

In practice, rectangular beams with tensile steel only are designed with reference to an auxiliary table (Table 9.1). Formulas (9.3) and (9.4) are transformed as follows

$$M = \alpha_m b h_0^2 R_c, \quad (9.7)$$

$$A_s = M / \zeta h_0 R_s \text{ where} \quad (9.8)$$

$$\alpha_m = (x/h_0)(1 - 0,5x/h_0) = \xi (1 - 0,5\xi), \quad (9.9)$$

$$\zeta = z_c / h_0 = 1 - 0,5x/h_0 = 1 - 0,5\xi. \quad (9.10)$$

From Eq. (9.7), we find the effective depth of the section

$$h_0 = \sqrt{M / \alpha_m b R_c}. \quad (9.11)$$

The coefficients  $\alpha_m$  and  $\zeta$  derived from Eqs. (9.9) and (9.10) are presented in Table 9.1 which significantly cuts down the computation.

The dimensions  $b$  and  $h$  are chosen as follows: we assume the section width  $b$  and the recommended  $\xi$  for which we look up the coefficient  $\alpha_m$  in Table 9.1; then, using formula (9.11), we determine the effective depth  $h_0$  and the overall depth  $h = h_0 + a$ , and assign the respective standard dimension. If the values of  $b$  and  $h$  thus found do not meet the required conditions of design or manufacture, they are refined by repeated calculations.

The necessary steel area,  $A_s$ , is found as follows: first, deduce  $\alpha_m$  from Eq. (9.7); then get  $\zeta$  and  $\xi$  from Table 9.1 for the  $\alpha_m$  thus found; determine  $A_s$  by formula (9.8); and, finally, check to see  $H$  condition  $x \leq \xi_R h_0$  is satisfied.

Table 9.1 may also be used to check a member for strength. In this case, we calculate  $\mu = A_s / b h_0$  using the known data about the action, and also  $\xi$  from formula (9.6) checking it for compliance with condition  $x \leq \xi_R h_0$ . Then, using  $\xi$ , we find  $\alpha_m$  from Table 9.1, and calculate the maximum bearable bending moment by formula (9.7).

**Rectangular Members with Double Reinforcement.** In practice, we may come across members congested with tensile and compressive steel (Fig. 9.10), although the steel in the compression zone is less effective than that in the tension zone.

If in a member in bending the design provides for longitudinal steel (with  $R_{s,c} \leq 400$  MPa) in the zone which is in compression under load, the longitudinal bars should be prevented from buckling by transverse reinforcement.

Table 9.1

**Auxiliary Table for Design of Bending Rectangular  
Beams with Tensile Steel Only**

$\xi = x/h_0$	$\zeta = z_c / h_0$	$\alpha_m$	$\xi = x/h_0$	$\zeta = z_c / h_0$	$\alpha_m$
0,01	0,995	0,01	0,37	0,815	0,301
0,02	0,99	0,02	0,38	0,81	0,309
0,03	0,985	0,03	0,39	0,805	0,314
0,04	0,98	0,039	0,4	0,8	0,32
0,05	0,975	0,048	0,41	0,795	0,326
0,06	0,97	0,058	0,42	0,79	0,332
0,07	0,965	0,067	0,43	0,785	0,337
0,08	0,96	0,077	0,44	0,78	0,343
0,09	0,955	0,085	0,45	0,775	0,34d
0,1	0,95	0,095	0,46	0,77	0,354
0,11	0,945	0,104	0,47	0,765	0,359
0,12	0,94	0,113	0,48	0,76	0,3C5
0,13	0,935	0,121	0,49	0,755	0,37
0,14	0,93	0,13	0,5	0,75	0,375
0,15	0,925	0,139	0,51	0,745	0,38
0,16	0,92	0,147	0,52	0,74	0,385
0,17	0,915	0,155	0,53	0,735	0,39
0,18	0,91	0,164	0,54	0,73	0,394
0,19	0,905	0,172	0,55	0,725	0,399
0,2	0,9	0,18	0,56	0,72	0,403
0,21	0,895	0,188	0,57	0,715	0,408
0,22	0,89	0,196	0,58	0,71	0,412
0,23	0,885	0,203	0,59	0,705	0,416
0,24	0,88	0,211	0,6	0,7	0,42
0,25	0,875	0,219	0,61	0,695	0,424
0,26	0,87	0,226	0,62	0,69	0,428
0,27	0,865	0,236	0,63	0,685	0,432
0,28	0,86	0,241	0,64	0,68	0,435
0,29	0,855	0,248	0,65	0,675	0,439
0,3	0,85	0,255	0,66	0,67	0,442
0,31	0,845	0,262	0,67	0,665	0,446
0,32	0,84	0,269	0,68	0,66	0,449
0,33	0,835	0,275	0,69	0,655	0,452
0,34	0,83	0,282	0,7	0,65	0,455
0,35	0,825	0,289			
0,36	0,82	0,295			

The latter should be spaced not more than  $20d$  apart in welded bar mats and not more than  $15d$  in tied bar mats (where  $d$  is the least diameter of the longitudinal compressive bars), nor more than 500 mm.

Substituting  $A_c$  and  $z_c$  from Eq. (9.1) into the formula gives the condition for the strength of a bending non-prestressed rectangular member with tensile and compressive steel

$$M \leq R_c b x (h_0 - 0,5x) + R_{sc} A_s' (h_0 - a') \quad (9.12)$$

and substituting  $A_s$  into the formula gives an equation for the depth of the compression zone

$$R_c x b = R_s A_s - R_{sc} A_s' \quad (9.13)$$

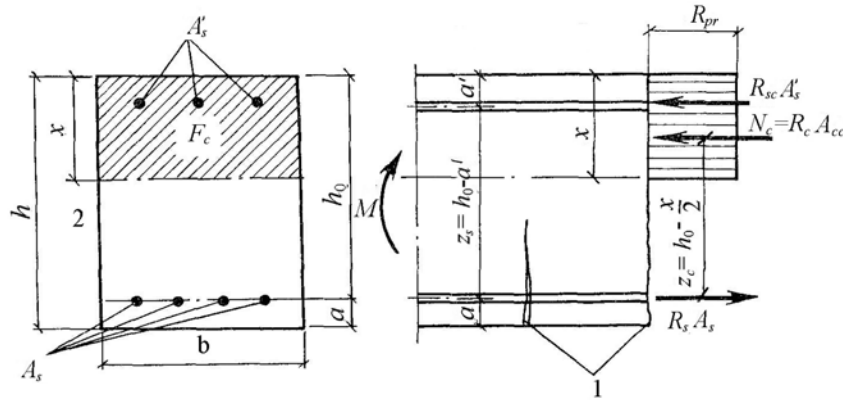


Fig. 9.10. Rectangular beam with double reinforcement and the loading system for the normal-section strength analysis:  
1 — normal cracks; 2 — neutral axis

Here,  $x$  should not exceed  $\xi_R h_0$ . If with tensile steel only,  $x > \xi_R h_0$  compressive steel is required by the design. In this case, the condition for strength is as follows

$$M \leq \alpha_R R_c b h_0^2 + R_{sc} A_s' (h_0 - a') \quad (9.14)$$

where  $\alpha_R = \alpha_0$  is taken from Table 9.1 for  $\xi = \xi_R$ .

In the design of members with tensile and compressive steel according to the specified moment, concrete brand and steel class two cases are possible.

**Case 1.** Given:  $b$  and  $h$ .

To find:  $A_s$  and  $A_s'$

*Solution.* Recalling Eq. (9.12), we find from condition (9.9) at  $x = \xi_R h_0$

$$A_s' = (M - \alpha_R R_c b h_0^2) / R_{sc} z_s; \quad (9.15)$$

and from Eq. (9.13)

$$A_s = A_s' R_{sc} / R_s + \xi_R R_c b h_0 / R_s. \quad (9.16)$$

**Case 2.** Given:  $b$ ,  $h$  and  $A_s'$

To find:  $A_s$

*Solution.* Recalling Eq. (9.12), we find from condition (9.9) that

$$\alpha_m = (M - R_{sc} A_s' z_s) / (R_c b h_0^2). \quad (9.17)$$

If  $\alpha_m \leq \alpha_R$  we find  $\xi$  from Table 9.1, and from Eq. (9.13) we get

$$A_s = A_s' R_{sc} / R_s + \xi R_c b h_0 / R_s. \quad (9.18)$$

If  $\alpha_m > \alpha_R$  the assumed  $A_s'$  is insufficient.

In the strength analysis of a section (with all parameters known), we calculate the depth of the compression zone from Eq. (9.13) and then check to see if condition (9.12) is satisfied.

Prestressed members with prestressed steel of area  $A_{sp}$ , and  $A_{sp}'$  are designed in a similar manner.

**T-Section Members.** These are used rather frequently in separate reinforced concrete T-beams (Fig. 9.11, *a* and *b*) and as parts of in-situ and precast panel floors (Fig. 9.11, *c* and *d*). A T-section consists of a flange and a rib.

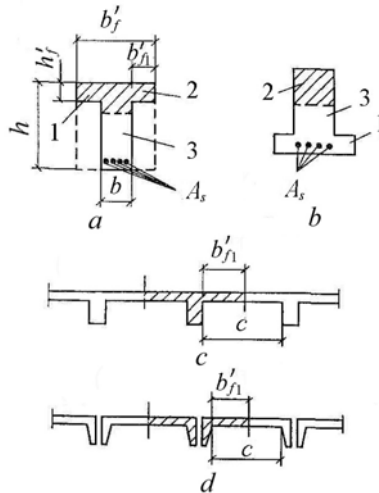


Fig. 9.11. T-sections:

*a* — beam with the flange in compression; *b* — beam with the flange in tension;  
*c* — T-section in an in-situ floor; *d* — T-section in a precast floor:

1 — flange; 2 — compression zone; 3 — rib



The design formulas for non-prestressed members are

$$R_c b_f' x = R_s A_s, \quad (9.19)$$

$$M \leq R_c b_f' (h_0 - 0,5x), \text{ or} \quad (9.20)$$

$$M \leq \alpha_m R_c b_f' h_0^2 \quad (9.21)$$

where  $\alpha_m$  is the coefficient taken from Table 9.1.

The latter case where  $x > h_f'$  covers members with narrow overhangs. Here, the compression zone of the section includes the compression zone of the rib and the overhangs.

The depth of the compression zone is found from the following equation

$$R_s A_s = R_c b x + R_c (b_f' - b) h_f'. \quad (9.22)$$

With the moments taken about the axis normal to the plane of flexure and passing through the point of application of the resultant force in the tensile steel, the condition for strength is as follows

$$M \leq R_c b x (h_0 - 0,5x) + R_c (b_f' - b) h_f' (h_0 - 0,5h_f') \quad (9.23)$$

For T-section members,  $x$  should not exceed  $\xi_R h_0$ .

Experience shows that the overall depth of a T-beam may approximately be determined from the following formula

$$h = (7 \text{ to } 9) \sqrt[3]{M} \quad (9.24)$$

where  $h$  is in centimeters and  $M$  is in kNm. The width of the rib is usually taken as

$$b = (0.4 \text{ to } 0.5)h \quad (9.25)$$

The flange dimensions  $b_f'$  and  $h_f'$  are most frequently assigned when proportioning a structure. The steel area  $A_s$ , necessary to fit the design moment is found according to the position of the neutral axis. If the neutral axis lies within the flange,  $A_s$  is determined from Table 9.1, assuming that the section is rectangular, with the width  $b$  and depth  $h$ , and has tensile steel only.

Which of the two cases is involved may be found as follows: if all of the section parameters including  $A_s$ , are known, then at

$$R_s A_s \leq R_c b_f' h_f', \quad (9.26)$$

the neutral axis lies within the flange; otherwise, it is located in the rib; if the dimensions  $b_f'$ ,  $h_f'$ ,  $b$  and  $h$  and the design bending moment are specified, but  $A_s$  is unknown, then at



$$M \leq R_c b_f' h_f' (h_0 - 0,5 h_f') \quad (9.27)$$

the neutral axis lies within the flange; otherwise, it crosses the rib.

Where the neutral axis lies below the flange, formulas (9.21) and (9.20) may be transformed, recalling that  $x = \xi h_0$  and taking into consideration Eq. (9.9), thus

$$R_s A_s = \xi R_c b h_0 + R_c (b_f' - b) h_f', \quad (9.28)$$

$$M \leq \alpha_m R_c b h_0^2 + R_c (b_f' - b) h_f' (h_0 - 0,5 h_f') \quad (9.29)$$

where the coefficients  $\xi$  and  $\alpha_m$  are taken from Table 9.1.

These expressions may be used to choose a section. If it is necessary to determine  $A_s$ , from Eq. (9.29) we calculate

$$\alpha_m = [M - R_c (b_f' - b) h_f' (h_0 - 0,5 h_f')] / R_c b h_0^2 \quad (9.30)$$

then we find  $\xi$  corresponding to the computed  $\alpha_m$  from Table 9.1, and determine the steel area from formula (9.28)

$$A_s = [\xi b h_0 + (b_f' - b) h_f'] R_c / R_s \quad (9.31)$$

If it is necessary to check a section for strength with all data known, which of the two cases applies is better determined from formula (9.27). Then (if the neutral axis lies below the flange), the depth or the compression zone is found from formula (9.22); further calculations are carried out according to Eq. (9.23).

#### ASSIGNMENTS TO CHAPTER 9

1. What is the limiting relative height of the compressed zone? How can we represent empirical dependence of limiting conditions in armature on relative height of the compressed zone in III stage?
2. What are preconditions of calculation of sections strength, normal to RCC axis – at a bend, eccentrically compressed and stretched?
3. How are conditions of the general way of calculation of section strength at various external influences (a bend, eccentric compression and stretching) written down?
4. What factors does the beginning of destruction of bending RCC depend on the stretched zone — in case 1, on the compressed zone in case 2?
5. What is the factor of operating conditions for high-strength armature  $\gamma_{s6}$ , equal to? What does its physical essence consist in?
6. How is stress in non-stressed armature from conditional border of fluidity determined at mixed reinforcing of a bending element?

7. Specify the modern method of RCC calculation accepted in normative documents.
8. Name stages of tense-deformed conditions of RCC at a bend.
9. What is the limiting condition of designs?
10. Name the groups of limiting conditions.
11. Name the basic advantage of a design calculation by limiting conditions.
12. How is loading divide according to time of action?
13. What loadings are reoffered to constant loadings?
14. What loadings are considered time loadings?
15. What is the normative and calculated loading?
16. What is the normative and calculated resistance of concrete?
17. What is the normative and calculated resistance of armature?
18. Speak on the advantages of RCC with previous stress?
19. Which ways of previous stress of reinforced concrete constructions exist?
20. What is the value of the previous tension?
21. What is the expense of the previous tension?
22. Name two cases of destruction of elements which are bent on normal sections?
23. What use scheme of local strengthening by armature of final sites preliminary — stressed beams?
24. How is stressed armature in cross-section of the stretched zone of pre-stressed beams placed?
25. How are constructive requirements for distance between collars in the longitudinal direction on support and span sites of bending elements established?
26. How should we write down durability conditions of the normal section of a bending element any structure with stressed and not stressed armature (cases 1 and 2)?
27. How should we write down durability conditions of the normal section of elements of a rectangular structure with single armature (cases 1,2)? What is the sequence of calculation for selection of bending elements sections a rectangular structure with the help of tables (2 types of tasks)?
28. What is the sequence of calculation for determining the bearing capacity of bending elements of a rectangular structure with single armature at the set sizes of sections and the area of armature?
29. What conditions determine the necessity of installation of compressed armature?
30. What conditions provide bending elements durability in a rectangular structure with double armature (2 types of tasks)?
31. What conditions provide durability of bending elements of a T-section structure?
32. How can we determine the bottom border position of the compressed zone? What requirements are established for introduction into calculation the durability of overhang width of the compressed shelf of elements in a T-section structure?

33. What requirements are established for introducing into calculation the durability of overhang width of the compressed shelf of elements of a T-section structure?
34. In what case the element appears to be subject to a slanting bend? How can we write down conditions of durability?
35. How can we determine the sizes of the compressed zone at a slanting bend of trapezoid form?
36. How are the place of theoretical breakage of longitudinal armature and the length of a core clipping determined?
37. Name two basic conditions of balance for an element that is bent, of rectangular section with single armature under critical condition.
38. Name two basic conditions of balance for an element that is bent, of rectangular section with single armature under critical condition.
39. Name the basic conditions of balance for an element which is bent, of rectangular section with double armature under critical condition.
40. Speak on the features of I and T sections calculation.
41. How can we construct the material diagram?

## 10. INCLINED-SECTION SHEAR STRENGTH ANALYSIS

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### 10.1. Basic Design Formulas

Bending moments and shearing forces applied simultaneously to members in bending give rise to inclined cracking which forces the members to fail. Consider the internal forces in the steel and the compression zone appearing at a crack. Fig. 10.1 shows a part of a reinforced concrete member at a support, containing longitudinal, transverse and diagonal reinforcement. This part is separated from the member by a section coinciding with the inclined crack.

The shear strength of a member should satisfy two conditions with regard to  $M$  and  $Q$  existing in the portion in question. In the design force diagram (Fig. 10.1), it is assumed that the member is subjected to the moment and the shearing force computed for design loads, and the stresses in the steel and the concrete are equal to their design strengths. Thus, the conditions for strength are as follows here  $A_s$ ,  $A_{sw}$  and  $A_{s,inc}$  are the areas of the longitudinal reinforcement, transverse reinforcement (stirrups) and diagonal reinforcement (bent bars), respectively;  $Q_c$  is the shearing force resisted by the concrete in the compression zone at the inclined section;  $\theta$  is the angle between the bent bars and the longitudinal axis of the member;  $z_s$ ,  $z_{s,inc}$  and  $z_{sw}$  are the lever arms for the forces in the longitudinal transverse and diagonal bars; and  $M_D$  and  $Q_D$  are the design bending moment and shearing force determined for point  $D$ .

$$M_D \leq R_s A_s z_s + \Sigma R_s A_{s,inc} z_{s,inc} + \Sigma R_s A_{sw} z_{sw}, \quad (10.1)$$

$$Q_D \leq \Sigma R_s A_{s,inc} \sin\theta + \Sigma R_s A_{sw} + Q_c \quad (10.2)$$

Equation (10.1) gives the condition for strength with regard to the bending moment. The shear strength of a member is considered adequate if the bending moment,  $M_D$ , due to all the forces applied to the part in question does not exceed the sum of the moments due to the internal design forces in the longitudinal, transverse and diagonal bars, taken about the same fulcrum.

Equation (10.2) gives the condition for the shear strength of a member. The shear strength of a member is considered adequate if the shearing force,  $Q_D$ , due to all the forces applied to the part in question

does not exceed the sum of the projections of the internal design forces in the transverse and diagonal bars crossed by the inclined section onto the axis normal to the longitudinal axis of the member and the shearing force carried by the concrete of the compression zone.

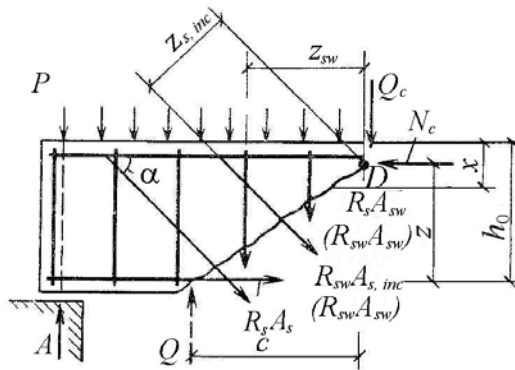


Fig. 10.1. Design loading diagram at an inclined section (quantities in the brackets give the forces in the transverse and diagonal bars, taken for the analysis of the inclined section for resistance to the applied bending moment)

Some transverse and diagonal bars located at the inclined crack closer to the compression zone cannot act at their full strength because the crack is narrow there. As a consequence, the forces in the transverse and diagonal bars in Eq. (10.2) are determined for the design strength  $R_{sw}$  which is somewhat smaller than  $R_s$ . In Eq. (10.1) the design strength is not reduced because the moments of the forces in the transverse and diagonal bars located close to the fulcrum are small and do not significantly affect the final results.

The design shearing force carried by the concrete in the compression zone is given by the following empirical formula

$$Q_c = \varphi_{b2}(1 + \varphi_f + \varphi_n)R_{ct}bh_0^2/c_0 \quad (10.3)$$

where  $B = \varphi_{b2}(1 + \varphi_f + \varphi_n)R_{ct}bh_0^2$ ;  $\varphi_{b2}$  is an experimental coefficient, taken as 2 for heavy concrete, 1.9 for concrete using coarse porous aggregate, and 1.7 for concrete using fine and coarse porous aggregate;  $h_0$  is the effective depth of the member;  $b$  is the width of a rectangular member, or rib or web width of a T- or I-beam; and  $c$  is the projection of the inclined section onto the axis of the member.

The condition described by Eq. (10.1) is usually met without calculations by adequate proportioning which will be discussed later. The condition for shear strength generally requires an appropriate computation.

Practical recommendations require that the maximum shearing force for rectangular, T- and other similar sections should be

$$Q_{\text{emin}} = \varphi_{b3}(1 + \varphi_f + \varphi_n)R_{\text{ct}}bh_0. \quad (10.4)$$

If above condition is met, the web concrete between inclined cracks will protect the wall concrete strength from the inclined compressive stress.

For prestressed members, which, in the general case, contain prestressed longitudinal, transverse and diagonal reinforcement, Eqs (10.1) and (10.2) should be extended to include the respective terms.

If no inclined cracking is expected to occur in a member (according to the design calculations), no shear strength analysis is carried out. The required strength is given by the following empirical formula

$$Q \leq 2,5R_{\text{ct}}bh_0. \quad (10.5)$$

## 10.2. Transverse Bar Design

Let us examine a member in bending containing transverse reinforcement without bent bars, which is the most commonly used type of reinforcement. Of all the possible inclined sections originating from point *B* (Fig. 10.2), we shall consider for design purposes the section with the least load-bearing capacity. We shall take into account that

$$\left. \begin{array}{l} Q_D = Q - pc_0 \\ \sum R_{\text{sw}}A_{\text{sw}} = q_{\text{sw}}c_0 \end{array} \right\} \quad (10.6)$$

where  $Q$  is the shearing force at the beginning of the inclined section (Fig. 10.2), and  $q_{\text{sw}}$  is the force carried by the transverse bars per unit length of the member. Substituting Eqs. (10.3) and (10.6) into Eq. (10.2) gives

$$Q \leq (q_{\text{sw}} + p)c_0 + B/c_0. \quad (10.7)$$

The least load-bearing capacity of the inclined member will obviously be given by

$$dQ/dc_0 = (q_{\text{sw}} + p)c_0 - B/c_0^2 = 0. \quad (10.8)$$

Hence the projection of the design inclined section is

$$c = \sqrt{B/(q_{\text{sw}} + p)} \sqrt{\varphi_{b2}(1 + \varphi_f + \varphi_n)R_{\text{ct}}bh_0^2/(q_{\text{sw}}p)} \quad (10.9)$$

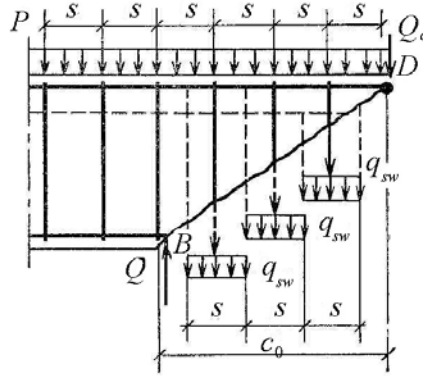


Fig. 10.2. Forces in transverse bars, taken for the inclined section analysis of a beam

Substituting this value into Eq. (10.7) gives the condition for the shear strength in terms of the least load-bearing capacity of the inclined section

$$Q \leq 2\sqrt{B(q_{sw} + p)} \quad (10.10)$$

Substituting the value of  $B$  given by formula (10.3) into the above expression gives the shearing force carried by the stirrups and the concrete at the design inclined section

$$Q_{sw,c} = 2\sqrt{\varphi_{b2}(1 + \varphi_f + \varphi_n)R_{ct}bh_0^2(q_{sw} + p)}. \quad (10.11)$$

Referring to the diagram of Fig. 10.2, we may write

$$q_{sw}s = R_{sw}A_{sw}; \quad A_{sw} = f_{sw}n \quad (10.12)$$

where  $s$  is the spacing between the transverse bars (stirrups);  $f_{sw}$  is the cross-sectional area of one transverse bar (stirrup);  $n$  is the number of transverse bars within the section of the member.

In the design calculations, the diameter of transverse bars and their number within the cross section of the member are usually specified in advance, so  $f_{sw}n = A_{sw}$  is regarded as a term known beforehand.

An idea about the required amount of reinforcement may be formed

$$q_{sw} = Q^2 / 4\varphi_{b2}(1 + \varphi_f + \varphi_n)bh_0^2R_{ct} \text{ or } q_{sw} \geq R_{ct}b/2 \quad (10.13)$$

whichever is the greater. This value of  $q_{tr}$  should be matched by the force in stirrups per unit length of the member

$$q_{sw} = R_{sw}A_{sw}/s. \quad (10.14)$$

It should be kept in mind that the spacing between the transverse bars ought not to exceed a value for which an inclined section would lie anywhere between two adjacent bars where the member owes its strength solely to the strength of the concrete in the compression zone. In this case, the condition  $Q \leq Q_c$  holds. The design shearing force resisted by the concrete of the compression zone (see Eq. (10.3)) is multiplied by the coefficient 0.75 which takes care of possible departure of cracks from their design direction due to the non uniformity of the concrete, and also of possible inaccuracy in the position of the stirrups. Then, the spacing between the transverse bars should not exceed

$$s_{max} = 0,75\varphi_{b2}(1 + \varphi_f + \varphi_n)bh_0^2R_{ct}/Q. \quad (10.15)$$

When determining the spacing between the transverse bars, we should also provide for some constructional features.

### 10.3. Bent Bars

Nowadays, bent bars are used rather seldom and then mostly to strengthen separate parts of a beam subjected to large shearing forces. The locations for bends (places where some of the longitudinal reinforcement is carried from the tension to the compression zone) are determined by the normal- and inclined-section analysis (Fig. 10.3; 10.4). Bends are provided where  $Q > Q_{sw,c}$ .

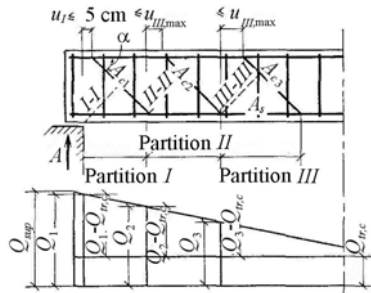


Fig. 10.3. Bent bar design

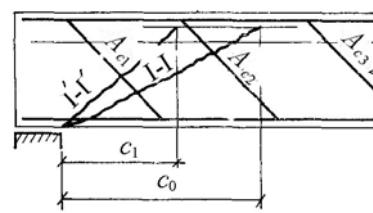


Fig. 10.4. Bent bar design

According to Eq. (10.2), the condition for the shear strength with allowance for bends may be written as

$$Q \leq R_{sw}A_{s,inc}\sin\alpha + Q_{sw,c}. \quad (10.16)$$



Hence, the cross-sectional area of the bends is

$$A_{s,inc} = (Q - Q_{sw,c})/R_{sw}\sin\alpha \quad (10.17)$$

The trial section for each portion (see Fig. 10.3) is that whose origin occurs at the maximum shearing force – this is the section at the face of the support and also where a bend starts. With uniform transverse reinforcement, the design ordinates of shearing forces for the bends are deduced by subtracting  $Q_{sw,c}$  from  $Q$ . The cross-section of areas of the bent bars for portions *I*, *II* and *III*, are determined by the analysis of inclined sections *I-I*, *II-II* and *III-III*.

#### 10.4. Constructional Features Ensuring Bending Moment Strength for Inclined Sections

As regards the applied bending moment, the load-bearing capacity of an inclined section should not be less than that of the normal section passing through the same point *D* (Fig. 10.5). Given certain constructional features (to be discussed later), this requirement can be met, and the analysis of inclined sections for bending moment may be omitted.

If the longitudinal reinforcement at a free support is properly anchored, so that the longitudinal steel acts at full strength in the span, the conditions for the member to develop the necessary strength at any inclined section beginning at the support face are guaranteed.

Without anchorage, the strength of the longitudinal reinforcement at the support as found by Eq. (10.1) is reduced.

If the longitudinal bars are insufficiently anchored for them to act at full strength at the section in question, they may be additionally reinforced in the anchorage zone: equipped with bearing plates or embedded parts welded to their ends, or have bent anchoring bars. In this case, the grip length of the bars should be not less than  $10d$ .

Beams are most often reinforced without bent bars. If all of the tensile longitudinal steel designed on the basis of the normal section carrying the maximum bending moment is extended as far as the support and properly anchored. Any inclined section will resist the applied bending moment well, owing to the longitudinal reinforcement alone, without the transverse steel. Under the circumstances, the analysis of the inclined section for bending moment is unnecessary.

To reduce steel consumption, some longitudinal steel (not more than 50 % of the design area) may be cut off in the span at a point where it is no longer required by the normal-section strength analysis.

These bars should be extended beyond their theoretical cutoff point determined from the bending moment diagram (section *I-I* in Fig. 10.5.) for a length  $w$  where (to guarantee the required resistance to bending moments) the transverse steel compensates for missing longitudinal bars at inclined sections (section *III-III* in Fig. 10.5, *a*).

As has been shown by experiments and design practice, if the length  $w$  is to meet the above considerations and anchorage conditions of cutoff bars, it must be taken equal to the greater of the two following values

$$\left. \begin{aligned} w &= (Q - Q_c) / 2q_{sw} + 5d \\ w &= 20d \end{aligned} \right\} \quad (10.18)$$

Here,  $Q$  is the design shearing force at the theoretical cutoff point (section *I-I* in Fig. 10.5), corresponding to the load at which this point is determined;  $Q_c$  is the shearing force carried by the diagonal bars at the theoretical cutoff point, if the member contains bent bars in addition to transverse steel;  $q_{sw}$  is the force per unit beam length, carried by the transverse bars, found from the bending moment at an inclined section (section *III-III* in Fig. 10.5, *a*); and  $d$  is the diameter of the cutoff bar.

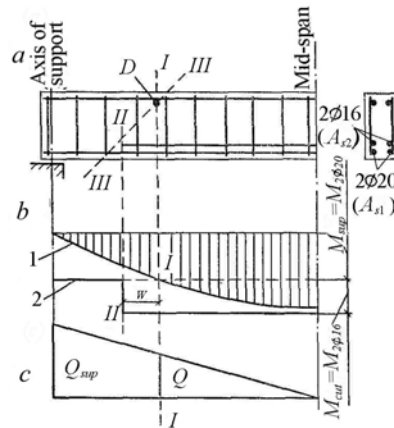


Fig. 10.5. Location of the cutoff point in the span:  
*a* — reinforcement scheme; *b* — moment diagram; *c* — shearing force diagram;  
 I-I — theoretical cutoff point for two bars 16mm in diameter;  
 II-II — actual cutoff point of these bars; *l* — distribution of the moments due to the load; *2* — distribution of the moments resisted by the normal sections of the member (that is known as the material diagram)

The values of  $Q_c$  and  $q_{sw}$  are determined from the following expressions

$$Q_c = R_s A_c \sin \alpha, \quad (10.19)$$

$$q_{sw} = R_s A_s / s. \quad (10.20)$$

If there are no bent bars in the zone where the longitudinal steel is cut off,  $Q_c = 0$  in the first line of Esq. (10.18). Fig. 10.6 illustrates the positioning of the cutoff bars in the beam span.

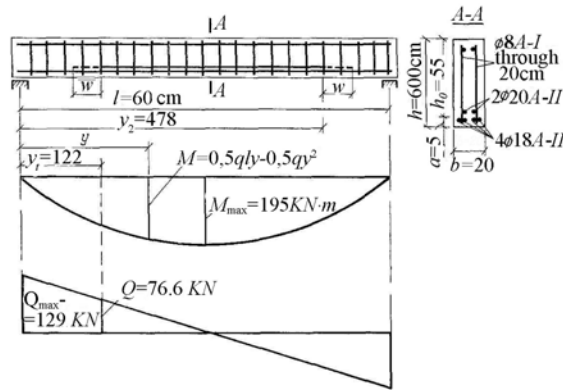


Fig. 10.6. Positioning of the cutoff bars in the beam span

The moment diagram plotted for the external design loads shows the ordinates of the moment resisted by the normal section of the reinforced concrete member containing the steel which is extended as far as the support without cutting off (in Fig. 10.5,  $A_s^{\text{sup}}$  is for two bars 20 mm in diameter,  $M_{\text{sup}} = M_{2\phi 20}$ ) This ordinate is found as

$$M_{\text{sup}} = R_s A_s^{\text{sup}} z_c \quad (10.21)$$

The intersections between the ordinate  $M_{\text{sup}}$  and the design moment diagram define the theoretical cutoff points,  $I-I$ . The real cutoff points,  $II-II$ , lie within the distance  $w$  of the theoretical cutoff points. The shearing force diagram shows the ordinate  $Q$ , which is included in formula (10.18) to determine  $w$ .

#### ASSIGNMENTS TO CHAPTER 10

1. What causes the formation of advanced cracks on support sites of bending elements? What schemes of destruction on an advanced section do you know?
2. What is the condition of durability of an element of inclined section for the action of the cross-section force, bending moment?
3. What are technology requirements to the least diameter of cross-section cores (collars) and constructive requirements to the greatest size between them?

4. What is cross-section force  $Q_{sw}$  perceived by collars in the calculated inclined section equal to?
5. What does the cross-section force  $Q_b$  perceived by concrete of the compressed zone above inclined section depend on?
6. How can we write down conditions of durability on inclined section in beams with cross-section armature? Without cross-section armature?
7. How can we carry out check of action of cross-section force on the inclined compressed strip?
8. What constructive requirements provide durability of inclined section on the bending moment?
9. In what cases is the calculation check on inclined sections necessary?
10. When is there a necessity for establishment of cross-section armature?
11. Specify calculation formulas of durability on inclined section to the action of transverse forces.
12. Specify calculation formulas of durability of an element on inclined section to the action of moment forces.

## 11. MEMBERS IN COMPRESSION

### 11.1. Constructional Features

Members in axial compression are assumed to include intermediate columns in buildings and other structures, top chords of trusses loaded at their joints, uprising diagonal and vertical web members (Fig. 11.1), and so on. In reality, however, the imperfect shape of structural members, departure of their actual dimensions from those specified by design, no uniform inner structure of concrete, and some other factors prevent members from working in pure axial compression; instead, there exists what may be called eccentric compression with accidental eccentricities.

As a rule, members intended to resist axial compression are quadrangular or rectangular in cross-section. The cross-sectional dimensions of columns are determined by computation; for the standardization of formwork and reinforcing cages, they are assigned as multiples of 50mm. For better concreting, incite columns with a side of less than 25cm are not recommended.

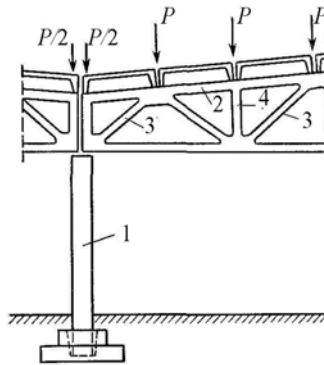


Fig. 11.1. Members in axial compression:

- 1 — intermediate columns (equally loaded from either side);
- 2 — top chord of a truss (with the load applied at the joints);
- 3 — uprising diagonals; 4 — verticals

Eccentric compression is induced in columns in one-storey industrial buildings subjected to crane load (Fig. 11.2, *a*), top chords in trusses

without diagonal web members (Fig. 11.2, *b*), and walls of underground tanks rectangular in plan, which carry the lateral pressure of ground or liquid and the vertical pressure of the roof (Fig. 11.2, *c*). In this case, members are subjected to the compressive force,  $N$ , and the bending moment,  $M$ .

The distance between the point of application of a direct load to a member and its longitudinal axis is termed the eccentricity, designated  $e_0$ . In the general case, the eccentricity at any point of a member of statically determinate structures is determined as

$$e_0 = M/N + e_a \quad (11.1)$$

where  $e_a$  is the accidental eccentricity. For members of statically indeterminate structures  $e_0 = M/N$ , but it should be not less than  $e_a$ .

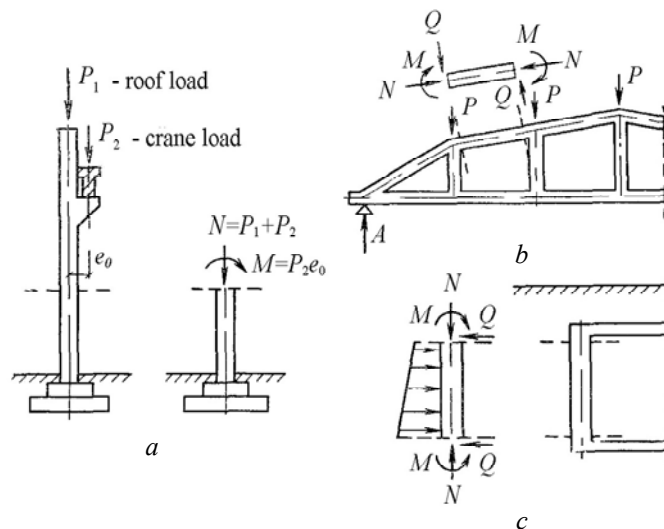


Fig. 11.2. Members in eccentric compression:  
*a* — column of an industrial building; *b* — top chord of a truss without diagonals; *c* — wall of a buried tank

It is advisable to proportion eccentrically loaded members so that their largest dimensions lie in the plane of the applied bending moment. They may be rectangular in cross-section or have an I- or T-section.

The concrete brand for members in compression should be at least B15 and at least B25 for heavily loaded members. Columns are reinforced by longitudinal bars 12 to 40mm in diameter (load-bearing reinforcement) mostly of class A-III hot-rolled steel and transverse bars of class A-I hot-rolled steel or cold-drawn low-carbon wire (Fig. 11.3). The longitudinal

and transverse reinforcement is combined into bar mats or reinforcing cages which may be either welded or tied (Fig. 11.4).

The amount of longitudinal steel in the cross-section of members in axial compression is expressed in terms of the reinforcement ratio or the percentage of reinforcement

$$\mu = (A_s/A)100 \% \quad (11.2)$$

where  $A_s$  is the total longitudinal steel area, and  $A$  is the cross-sectional area of the member. In practice, the percentage of reinforcement for members in axial compression does not exceed 3 %.

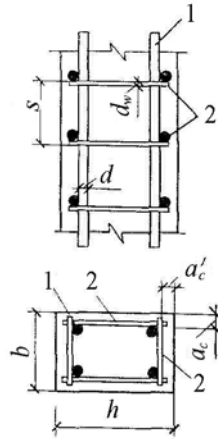


Fig. 11.3. Reinforcement of compressed members:  
 1 — longitudinal bars;  
 2 — transverse bars;  $a_w$  — concrete cover for transverse reinforcement;  
 $a$  — concrete cover for transverse reinforcement

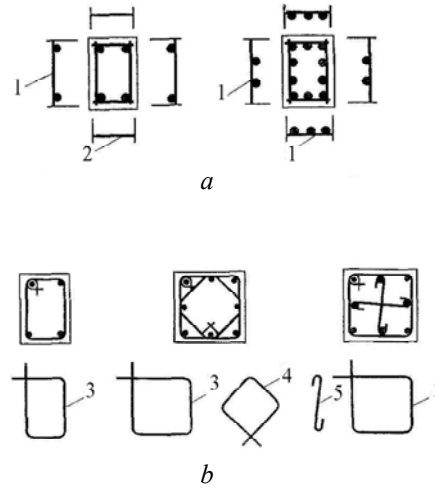


Fig. 11.4. Reinforcement of members in axial compression:  
 a — welded-bar mats; b — tied mats;  
 1 — welded bar mats;  
 2 — erection bars; 3 — stirrups;  
 4 — additional stirrups; 5 — hooks

In compression members with design eccentricities longitudinal bars are distributed near the short sides of the cross-section as shown in Fig. 11.5:  $A_s$  with the cross-sectional area  $A_s$  is located near the face most distant from the compressive force, and  $A_s$  with the cross-sectional area  $A_s$  is located at the face closest to the compressive force. The amount of steel in eccentrically compressed members is expressed in terms of the longitudinal reinforcement ratio at one of the short sides. In practice, the steel area for eccentrically loaded columns ranges between 0,5 and 1,2 % of the cross-sectional area of the member. If the cross-

sectional areas of  $A_s$  and  $A'_s$  are not the same we speak of asymmetrical reinforcement; symmetrical reinforcement is preferable.

According to relevant specifications, the minimum  $A_s$  and  $A'_s$  for member in compression is as follows:

- 0,05 for members with  $l_0/i < 17$ ;
- 0,1 for members with  $l_0/i < 35$ ;
- 0,2 for members with  $l_0/i < 83$ ;
- 0,25 for members with  $l_0/i > 83$ .

Here  $i$  is the radius of gyration of the cross-section in the plane of the longitudinal force eccentricity, and  $l_0$  is the calculated length of the compressed member.

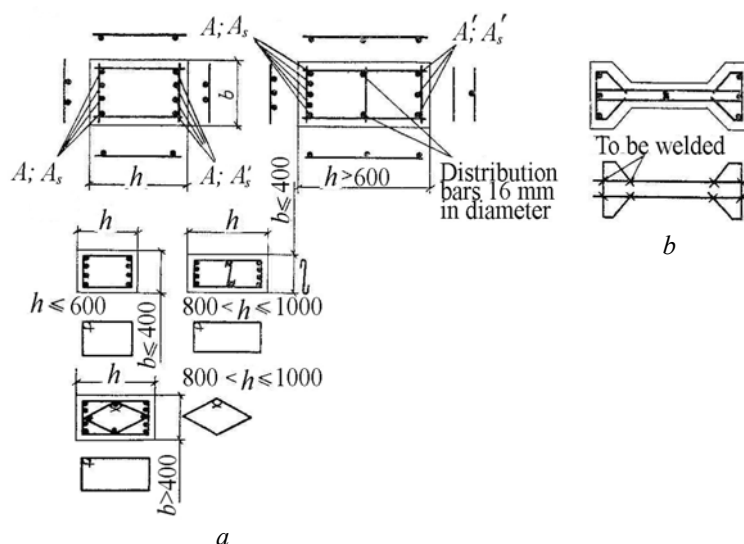


Fig. 11.5. Reinforcement of member's compression:  
*a* — welded-bar mats; *b* — tied mats

In the cross-section of a column, the load-bearing bars should be placed as close to the surface as possible, that is with a minimum concrete cover,  $a_l$  which, according to specifications, should be not less than the bar diameter or 20mm (see Fig. 11.3) whichever is the greater.

Columns up to 40×40 cm in cross-section may be reinforced by four longitudinal bars (see Fig. 11.4). In which case, the clear space-between the load-bearing bars amounts to the maximum allowed value. The minimum clear space between these bars is 50 mm if the bars are vertical during concreting and 25 mm for bottom and 30 mm for top bars if they are horizontal during concreting. In any case, the clear space should be not less than the maximum bar diameter. If load-bearing bars



are spaced wider than 400 mm apart intermediate bars should be distributed round the perimeter so that the clear space between the longitudinal bars will not exceed 400 mm.

Transverse bars are placed without design calculations, but subject to appropriate specifications. The maximum space between these bars at which the longitudinal bars would not buckle when loaded  $s$  (see Fig. 11.3), is  $20d$  for welded reinforcing cages and  $15d$  (but not more than 500 mm) for tied cages (where  $d$  is the least diameter of the longitudinal compression bars). The value of  $s$  is rounded to a multiple of 50 mm.

The transverse bar diameter,  $d_w$  in welded reinforcing cages should meet the requirements for weld ability. The stirrup diameter for tied cages should be not less than 5 mm nor less than  $0,25d$ , where  $d$  is the maximum longitudinal bar diameter.

The concrete cover for transverse steel,  $a_w$ , should be at least 15 mm. It is not recommended to use spliced longitudinal bars within the length of a member.

At splices in reinforcing mats, the space between transverse bars within the overhang should not exceed  $10d$  (where  $d$  is the diameter of the bars being spliced).

If the total percentage of reinforcement exceeds 3 %, transverse bars should be placed not wider than  $10d$  or 300 mm apart.

Welded bar mats are combined into reinforcing cages by transverse bars spot-welded to the edge bars of the mats (Fig. 11.5, *a*). If the welded bar mats located near the larger faces of a member contain intermediate bars, the bars (belonging to the opposite mats) are welded together, using hooks disposed along the length of the member with spacing equal to that between the transverse bars in the mats.

The longitudinal bars of tied reinforcing cages are held in place by stirrups placed at every other stirrup bend, as a minimum. If the face of a member is not wider than 400 mm, and the number of longitudinal bars located at this face does not exceed four, all the longitudinal bars may be embraced by a single stirrup (see Fig. 11.5, *b*). Prestress is used for eccentrically compressed members where the compressive force is applied with a large eccentricity and large bending moments induce tension in some part of the cross-section. It is also used in very slender members. In the former case prestress improves the crack resistance and stiffness of a member in service: in the latter case it is used to improve the above properties during manufacture, transportation and erection.

It does not pay to use very slender members in axial compression because their load-bearing capacity is considerably reduced by poor stress-

strain behavior. In any case, the slenderness of heavy-concrete and porous-aggregate-concrete members in all directions should be  $\lambda = l_0/i \leq 200$ , and that of columns in buildings should be  $\lambda = l_0/i \leq 120$ .

### **11.2. Design of Members of an Arbitrary Symmetrical Cross-Section, Eccentrically Compressed in the Plane of Symmetry**

When loaded to the limit of load-bearing capacity, members of an arbitrary cross-section subjected to eccentrical compression in the plane of symmetry may fail in any one of two ways during Stage III.

**Case 1.** This applies to eccentrically loaded members to which a longitudinal force is applied at a relatively large eccentricity. The stress-strain state (as well as the failure of a member) is close in nature to that of conservatively reinforced members in bending (Fig. 11.6, *a*). The part of the section most distant from the point at which the force is applied is in tension and contains cracks normal to the longitudinal axis of the member. The tensile force in this zone is carried by the reinforcing steel. The part of the section closer to the compressive force is in compression together with its reinforcing steel. The member begins to fail as soon as the stress in the tensile steel reaches the yield point (or proof yield stress). The failure is complete when the stresses in the concrete and steel of the compression zone reach their ultimate strengths, with the stress in the tensile steel remaining constant, if the latter has a definite yield point or rising if it has no definite yield point. The failure is gradual in nature.

**Case 2.** This refers to eccentrically loaded members to which a compressive force is applied at a relatively small eccentricity. This case covers two types of the stress-strain state, namely with all of the cross-section in compression (Fig. 11.6, *b*, diagram 1 shown with the dashed line), or with most of the section closer to the longitudinal force in compression and the opposite part exposed to a relatively weak tension (Fig. 11.6, *b*, diagram 2). The member fails as the stress in it exceeds the ultimate strength of the concrete and steel in the part of the section closer to the applied force. As this takes place, the stress (compressive or tensile) in the distant part of the section remains low, and the material does not act at its full strength.

In the plane of the bending moment, eccentrically compressed members are designed with allowance for the design eccentricity of longitudinal forces and accidental eccentricity,  $e_a$  (Eq. 11.1).

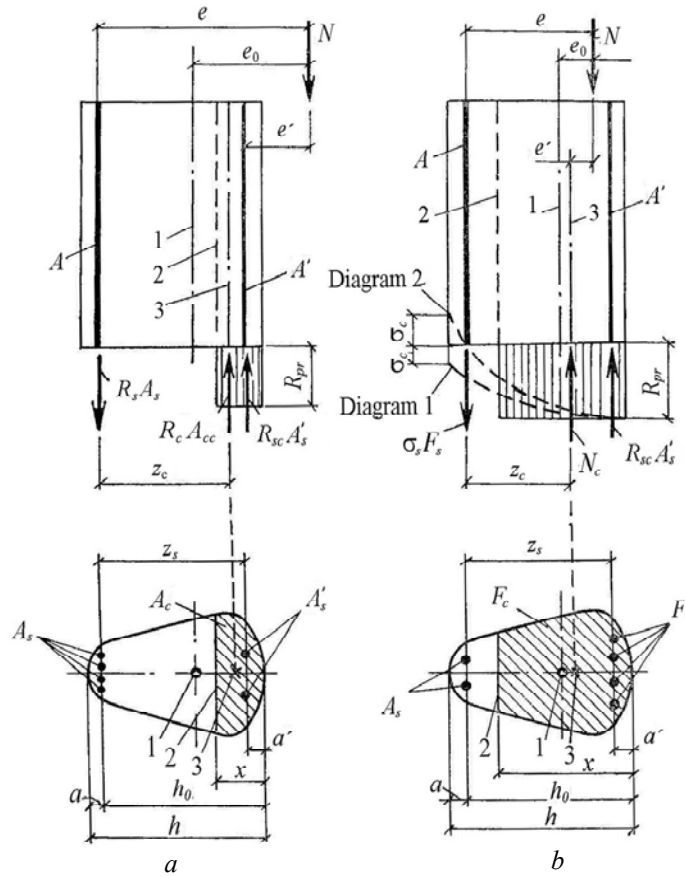


Fig. 11.6. Disposition of loads members in eccentric compression:

*a* — at  $\xi = x/h_0 \leq \xi_R$ ; *b* — at  $\xi = x/h_0 > \xi_R$ ;

1 — geometrical axis of the member in the design loading system;

2 — neutral axis; 3 — centroid of the concrete in the compression zone;

$A_s$  — steel more distant from the compressive force;

$A'_s$  — steel closer to the compressive force

In the plane normal to the plane of flexure, members are checked to see if they resist the longitudinal force applied only at accidental eccentricity  $e_a$ .

Fig. 11.6 shows the loading diagrams for the strength analysis of members having an arbitrary cross-section, subjected to compression with an eccentricity in the plane of symmetry in Cases 1 and 2. For members falling in Case 1, the design strength of the concrete in the compression zone is taken to be constant and equal to  $R_c$ , and that in the

tensile and compressive steel is taken equal to  $R_s$  and  $R_{sc}$ , respectively. For members falling in Case 2, the actual compressive stress diagram shown by the dashed line in Fig. 11.6,b is replaced by a rectangular diagram with the ordinate equal to  $R_c$  and the design strength of the compressive steel  $S'$  of cross-section area  $A_{S'}$  is taken to be  $R_{s,com}$ . The stress in the steel  $S$  of cross-sectional area  $A_S$  is below the design value. The loading diagram of Fig. 11.6, *a* is valid when  $\xi = x/h_0 < \xi_R$ , and that of Fig. 11.6, *b*, when  $\xi = x/h_0 > \xi_R$  (where  $\xi_R$  is the ultimate relative depth of the compression zone). When  $\xi = x/h_0 < \xi_R$  (Fig. 11.6, *a*), the position of the neutral axis is located by equating the design longitudinal force due to the external design loads,  $N$ , with the sum of the projections of all the internal design forces in the steel and the concrete in compression on the longitudinal axis of the member

$$N = R_c A_{cc} + R_{sc} A_{S'} - R_s A_s. \quad (11.3)$$

The load-bearing capacity of a member is said to be adequate if the bending moment,  $M = Ne$ , due to the external design loads does not exceed the sum of the moments due to the above internal forces about the axis normal to the plane of the bending moment and passing through the point at which the tensile resultant is applied to the steel  $A$

$$Ne \leq R_c A_{cc} z_c + R_{sc} A_{S'} z_s. \quad (11.4)$$

In Eq. (11.4) 
$$Z_s = h_0 - a'. \quad (11.5)$$

In Fig. 11.6, *a*,  $e$  and  $e'$  are the distances between the longitudinal force,  $N$ , and the centroids of the tensile steel,  $A_s$ , and compressive steel,  $A_{S'}$  respectively. When  $\xi = x/h_0 > \xi_R$  (Fig. 11.6, *b*), the strength of compression members is calculated according to Eq. (11.4), and the depth of the compression zone is determined from the following equation

$$N = R_c A_{cc} + R_{sc} A_{S'} - \sigma_s A_s. \quad (11.6)$$

Here, the stress in the less stressed reinforcing steel,  $\sigma_s$  is specified according to the materials used. For example, for members made of B30 (or lower class number) concrete and using class A-I, A-II or A-III non prestressed steel,  $\sigma_s$  is determined from the following relation

$$\sigma_s = [2(1-x/h_0)/(1-\xi_R) - 1]R_s \quad (11.7)$$

in which the stress in the steel is linearly related to  $\xi$  in the range from  $\xi = x/h_0 = 1$  to  $\xi = \xi_R$ , which has been proved experimentally. For members made of B30 (and higher class number) concrete and using

class A-III prestressed or non-prestressed steel,  $\sigma_s$  in MPa is found from the empirical expression

$$\sigma_s = \sigma_{sp} + \sigma_{scu}(\omega/\xi - 1)(1 - \omega/1,1). \quad (11.8)$$

Under the action of the bending moment, a slender member buckles, thereby increasing the initial eccentricity,  $e_0$ , of the longitudinal force,  $N$  (Fig. 11.7). As this takes place, the bending moment rises, too, and the member fails at a lower longitudinal force than does a short (stub) column.

If the slenderness ratio is  $l_0/r > 14$ , slender members subjected to eccentric compression may be designed by the above expressions, but taking into consideration the increased eccentricity which is obtained by multiplying the initial eccentricity  $e_0$ , by the coefficient  $\eta > 1$ . The value of this coefficient is found from the following relation

$$\eta = 1/(1 - N/N_{cr}). \quad (11.9)$$

Here,

$$N_{cr} = (6,4E_c/l_0^2)(I/\varphi_l[0,11/(0,1 + \delta_e/\varphi_p) + 0,1] + \alpha I_c). \quad (11.10)$$

Equation (11.10) takes care of the distinctions of reinforced concrete, namely presence of reinforcing steel in the cross-section, inelastic properties of concrete in compression, cracks in the tension zone and the effect of the long-time loading on the stiffness of a member in the limit state.

In Eq. (11.10)  $E_c$  is the tangent, modulus of elasticity of concrete;  $l_0$  is the effective length of the member;  $I$  is the moment of inertia of the concrete section; and  $I_s$  is the transformed moment of inertia of the steel about the centroid of the concrete section.

The coefficients  $\varphi_l$  (given the effect which the long-time loading has on the buckling of a member in the limit state) and  $\varphi_p$  (given the effect that the prestress in the steel has on the stiffness of a member in the limit state; the steel is assumed to be uniformly prestressed) are found from the following empirical relations

$$\varphi_l = 1 + \beta M_l/M, \quad (11.11)$$

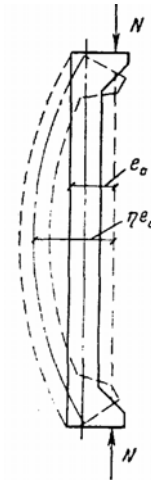


Fig. 11.7. Eccentricity due to buckling

$$\varphi_p = 1 + 12(\sigma_{cp} / R_c)(e_0/h). \quad (11.12)$$

In the general case,  $M$  and  $M_1$  in Eq. (11.11) are the moments about the axis parallel to the neutral axis and passing through the centre of the most tensioned or least compressed reinforcing bar (with the section completely in compression); they are due to the combination of all loads and the dead and long-time live loads, respectively; the coefficient  $\beta$  is taken from Table 11.1.

At  $l_0/r_u < 14$ ,  $\eta$  is taken equal to unity.

If  $N$  turns out to exceed  $N_{cr}$  the cross-sectional dimensions should be increased.

The transverse reinforcement in members subjected to eccentric compression is designed to resist shearing forces by the formulas for members in bending.

Table 11.1

**Coefficient  $\beta$  in Eq. (11.11)**

Concrete	$\beta$
Heavy	1
Manufactured light-aggregate: Ceram site, agloporite, slag pumice, with quartz sand Same with porous sand	1 1,5
Solid porous	2,5
Natural porous-aggregate: Tuff, pumice, lava slag, shell rock limestone (irrespective of sand) Fine-grained, group A B C	2,5 1,3 1,5 1,0

### 11.3. Design of Eccentrically Loaded Rectangular Members

For the rectangular section of Fig. 11.8

$$\left. \begin{aligned} A_{cc} &= bx, & Zc &= h_0 - x/2 \\ N_c &= R_c bx \end{aligned} \right\}. \quad (11.13)$$

Taking into consideration the above expressions, we may rewrite Eq. (11.4) for the load-bearing capacity as follows

$$Ne \leq R_c bx(h_0 - x/2) + R_{sc} A_s'(h_0 - a'). \quad (11.14)$$

The depth of the compression zone is given by any one of the following equations:

$$\text{a) at } \xi = x/h_0 \leq \xi_R \quad N = R_c b x + R_{sc} A_s' - R_s A_s, \quad (11.15)$$

$$\text{b) at } \xi = x/h_0 > \xi_R \quad N = R_c b x + R_{sc} A_s' - \sigma_s A_s \quad (11.16)$$

where  $\sigma_s$  is determined by Eq. (11.7) or (11.8), depending on the materials used.

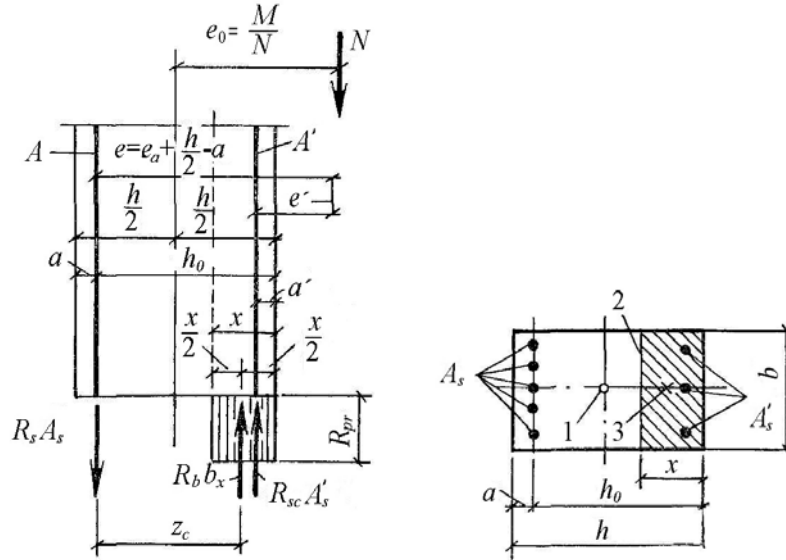


Fig. 11.8. Design of rectangular members in eccentric compression:  
 1 — geometrical axis of the member; 2 — neutral axis;  
 3 — centroid of the concrete in the compression zone

### Load-Bearing Capacity Check

In checking a member for load-bearing capacity, when its characteristics are known, the depth of the compression zone is found from Eq. (11.15) on the assumption that  $\xi = x/h_0 \leq \xi_R$

$$x = (N - R_{sc} A_s' + R_s A_s) / R_c b. \quad (11.17)$$

Then,  $\xi_R$  is determined. After that, the condition  $x \leq \xi_R h_0$  is checked; if it is met, the member is checked for load-bearing capacity by Eq. (11.14) for the found value of  $x$ .

If the condition  $x/h_0 \leq \xi_R$  is not met  $x$  should be determined by Eq. (11.16) at  $\xi = x/h_0 > \xi_R$ .

When  $x > \xi_R h_0$  and use is made of concrete with a class number B30 and class A-I, A-II or A-III non-prestressed steel  $\sigma_s$  found by Eq. (11.8) should be substituted into Eq. (11.16) whence  $x$  is determined.

The value of  $x$  found from the above expression should be substituted into Eq. (11.14) to check the member for load-bearing capacity.

If  $x > \xi_R h_0$ , and use is made of concrete with a class number of higher than B30 and class A-IV and higher non-prestressed reinforcing steel  $\sigma_s$  found by Eq. (11.8) should be substituted into Eq. (11.14) to check the member for loading-bearing capacity.

On finding  $\xi$  we may calculate  $x = \xi_R h_0$  which is finally used in Eq. (11.14) to check the member for load-bearing capacity.

### Steel Area Calculation

When calculating the necessary steel area  $A_s$  and  $A_s'$  ( $N$ ,  $l_0$ ,  $b$  and  $h$  are assumed to be known), the design formulas should be transformed as follows.

The condition governing steel area is  $\xi = x/h_0 \leq \xi_R$ . It is obvious that the steel  $A_s'$  will be required by design only if the relative depth of the compression zone is calculated with only the tensile steel. Present exceeds the limiting value of  $\xi_R$ . Using this depth of the compression zone and the respective value of  $\alpha_m$  we obtain on the basis of Eqs. (11.14) and (11.15)

$$A_s' = (Ne - \alpha_m R_c b h_0^2) / R_{sc} z_s, \quad (11.18)$$

$$A_s = (\xi_R R_c b h_0 - N) / R_s + A_s' R_{sc} / R_s. \quad (11.19)$$

The steel area  $A_s'$  should not be less than the minimum value.

When the steel area  $A_s'$  is specified in advance (from constructional or any other considerations, for example, in the presence of moments of both signs), Eq. (11.14) yields

$$x(h_0 - x/2) = [Ne - R_{sc} A_s' (h_0 - a')] / R_c b. \quad (11.20)$$

All terms on the right-hand side of this expression are known. On the other hand we find that

$$\alpha_m = \xi(1 - \xi/2) \text{ where } \xi = x/h_0, \quad (11.21)$$

is known, so

$$\alpha_m = [Ne - R_{sc} A_s' (h_0 - a')] / R_c b h_0^2. \quad (11.22)$$

With  $\alpha_m$  known, we can calculate it from the following expression

$$\xi = 1 - (1 - 2/\alpha_m)^{1/2}. \quad (11.23)$$



Thus, knowing  $x = \xi h_0$ , we can deduce the necessary steel area from Eq. (11.15) as

$$A_s = (\xi R_c b h_0 - N) / R_s + A_s' R_{sc} / R_s. \quad (11.24)$$

In practice, members, especially those subjected to bending moments differing in sign but close in value, are often reinforced symmetrically.

With symmetrical reinforcement where  $A_s = A_s'$  and  $R_{sc} = R_s$ , that is,  $R_{sc} A_s' = R_s A_s$ , we may get from Eq. (11.15)

$$x = N / R_c b \quad (11.25)$$

then, using this  $x$ , we get from Eq. (11.14)

$$A_s = A_s' = N(e - h_0 + N/2R_c b) / R_{sc}(h_0 - a'). \quad (11.26)$$

The condition governing steel area is  $\xi = x/h_0 > \xi_R$ .

#### 11.4. Design of T- and I-section members

T- and I-section members subjected to eccentric compression are often used in arches, columns, and similar applications.

In T-section members, the flange is usually placed near the face which carries the greater compressive force (Fig. 11.9, *a*). Here, two design cases are of interest, namely with the neutral axis lying within the flange, and with the neutral axis crossing the rib. In the former case, the section is regarded as rectangular with the width  $b_f'$  in the latter, the compression in the rib is taken into account. The flange width used in the design calculations is assumed to be the same as in bending.

If the flange of a T-section is located near the tensile (or less compressed) face, it is not taken into consideration, and the section is regarded as rectangular, having the design width equal to the width of the rib.

I-section members are treated as T-sections with the flange in the compression zone, since the flange in the tension zone does not contribute to the strength of the member (Fig. 11.9, *b*).

Eccentrically loaded T-section members with the flange in the compression zone, as well as any other members with a symmetrical cross-section, are treated, according to the following condition

$$\xi = x/h_0 \leq \xi_R. \quad (11.27)$$

At first, it is necessary to locate the position of the neutral axis. If the condition

$$N > R_c b_f' h_f' \quad (11.28)$$

is met, the neutral axis lies below the flange.

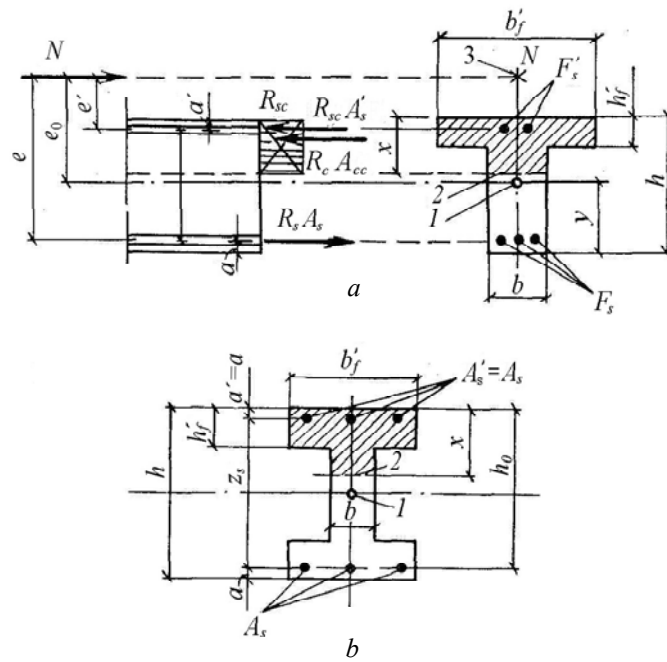


Fig. 11.9. Design of T- and I — section members:  
*a* — T-section member with the flange in the compression zone;  
*b* — symmetrical I-section member; *l* — geometrical axis of the member;  
 2 — neutral axis; 3 — longitudinal compressive force

If  $x > h_f'$  the strength analysis of the section is carried out on the basis of the following condition

$$Ne < R_c bx(h_0 - 0,5x) + R_c(b_f' - b)h_f'(h_0 - 0,5h_f') + R_{sc}A_s'(h_0 - a'). \quad (11.29)$$

The depth of the compression zone is found from the following expressions

a) at  $\xi = x/h_0 \leq \xi_R$ ,

$$N = R_c bx + R_c(b_f' - b)h_f' + R_{sc}A_s' - R_s A_s; \quad (11.30)$$

b) at  $\xi = x/h_0 > \xi_R$ ,

$$N = R_c bx + R_c(b_f' - b)h_f' + R_{sc}A_s' - \sigma_s A_s, \quad (11.31)$$

where  $\sigma_s$  is determined from Eq. (11.7) or Eq. (11.8), depending on the materials used. The same expressions hold for symmetrically reinforced I-section members (Fig. 11.9, *b*). In the design expressions, the distance  $e$  (Fig. 11.9, *a*) is determined as

$$e = \eta e_0 + y - a, \quad (11.32)$$

where  $y$  is the distance from the centroid of the section to the tensile face of the rib.

#### ASSIGNMENTS TO CHAPTER 11

1. What are the main methods of reinforced concrete structures calculation?
2. Explain the stressed-deformed state of reinforced concrete beam in bending.
3. What is the structure limit state? Name the group of calculation as to the limit state.
4. What is the standard and design loading materials resistance?
5. Comment upon the idea of reliability factor (coefficient) as to material and soil structure working conditions.
6. Speak on the advantages of structures with prestressed reinforcement and field of their application.
7. What methods of making prestressed reinforcement in reinforced concrete structures do you know?
8. What are the peculiarities of T- and I-cross sections calculations?
9. In what cases is check as to inclined cross-section necessary?
10. How is the diagram of materials plotted?
11. Name two cases of eccentrically stressed reinforced concrete elements destroyal, boundary between them.
12. How does the scheme of reinforcing a reinforced concrete column look?
13. What are collars used for? How are diameter and a step of collars selected?
14. How are casual eccentricities of longitudinal compressing force  $e_a$  determined?
15. How is the influence of deflection in calculation of flexible non-central compressed elements taken into account?
16. How are conditions of strength of elements of any symmetric section at non-central compression for case 1 at  $\xi \xi_R$ , for a case 2 at  $\xi \xi_R$  written down?
17. What two cases of destruction occur in stage III of non-central compressed elements?
18. How can we write down conditions of strength of elements of rectangular section at non-central compression?
19. How can we receive the formula for selection of armature for elements of rectangular section at non-central compression and  $A_s = A_{s'}$ ?
20. What is the sequence of selection of armature for elements of rectangular section at non-central compression and  $A_s = A_{s'}$ ?

21. What is the scheme of strengthening columns with transverse reinforcing like?
22. What does transverse reinforcing consist in?
23. What is the sequence of strength calculation of non-central compressed elements strengthened by grid or spiral armatures?
24. What elements of RCC are referred to non-central compressed ones?
25. How is the eccentricity determined?
26. Name two cases of destruction of non-central compressed elements, a border between them.
27. How are influence of flexibility of non-central compressed elements and duration of loading action taken into account?
28. What is the indirect reinforcing of compressed elements?

## 12. CRACK RESISTANCE OF REINFORCED CONCRETE MEMBERS

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As already defined, the crack resistance of members refers to their ability to resist incipient cracking in Stage I or opening of cracks in Stage II of the stress-strain state. Members are checked for crack resistance at sections normal to their longitudinal axis, and also at inclined sections if they are subjected to shearing forces. Analysis for crack resistance and deflection belongs to analysis on the basis of the second group of limit states.

The design calculations are carried out on the following assumptions: (1) the stress in the concrete of the tension zone prior to cracking is equal to  $R_{ct,ser}$ ; (2) the stress in the prestressed steel is equal to  $\sigma_p = \sigma_{sp} + 2\alpha R_{ct,ser}$ , that is, to the sum of the prestress (with allowance for losses and multiplied by the tensioning accuracy factor) and the increment in the stress caused by the increment in the strain of the surrounding concrete after all of the prestress has been cancelled; (3) the stress in the non-prestressed steel of prestressed members is equal to the sum of the compressive stress due to shrinkage and creep in the concrete, and the increment in the tensile stress due to the increment in the strain of the concrete.

### 12.1. Incipient-Cracking Resistance Analysis

The analysis is based on the assumption that no cracks form at sections normal to the longitudinal axis of a member if the longitudinal force,  $N$ , induced by the external loading does not exceed the longitudinal cracking force,  $N_{cr}$  that is

$$N \leq N_{cr}. \quad (12.1)$$

The longitudinal force,  $N_{cr}$ , is found from the stresses existing in the material immediately before cracking

$$N_{cr} = R_{ct,ser}(A + 2\alpha A_s) + P \quad (12.2)$$

where  $A$  is the cross-sectional area of the member;  $A_s$  is the total cross-sectional area of the prestressed and non-prestressed steel; and  $P$  is the prestressing force.

When determining  $N_{cr}$  for non-prestressed members,  $P$  should be taken as  $-\sigma_s A_s$ . The stress in the non-prestressed steel,  $\sigma_s$  induced by

creep and shrinkage in the concrete reduces the incipient-cracking resistance of the member.

## 12.2. Incipient-cracking Resistance of Members in Bending, Eccentric Compression and Eccentric Tension

### Normal Cracking Analysis

This is based on the assumption that no cracks will form in sections normal to the longitudinal axis of a member if the external moment,  $M$ , does not exceed the internal or cracking moment, that is,

$$M \leq M_{\text{crc}}. \quad (12.3)$$

In bending, the external moment is determined in the regular way; whereas in eccentric compression and eccentric tension with some of the cross section in compression, it is

$$M = Nc_1 \quad (12.4)$$

where  $c_1$  is the distance from the external longitudinal force,  $N$ , to the same axis about which the moment due to the internal forces is taken (Fig. 12.1)

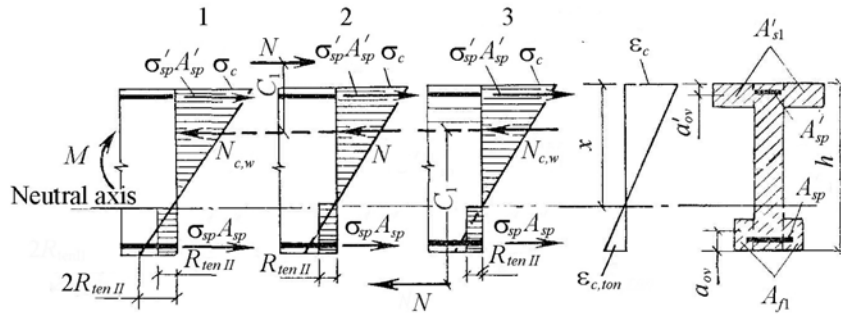


Fig. 12.1. Determination of the cracking resistance of prestressed members in: 1 — bending, 2 — eccentric compression and 3 — eccentric tension in terms of Stage I with the concrete of the compression zone behaving elastically

### 12.3. Determining $M_{\text{crc}}$ by the Kern Moment Method

Relevant standards recommend determining  $M_{\text{crc}}$  approximately by what is known as the kern moment method. The stress-strain state of a section in Stage I before cracking due to the composite action

of external loads and prestressing force may be approximated to linear eccentric compression, on the assumption that all forces act independently. Then, the cracking moment is determined as

$$M_{crc} = R_{cb,ser}W_{pl} + M_{rp} \quad (12.5)$$

where  $M_{rp}$  is the moment due to the prestressing force,  $N_0$ , taken about the axis passing through the kern point most distant from the tension zone, that is,

$$M_{rp} = P(e_{op} + r), \quad (12.6)$$

$W_{pl}$  is the elastic-plastic moment of resistance of the tension zone in the reinforced concrete section with zero longitudinal force;  $e_{op}$  is the eccentricity of the prestressing force,  $P$ , with respect to the centroid of the transformed section;  $r$  is the distance from the kern point farthest from the tension zone to the centroid of the transformed section (Fig. 12.2).

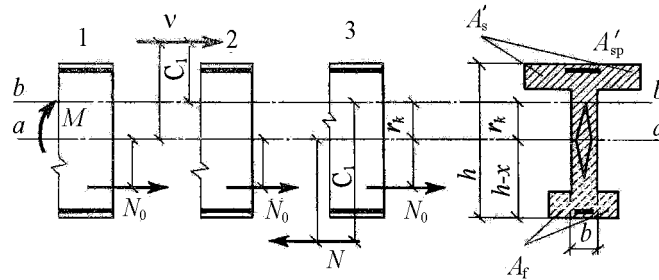


Fig. 12.2. Cracking-resistance analysis of members in:  
1 — bending, 2 — eccentric compression and 3 — eccentric tension  
by the kern moment method,  $a$  — centroid of the transformed section;  
 $b$  — boundary of the kern

To allow for inelastic strain in the concrete of the compression zone, the value of  $r$  is found as follows:  $N \leq P$

– for prestressed members in bending and eccentric compression

$$r = P(W_{red}/A_{red}); \quad (12.7)$$

– for members in eccentric tension

$$0,7 \leq P = 1,6 - (\sigma_c/R_{c,ser}) \leq 1; \quad (12.8)$$

– provided that  $N > P$

$$r = W_{pl}/[A_{red} + \alpha(A_s + A_s')]; \quad (12.9)$$

– for non-prestressed members in bending

$$r = W_{red}/A_{red}. \quad (12.10)$$

If the condition (12.9) is not met, where  $W_{\text{red}}$  is the elastic moment  $W$  of resistance of the tension zone in the transformed section;  $A_{\text{red}}$  is the cross-sectional area of the transformed section;  $e_0$  is the eccentricity of the longitudinal force,  $N$ , with respect to the centroid of the transformed section. The value of  $W_{\text{pl}}$  may be determined by the following formula

$$W_{\text{pl}} = [2(I_{\text{co}} + \alpha I_{\text{so}} + \alpha I_{\text{so}'})](h - x) + S_{\text{ct}}. \quad (12.11)$$

Here  $I_{\text{co}}$ ,  $I_{\text{so}}$ ,  $I_{\text{so}'}$  are the moments of inertia of the concrete in compression and the steel in compression and tension about the neutral axis, respectively;  $S_{\text{ct}}$  is the static moment of the concrete in tension about the same axis;  $(h - x)$  is the distance from the neutral axis to the tensile face. The position of the neutral axis is located from the following condition

$$S_{\text{co}} + \alpha S_{\text{so}'} - \alpha S_{\text{so}} = [(h - x)A_{\text{ct}}] / 2, \quad (12.12)$$

where  $S_{\text{co}}$ ,  $S_{\text{so}}$  and  $S_{\text{so}'}$  are the static moments of the concrete in compression and the steel in compression and tension about the neutral axis, respectively;  $A_{\text{ct}}$  is the cross-sectional area of the concrete in tension. The value of  $W_{\text{pl}}$  may also be determined in terms of the elastic moment of resistance,  $W_{\text{red}}$  using the following formula

$$W_{\text{pl}} = \gamma W_{\text{red}}. \quad (12.13)$$

Here, the factor  $\gamma$  takes care of inelastic strain in the concrete of the tension zone according to the shape of the cross-section. For rectangular sections and  $T$ -sections with their flanges in the compression zone,  $\gamma = 1,75$ ; for other cross-sections the value of  $\gamma = 1,5$ .

The external moment about the axis passing through an assumed kern point is determined by Eq. (12.4). In the case of eccentric compression, the lever arm is  $c_1 = e_0 - r$ . Then,

$$M = M_r = N(e_0 - r) \quad (12.14)$$

For eccentric tension,

$$M = M_r = N(e_0 + r) \quad (12.15)$$

In manufacture and erection a zone designed to work in compression under external loads may happen to be actually in tension. In such a case

$$M_{\text{crc}} = R_{\text{ct,ser}} W_{\text{pl}} - P(e_{0p} - r) \quad (12.16)$$

Here, the values of  $W_{\text{pl}}$  and  $r_k$  are taken for the face subjected to tension due to the prestressing force,  $P$ ; and  $R_{\text{ct,ser}}$  is determined depen-



ding on the transfer strength of concrete,  $R_{cr}$ . In these calculations, the external moment is found for the loads acting in a particular stage (for example, the member self-weight).

#### 12.4. Crack-opening Resistance. General Width of Normal Cracks

After the onset of cracking in the tension zones of reinforced concrete members, further loading causes the cracks to open. This is Stage II of the stress-strain state. Experiments show that in tension the spacing between cracks may be larger or smaller than the average value by about 50 % due to the non-uniform inner structure of concrete.

The width of cracks normal to the longitudinal axis of a member is determined as the difference between the elongations in the steel and tensile concrete over the spacing  $l_{cr}$  between the cracks

$$a_{cr} = \varepsilon_{sm} l_{cr} - \varepsilon_{cm} l_{cr}. \quad (12.17)$$

The average strain of the tensile concrete,  $\varepsilon_{cm}$  is much smaller than the average strain of the tensile steel so it is usually ignored. As a result

$$a_{cr} = \varepsilon_{sm} l_{cr} \quad (12.18)$$

The crack width depends on  $\psi_s$  (which, in turn, depends on the bond between the steel and the concrete), the stress in the steel at the section containing the crack,  $\sigma_s$ , and the crack spacing,  $l_{cr}$ . The values of the above factors are found by calculation.

According to relevant standards, the width of cracks normal to the longitudinal axis of a member should be determined at the level of the tensile steel axis by the following empirical formula

$$a_{cr} = 20(3,5 - 100\mu)\delta\eta\varphi_1(\sigma_s / E_s)^3 \sqrt{d} \quad (12.19)$$

where  $\mu = A_s / bh_0$  is the reinforcement ratio of the section (the rib of a T-section) taken as not more than 0,02 in the design calculations;  $A_s$  is the cross-sectional area of the tensile steel;  $\delta$  is taken as 1 for members in bending and eccentric compression, and 1.2 for members in eccentric tension;  $\eta$  is taken as 1 for deformed bars, 1.2 for class Bp-1, Bp-II wire and strands, 1,3 for hot-rolled plain bars, and 1,4 for class B-II wire;  $\varphi_1$  is the factor taking care of the duration of loading and assumed as 1 for short-time loading and 1,5 for long-time loading;  $\sigma_s$  is the stress or its increment in tensile steel section containing a crack after canceling compression;  $d$  is the tensile steel bar diameter in mm; if the bars differ in diameter, use should be made of an average bar diameter.

The short-time crack width in members the crack resistance of which should meet the requirements of Category Two is determined for the short-time action of dead, long-time and short-time loads with  $\varphi_c=1$ . For members the crack resistance of which should be in compliance with the requirements of Category Three, it is determined, using a nonlinear relationship, as the sum of the increment in the crack width ( $a_{crc1} - a_{crc1}$ ) caused by a short-time increase in load up to the total load value, and the crack width ( $a_{crc2}$ ) due to dead and long-time live loads

$$a_{crc} = a_{crc1} - a_{crc1} + a_{crc2}. \quad (12.20)$$

### 12.5. Crack Closure

In prestressed members designed to meet the requirements of Category Two, normal and inclined cracks should safely close after the load has been reduced to a certain value. This is important because long-time opening of cracks is most dangerous for reinforcing steel as it may affect its corrosion resistance. If the total loading, constituted by short- and long-time loads, results in cracking, then, after it has been reduced to the long-time load only, the cracks will not close unless the steel behaves elastically and no permanent set takes place.

Normal cracks will safely close if:

$$\sigma_{sp} + \sigma_s \leq 0,8 R_{s,ser} \quad (12.21)$$

where  $\sigma_{sp}$  is the prestress in the steel with allowance for all losses;  $\sigma_s$  is the increment in the stress in the steel due to the external loads; under dead and long-time live loads, the tensile face of the member in a section containing a crack in the tension zone remains in compression at the normal stress  $\sigma_c \geq 0,5\text{MPa}$ .

For an elastic transformed section, the compressive stress,  $\sigma_b$ , is determined with allowance for the external loads and the prestressing force,  $P$ .

Inclined cracks will safely close if, at the level of the centroid of the transformed section,  $\sigma_{mt} = \sigma_{mc}$  they both are compressive and are at least equal to 0,5 MPa. For this purpose, it may prove necessary to induce a biaxial prestress (by using prestressed stirrups or bent bars).

### ASSIGNMENTS TO CHAPTER 12

1. What forces is the calculation of reinforced concrete elements with crack formation made for?
2. What is the limit width of crack opening in reinforced concrete elements?

3. What structures are calculated for crack closing?
4. What purposes is the calculation for deformation of elements made for?
5. How are elements deformations without cracks indicated?
6. What makes calculation of reinforced concrete elements with cracks in tensile zone so complicated?
7. What is crack resistance of reinforced-concrete elements?
8. How are starting positions of calculation for cracks emergence formulated at central stretching, at a bend?
9. What is internal effort before cracks occurrence equal to in centrally stretched element?
10. What are the main provisions of calculation of the moment of cracks occurrence at elastic work of concrete of the compressed zone of elements?
11. What are the main provisions of calculation of the moment of cracks occurrence at non-elastic work of concrete of the compressed zone of elements?
12. What are the main provisions of calculation of the moment of cracks occurrence according to the method of core moments?
13. What does the calculation of cracks occurrence inclined to a longitudinal axis of an element, consist in?
14. What does physical interpretation of the width of cracks disclosing in concrete of the stretched zone consist in?
15. What factors does the width of cracks disclosing normal to the axis depend on according to the empirical formula of norms?
16. What are the preconditions of calculation of factor, which characterizes concrete work for stretching on a site between cracks at central stretching, at a bend?
17. How is stress in concrete and armature determined in sections with a crack?

### 13. CURVATURE OF THE DEFLECTED AXIS AND STIFFNESS OF REINFORCED CONCRETE MEMBERS IN BENDING

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#### 13.1. General Part

The analysis of reinforced concrete members for deflection (linear and angular) involves determining the curvature of the axis or stiffness (or flexuous rigidity) of members in bending.

Depending on the type of loading and the state of stress, the tension zone of a reinforced concrete member may or may not have portions free from cracks (or portions where cracks are closed). For the purpose of our discussion, we shall take members or their portions as crack-free in the tension zone if dead, long- and short-time loads do not cause cracking when taken with the load reliability factor  $\gamma_f = 1$  applied.

#### 13.2. Curvature of the Deflected Axis and Stiffness of Reinforced Concrete Members in Crack-Free Portions

The curvature of the deflected axis of bending and eccentrically loaded members in crack-free portions is determined as for a continuous transformed section in Stage I of the stress-strain state, using the following formula

$$(1/r)_1 = M/B \quad (13.1)$$

where  $M$  is the bending moment;  $B$  is the stiffness of the transformed section which, for heavy and coarse-aggregate quartz-sand concrete and short-time loading, is

$$B = 0,85E_cJ_{\text{red}}. \quad (13.2)$$

Here, the factor 0,85 takes care of the reduction in stiffness caused by plastic strain in the concrete of the tension zone; under long-time load the curvature is determined

$$(1/r)_2 = M\varphi/B \quad (13.3)$$

where  $\varphi$  – is the factor taking care of the reduction in stiffness (increase in curvature) due to creep in the concrete of the compression zone under

long-time loading, taken as 2 at an average relative humidity of more than 40 %, and 3 at an average relative humidity of 40 % and less.

The curvature,  $(1/r)_3$ , due to the short-time prestressing force is also found by Eq. (14.1) at the bending moment equal to

$$M = P l_{op}. \quad (13.4)$$

The curvature due to creep in the concrete produced by the prestressing force is taken equal to the slope of the strain distribution diagram

$$(1/r)_4 = (\varepsilon_c - \varepsilon_c')/h_0 \quad (13.5)$$

where  $\varepsilon_c$  and  $\varepsilon_c'$  are the strains in the concrete due to creep at centroid of the tensile steel and the extreme compressive fibre in the concrete:

$$\varepsilon_c = \sigma_s/E_s \text{ and } \varepsilon_c' = \sigma_s'/E_s. \quad (13.6)$$

Here, the losses are equal to  $\sigma_c = \sigma_6 + \sigma_9$  and  $\sigma_c' = \sigma_6' + \sigma_9'$ .

The total value of curvature is found as:

$$1/r = (1/r)_1 + (1/r)_2 - (1/r)_3 - (1/r)_4 \quad (13.7)$$

for non-prestressed members:  $(1/r)_3 = (1/r)_4 = 0$ .

If normal cracks in the compressed zone remain closed under the load in question, the curvatures determined may be increased by 25 %, the other curvatures — by 15 %.

### 13.3. Curvature of the Deflected Axis and Stiffness of Reinforced Concrete Members in Crack Containing Portions

In portions where normal cracks form in Stage II, the general stress-strain state is described in terms of the average strain in the tensile steel,  $\varepsilon_{s,m}$  the average strain in the concrete of the compression zone,  $\varepsilon_{c,m}$ , and the average position of the neutral axis with the radius of curvature  $r$  (Fig. 13.1). Let us examine the pure bending zone of a reinforced concrete member. The curvature of the deflected axis and the average strain in the steel and concrete are related as follows

$$l_{cr}/r = \varepsilon_{sm}l_{cr}/(h_0 - x_m) = \varepsilon_{cm}l_{cr}/x_m = (\varepsilon_{sm} + \varepsilon_{cm}) l_{cr}/h_0. \quad (13.8)$$

After dividing the above expression by  $l_{cr}$  curvature of the axis can be expressed as the slope of the average strain distribution diagram.

$$1/r = \varepsilon_{sm}/(h_0 - x_m) = \varepsilon_{cm}/x_m = (\varepsilon_{sm} + \varepsilon_{cm})/h_0. \quad (13.9)$$

Taking into account that

$$\varepsilon_{sm} = \psi_s \sigma_s / E_s \text{ and } \varepsilon_{cm} = \psi_c \sigma_c / \nu E_c \quad (13.10)$$

we may determine the curvature of the axis as

$$1/r = \psi_s \sigma_s / E_s (h_0 - x_m) = \psi_c \sigma_c / \nu E_c x_m \psi_s \sigma_s / (E_s h_0) + \psi_c \sigma_c / \nu E_c h_0. \quad (13.11)$$

On substituting the stresses in the steel and concrete

$$\sigma_s = M/W_s \text{ and } \sigma_b = M/W_c \quad (13.12)$$

into Eq. (13.11), we get the following expression for the curvature

$$\begin{aligned} 1/r &= M\psi_s/E_s W_s (h_0 - x_m) = M\psi_c/\nu E_c W_c x_m = \\ &= M(\psi_s/E_s W_s + \psi_c/\nu E_c W_c)/h_0 \end{aligned} \quad (13.13)$$

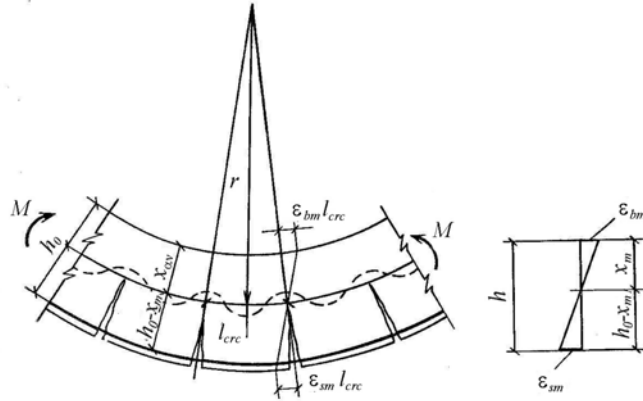


Fig. 13.1. Determination of the deflected axis curvature

The denominator in Eq. (13.13) is the stiffness of the reinforced concrete section. For the tension zone, it is

$$B = (E_s / \psi_s) W_s (h_0 - x_m), \quad (13.14)$$

for the compression zone,

$$B = \nu E_c W_c x_m / \psi_c, \quad (13.15)$$

and for both zones

$$B = h_0 / (\psi_s / E_s W_s + \psi_c / \nu E_c W_c). \quad (13.16)$$

Introducing the values of the elastic-plastic moments of resistance,  $W_s$  and  $W_c$  the expressions for curvature and stiffness may be re-written as follows

$$1/r = M [\psi_s/E_s A_s + \psi_c/(\varphi_f + \xi)vE_c b h_0]/h_0 z_1, \quad (13.17)$$

$$B = h_0 z_1 / [\psi_s/E_s A_s + \psi_c/(\varphi_f + \xi)vE_c b h_0]. \quad (13.18)$$

In the general case, for prestressed members in bending, eccentric compression and eccentric tension with  $e_{oN} \geq 0,8h_0$ , the external loads and the prestressing force are replaced by the equivalent moment  $M_s$  and the total longitudinal force,  $N_{tot}$ . Then, the stress in the concrete of the compression zone is

$$\sigma_c = M_s/(\varphi_f + \xi) b h_0 z_1 \quad (13.19)$$

and, the stress in the tensile zone is

$$\sigma_s = M_s/A_{sp} z_1 - N_{tot}/A_{sp}. \quad (13.20)$$

On substituting the values for  $\sigma_c$  and  $\sigma_s$ , the general expression for the curvature of the deflected axis takes up the following form

$$1/r = M_s [\psi_s/E_s A_s + \psi_c/vE_c b h_0]/h_0 z_1 - N_{tot} \psi_s/h_0 E_s A_s. \quad (13.21)$$

The curvature of the deflected axis,  $1/r$ , and stiffness,  $B$ , in cracked portions vary with time; so, in design, they are multiplied by various factors, such as  $\psi_s$  taking care of the tensile behavior of the concrete between cracks,  $\psi_c$  taking care of the fact the strain in the concrete of the compression zone is non-uniform between cracks, and  $v$  taking care of the non-elastic strain in the concrete of the compression zone. The values of  $\psi_s$  and  $v$  are determined according to the duration and loading.

Relevant standards specify the values of  $v$  for heavy and porous-aggregate concrete according to the type of loading and service conditions of the structure. With short-time loading,  $v = 0,45$ ; with long-time loading,  $v = 0,15$  for an average relative humidity of more than 40 %, and  $v = 0,1$  for an average relative humidity of 40 % and less. It should be noted that the standards specify the values, of the product  $\omega v$  rather than  $v$ . This is done because with the rectangular stress diagram adopted for the concrete of the compression zone in Stage II and with  $\omega = 1$  the product is numerically equal to  $v$ .

#### 13.4. Deflections of Reinforced Concrete Members

**Sag of Reinforced Concrete Members in Crack-Free Tension Zones.** This is determined in terms of the stiffness  $B$  of the transformed section with allowance for the factor  $\varphi$  adjusted for long-time loading. The total sag is

$$f = f_1 + f_2 - f_3 - f_4 \quad (13.22)$$

where  $f_1$  is the sag due to short-time loading;  $f_2$  is the sag due to dead and long-time live loads;  $f_3$  is the hog due to the short-time prestressing force with allowance for all losses; and  $f_4$  is the hog due to the creep in the concrete induced by prestressing.

In prestressed members of constant depth, the hog due to eccentrically prestressing is

$$f_3 = P_{\text{top}} l^2 / 8B \quad (13.23)$$

and the hog due to the creep in the concrete induced by prestressing is

$$f_4 = l^2 / 8(1/r)_4. \quad (13.24)$$

Sag of Reinforced Concrete Members in Cracked Tension Zones. This is determined in terms of the curvature of the deflected axis

$$f = \int_0^l \bar{M}(1/r)(x) dx \quad (13.25)$$

where  $\bar{M}$  is the bending moment at the  $x$  section due to the unit force applied in the direction of the deflection in question;  $1/r(x)$  is determined by Eq. (13.21)

When determining deflections in constant-section members, it is well to determine the curvature for the most stressed section within each portion where the bending moment does not change sign, and to assume it changing in proportion to the bending moment diagram elsewhere. This is equivalent to finding  $B$  for the most stressed section and assuming it to be constant elsewhere. For prestressed members designed to meet the crack-resistance requirements of Categories Two and Three, the above assumptions may sometimes result in overestimated sags, because cracked portions in the tension zone may be of a limited length. In such cases, the sag is

$$f = \sum_0^l \bar{M} 1/r(x) dx \quad (13.26)$$

Here, the curvature diagram,  $1/r(x)$ , is divided into several portions along the span as a piecewise — linear function, and the deflection integral is evaluated by multiplying the diagrams according to Vereshchagin's rule. In each crack — free and cracked portion, the curvature,  $1/r(x)$ , is found by Eqs. (13.1) (13.2) and (13.4).

The angular deflection of reinforced concrete members is likewise found by Eq. (13.25) or (13.26), but the integration is performed with respect to the moment  $\bar{M}$  at the  $x$  section due to the unit moment.



In the simplest cases, the sag of non-prestressed bending members (slabs, panels, beams and so on) due to distributed loads is

$$f = 5gl^4/384B \quad (13.27)$$

The sag of single-span beams and cantilevers induced by various loads is determined in terms of the curvature or stiffness at the section where the bending moment is maximum, using the following general formula

$$f = s^2/2 \text{ or } f = s^2M/B \quad (13.28)$$

where  $s$  depends on the type and disposition of loads. For a simply supported beam:  $s = 5/48$  with a distributed load;  $s = 1/12$  with a concentrated load applied at the midspan; and  $s = 1/8$  with two equal moments applied at the ends. For a cantilever beam  $s = 1/4$  with a distributed load;  $s = 1/3$  with a concentrated load applied at the cantilever arm; and  $s = 1/2$  with a moment applied at the cantilever arm.

For multispan beams whose sections are in both compression and tension, the stiffness of each portion is assumed to be the same and equal to that at the section where the moment is a maximum (Fig. 13.2).

The sag of short bending members with the ratio  $l/h < 10$  (such as crane beams, hammer beams, and the like) should be determined with allowance for shearing forces. In this case, the total sag is equal to the sum of sags due to bending strain and shear strain,  $f_Q$ . The sag is determined as

$$f_0 = \int 1,5\bar{Q}Q\varphi_{c2}\varphi_{cr} dx / Gbh_0 \quad (13.29)$$

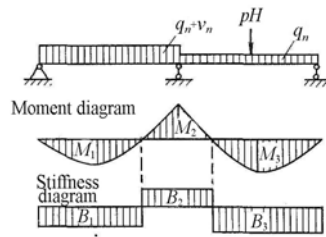


Fig. 13.2. Bending moment diagram and stiffness diagram of a two-span beam

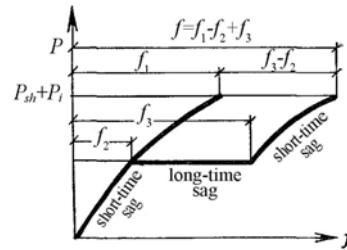


Fig. 13.3. Sag of a bending reinforced concrete member under short- and long-time loading

where  $\bar{Q}$  is the shearing force at the  $x$  section due to the unit force applied in the direction of the deflection in question;  $\varphi_{b2}$  is the loading-duration factor;  $\varphi_{cr}$  is the factor taking care of the effect that cracks

have on the shear strain, and assumed as 1 for portions free from normal and inclined cracks, 4,8 for portions containing inclined cracks only, and

$$\psi_{\text{crc}} = 3B/B_{\text{crc}} \text{ or } \psi_{\text{crc}} = 3B/M_x I/r(x) \quad (13.30)$$

for portions containing only normal cracks or both normal and inclined cracks;  $B_{\text{crc}}$  is the stiffness of the section upon cracking.

The total sag with allowance for the duration of loading is

$$f = f_1 - f_2 + f_3 - f_4 \quad (13.31)$$

where  $f_1$  is the sag due to all loads applied for a short time;  $f_2$  is the sag due to dead and long-time live loads applied for a short time;  $f_3$  is the sag due to dead and long-time live loads applied for a long time; and  $f_4$  is the hog due to prestressing and creep in the concrete caused by prestressing.

The values of  $f_1$  and  $f_2$  are calculated with  $\psi_s$  and  $\nu$  taken for short-time loading, and that of  $f_3$ , with  $\psi_s$ , and  $\nu$  taken for long-time loading.

The physical meaning of Eq. (13.31) is illustrated by the F-f diagram shown in Fig. 13.3.

The total sag of prestressed members with allowance for the duration of loading is determined as the total curvature

$$1/r = (1/r)_1 - (1/r)_2 + (1/r)_3 - (1/r)_4. \quad (13.32)$$

### ASSIGNMENTS TO CHAPTER 13

1. How do cracks affect rigidity and long life of reinforced concrete elements?
2. What purposes should calculation for deformation be made for?
3. How are elements deformations expressed if there are no cracks?
4. What makes the calculation of reinforced concrete elements with cracks in tensile zone so complicated?
5. What is the order of making calculation for determination curvature in sequence on at a bend of reinforced-concrete elements on sites with cracks made?
6. How can the formula of rigidity of a reinforced-concrete element on sites with cracks be written?
7. How can we express curvature of an axis at a bend of preliminary stressed element on sites with cracks?
8. What efforts is calculation of RCCs by formation of cracks made for?
9. What cracks formation is calculation of RCCs is made for?
10. What width of cracks disclosing is allowed by norms?
11. What constructions are calculated for disclosing cracks?
12. What is calculation of elements for deformations necessary for?
13. How are deformations of elements determined at absence of cracks?
14. What is the difficulty of calculation of RCE with cracks in the stretched zone?

## 14. FLOOR DESIGN

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### 14.1. Basic Floor Types

Floors made of reinforced concrete are most common in industrial and civil buildings and structures. This is because their manufacture can be highly industrialized, the material consumption and the cost of manufacture and erection are relatively low, and the structure itself is sufficiently rigid, fireproof and durable in service.

There are two basic types of floor design, namely beam and girder construction and flat-slab construction. The former refers to floors formed by slabs or floor panels supported by and acting together with beams running in one or two directions. In the flat-slab construction, flat slabs are carried directly by columns flared at the top to form what are called capitals. Either type may be built of precast members, cast in-situ or erected as a combination of precast and in-situ members. Precast and in-situ floors differ in design approach, so, when classifying them, it is necessary to mention the manner of manufacture. Accordingly, we may distinguish precast slab and girder construction, in-situ beam and girder construction, in-situ ribbed-slab construction with slabs spanning in two directions, precast in-situ beam and girder construction, precast flat-slab construction, in-situ flat-slab construction, and precast in-situ flat-slab construction.

According to the aspect ratio, floor slabs may be classed as follows. When  $l_2/l_1 > 2$  (Fig. 14.1, *a*), we speak of a one-way slab subjected to bending along its shorter side; the bending moment induced along the longer side is small and may be neglected. When  $l_2/l_1 \leq 2$  (Fig. 14.1, *b*), we speak of a two-way slab with two-way reinforcement, subjected to bending in two directions.

In mass construction, use is mostly made of precast floor types because their manufacture can be highly industrialized. In-situ floors are used rather seldom, chiefly in buildings erected to other than type designs.

The type of floor construction is chosen from economic considerations in each particular case according to the purpose of a building, magnitude and type of loading, local conditions, etc.

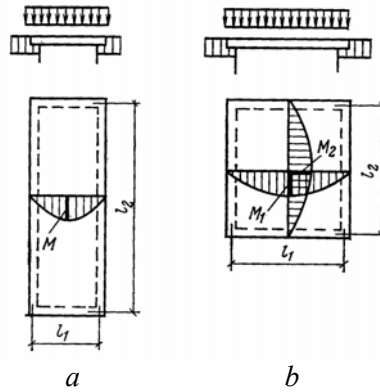


Fig. 14.1. One-way slab subjected to bending along its shorter side (*a*) and two-way slab subjected to bending in two directions (*b*)

## 14.2. Slab and Girder Construction

### Arrangement of Structural Members

The slab and girder floor construction consists essentially of floor slabs (or panels) supported by girders (Fig. 14.2, *a*). In turn, the girders are carried by columns or bearing walls. There may be longitudinal girders (those directed along the building) and transverse girders (Fig. 14.2, *b*). Together with the columns, the girders make up the skeleton of the building.

In the transverse direction, a floor may have two or three spans in civil buildings, and five or six spans in industrial buildings. In the latter case, the span of a girder depends on the general layout, floor type and dead load, being from 6 to 9m with columns spaced 6m apart. A girder span in civil buildings depends on the column grid size which may range between 2,8 and 6,8 m, being a multiple of 0,2 m.

The arrangement of structural members consists in selecting the direction of girders, assigning the span and spacing between girders, and choosing the type and size of slabs. Here, it is necessary to take into account:

1) the magnitude of live load, the purpose of a building and the general layout;

2) the general arrangement of all structural members in the building. In buildings where the transverse spatial rigidity is provided by frames with rigid joints, the girders should be run in the transverse direction and the slabs in the longitudinal direction. In civil buildings, the girders

may be directed along, and the slabs across, the building. In either case, the columns should be appropriately spaced;

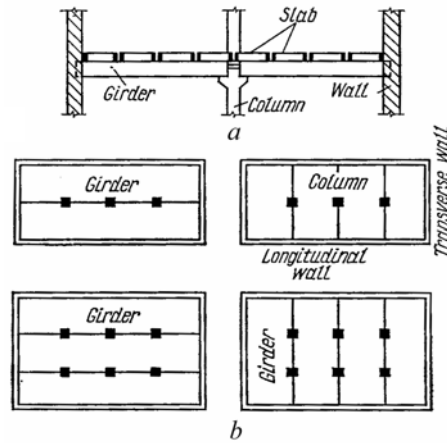


Fig. 14.2. Arrangement of slabs and girders in the slab and girder floor

3) the technical and economic parameters of the floor type. The amount of concrete and steel should be minimum and the members should have a maximum weight and size compatible with the lifting power of erecting cranes and the load-carrying capacity of vehicles.

As already noted, several alternative designs are usually worked out and compared, and the least expensive is then adopted for construction.

The overall amount of concrete and steel per reinforced concrete floor is the sum of the amounts of concrete and steel in slabs, girders and columns. Slabs require about 65 % of reinforced concrete, which is why primary attention should be focused on the optimum design of slabs.

### 14.3. Floor Slabs

**Selection of Optimal Cross-Section.** In order to save concrete and to reduce the weight of slabs, use is made of hollow-core or ribbed slabs (Fig. 14.3, *a*). A large part of the concrete on the tensile side of a slab is eliminated, leaving only the ribs or joists wide enough to contain reinforcing cages and resist shearing stresses. In the span between girders, such slabs behave in bending as T-beams (Fig. 14.3, *b*). The top flange of a slab is also subjected to local bending between ribs. If it is necessary to make a flat ceiling, the hollow cores are closed from the bottom.

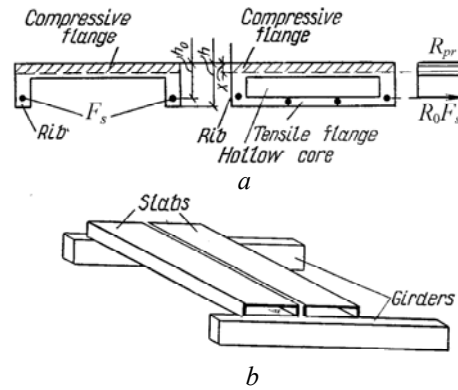


Fig. 14.3. Floor slabs

Hollow cores in slabs may be oval, round or of some other shape. Wide slabs may contain several adjacent hollow cores.

The general principle of design for any floor slabs may be stated as follows. Leave just enough concrete to form vertical ribs resisting shearing forces and remove as much of the rest as the maker's plant and capabilities permit.

According to their cross-section, floor slabs may be classed into those with oval, round and vertical voids, ribbed slabs with their joists facing up (with floor covering placed on joists) or down, and also solid slabs (Fig. 14.4, a through f).

In hollow-core slabs, the minimum thickness of topping ranges between 25 and 30mm, and the minimum thickness of ribs is from 30 to 35mm. In ribbed slabs with their joists facing down, the minimum thickness of topping is from 50 to 60 mm.

If the length of slabs is specified in advance, their width should be chosen so that their weight would not exceed the lifting power of erecting cranes (which is usually 3 or 5 tones, or, sometimes, greater). Slabs 3.2 m wide and 6 m long can cover a whole room; if they contain voids, the weight of such slabs is 5 or 6 tones. Hollow-core and solid slabs which form flat ceilings are used in civil buildings, whereas ribbed slabs with their joists down are used in industrial buildings for basic live loads of above 5 kN/m<sup>2</sup>.

The main parameters of floor slabs are the reduced concrete depth determined by dividing the volume of concrete in a slab by its surface area, and the amount of reinforcing steel (Table 14.1).

As regards the amount of concrete used, the best choice is oval-void slabs, the reduced concrete depth of which is 9.2 cm, whereas that of round-void slabs is 12 cm. Oval-void slabs, however, are rather difficult

to manufacture at a factory because, after the void formers have been removed, the voids in the finished product tend to collapse. That is why round-void slabs are adopted as a standard, and oval-void slabs may be made only if a factory has appropriate equipment and adequate know-how. With further progress in the factory manufacture of hollow-core slabs, much better slab designs are expected to be made available.

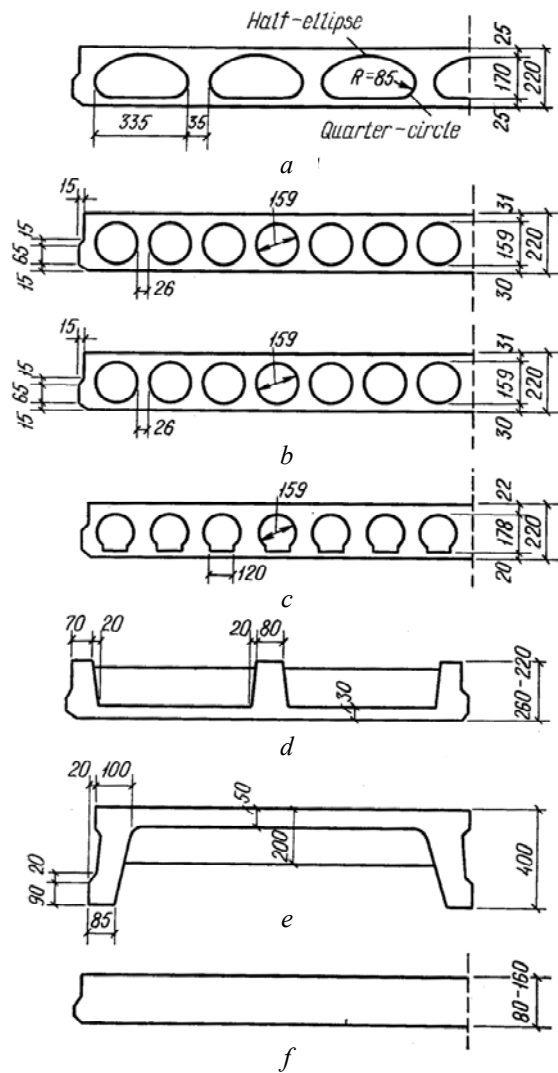


Fig. 14.4. Cross-sections of floor slabs

Table 14.1

**Floor Slab Parameters at the Nominal Span of 6m and the  
Basic Load of 6 to 7 kN/m<sup>2</sup>**

Slab type	Reduced depth, cm	Amount of steel per square metre, kg		
		Non- prestressed	Prestressed	
			bars	wire
With oval voids	9,2	8	4,3	3,4
With vertical voids	10,2	8,5	4,7	3,7
With round voids	12	8,5	4,7	3,7
Ribbed with ribs up	8	9,1	5	4
Solid	12-16	14-16	12-14	10-11

Ribbed slabs with their joists facing up having a relatively small reduced concrete depth of 8 cm are less adapted to industrial manufacture, because they require additional floor topping. As a result, the cost of the ribbed-slab floor is higher.

In ribbed slabs with their joists facing down, the reduced concrete depth is 10,5 cm, and the amount of reinforcing steel per square meters ranges between 8,3 and 21,5 kg, depending on live load.

Prestressed floor slabs are made of B15 and B25 class concrete, whereas non-prestressed slabs are manufactured of B15 and B20 class concrete.

**Slab Design.** The effective span of a slab,  $l_0$ , is taken as the centre-to-centre distance between its supports (Fig. 14.5, *a* through *c*); if the slab is supported by the top faces of girders,  $l_0 = l - b/2$  (where  $b$  is the width of a girder); and, if it is carried by girder flanges,  $l_0 = l - a - b$  (where  $a$  is the flange overhang). If one end of a slab is supported by a girder and the other by a bearing wall, its effective span is taken as the centre-to-centre distance between the support on the wall and that on the girder.

The depth of a slab,  $h$ , should be chosen to ensure the necessary strength and stiffness (so that limiting sags would not be exceeded).

With spans ranging between 5 and 7m, the depth of the slab section is mostly governed by rigidity requirements. The slab section depth satisfying both strength and rigidity requirements can approximately be determined by the following formula

$$h = cl_0 R_s (\theta g_n + v_n) / E_s (g_n + v_n) \quad (14.1)$$

where  $c$  equals 18 to 20 for hollow-core slabs, and 30 to 34 for ribbed slabs with the flange in the compression zone, the greater values being specified for class A-II reinforcement, and the lower for class A-III



reinforcement;  $g_n$  is the long-time basic live load per square meters of the floor;  $v_n$  is the short-time basic live load per square meters of the floor;  $\theta$  is the factor taking care of the increase in sag under long-time load, equal to 2 for hollow-core slabs and 1.5 for ribbed slabs with the flange in the compression zone.

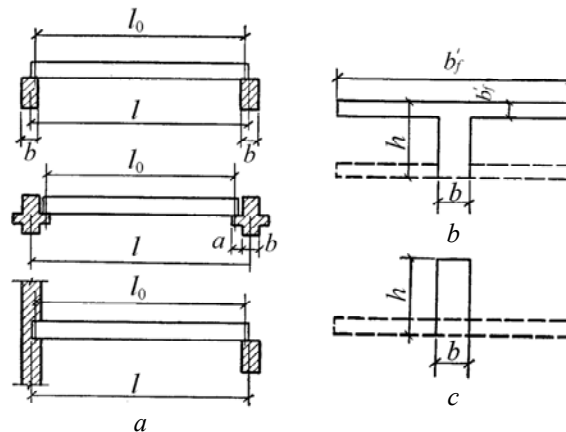


Fig. 14.5. Effective spans and cross-sections of floor slabs

The depth of prestressed slabs may also be approximately determined by Eq. or assumed as

$$h = (1/20 \text{ to } 1/30)l_0. \quad (14.2)$$

For strength analysis in terms of bending moment, the width of the rib,  $b$ , is assumed as the total width of all slab ribs, and the design width of the compressed flange as the total width of the slab. If the topping is thin, that is  $h'_f/h \leq 0.1$ , the flange width taken in calculations should not exceed

$$b'_f = 12(n - 1)h'_f + b \quad (14.3)$$

where  $n$  is the number of ribs in the cross section of the slab.

If in a ribbed slab with its joists facing down and containing lateral joists  $h'_f/h \leq 0.1$ , the flange width is taken as the total width of the slab.

Thus, the strength analysis of slabs reduces to that of a T-section with the flange in the compression zone. In most cases, the neutral axis lies within the compressed flange, so, on determining

$$\alpha_m = M/R_c b'_f h_0^2 \quad (14.4)$$

the values of  $\xi$  and  $\zeta$  are looked up in the table and a check is made to see if  $x = \xi h_0 \leq h'_f$ . Then, the cross-sectional area of the tensile steel is found as

$$A_s = M/R_s \zeta h_0 \quad (14.5)$$

when  $x = \xi h_0 > h'_f$  and the neutral axis crosses the rib, allowance should be made for compression in the rib.

The design width of a slab with its ribs facing up is taken equal to the total width of the ribs, and the section is treated as rectangular. The transverse reinforcement for a slab is designed to carry shearing forces, with the design rib width,  $b$ , assumed to be equal to the total width of all ribs in the section.

In hollow-core slabs with the depth of 300 mm or less, transverse reinforcement may be dispensed with, provided it is not structurally required.

Incipient cracking, crack-opening and sagging analyses are carried out according to the category of crack-resistance of a given slab.

In analysis for sagging, hollow-core sections are replaced by equivalent I-sections. For round-void slabs, the equivalent I-section is found by equating the area of a round hole of diameter  $d$  to the area of a square hole of side  $0,9d$  (Fig. 14.6, *a*). Oval-core sections are reduced to equivalent I-sections (Fig. 14.6, *b*) by replacing oval voids by rectangular voids of the same area and moment of inertia so that their centroids would coincide. For hollow-core slabs with  $h$  ranging between 15 and 25cm and cores up to 50 cm wide, the equivalent sections can be found as shown in Fig. 14.6, *c* and *d*.

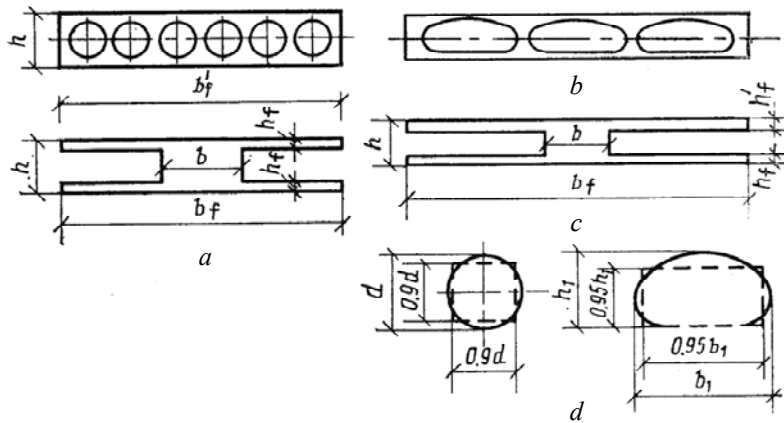


Fig. 14.6. Equivalent cross-sections for sagging analysis

The slab flange behaves in local bending as a slab of span  $l_0$  equal to the clear distance between the ribs, partly fixed on supports. In ribbed

slabs with their ribs down, such a fixation is provided by concreting the joints, which prevents the ribs from angular deflection (Fig. 14.7, *a*). Here, the bending moment is determined as

$$M = ql_0^2/11. \quad (14.6)$$

In ribbed slabs with intermediate transverse ribs, the bending moments in the flange may be determined as in a two-way slab supported on four sides (Fig. 14.7, *b*).

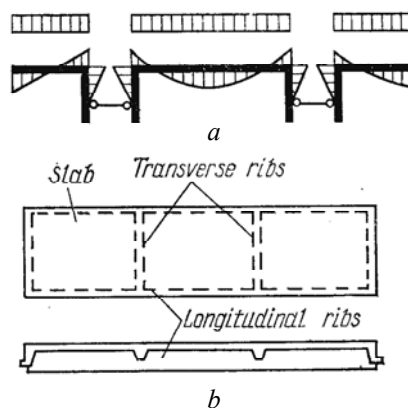


Fig. 14.7. Loading system for slab flanges

**Reinforcement and Layout of Slabs.** Slabs are reinforced with welded-wire fabric and reinforcing cages made of plain wire and hot-rolled deformed bars (Fig. 14.8). Longitudinal prestressed reinforcement may consist of class A-IV, A-V, A<sub>T</sub>-IVC or A<sub>T</sub>-V bars.

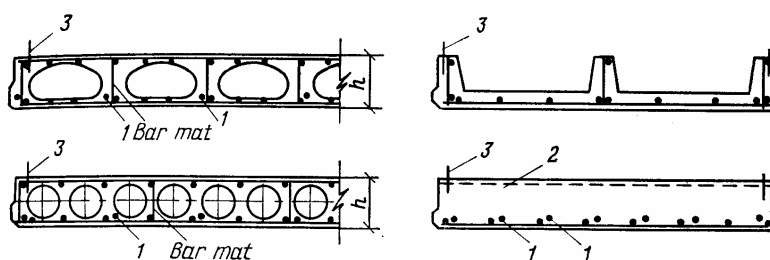


Fig. 14.8. Reinforcement of floor slabs:

1 — prestressed reinforcement; 2 — narrow fabric placed on the topside; 3 — lifting eyes high-strength wire or wire strands. If the slab span is less than 6 m, prestressed reinforcement may be omitted

The longitudinal load-bearing reinforcement is distributed over the entire width of the bottom flange in hollow-core slabs and in the ribs of ribbed slabs. Transverse bars are joined to longitudinal non-prestressed distribution and load-bearing bars to form welded-bar mats which are placed in the ribs of slabs. In round-core slabs, welded-bar mats may be located only near supports at every two or three ribs.

The ends of longitudinal non-prestressed bars in ribbed slabs are welded to anchors made of angles or plates to hold the bars in place at the supports.

Solid slabs of heavy or porous-aggregate concrete are reinforced with prestressed longitudinal steel and welded fabric.

Lifting eyes are placed at the four corners of a slab and welded to the main reinforcement. Near lifting eyes, solid slabs are reinforced with additional top fabric. A likely reinforcement for a ribbed slab used in industrial buildings is shown in Fig. 14.9. The nominal width of such a slab is assumed as 1,5 m.

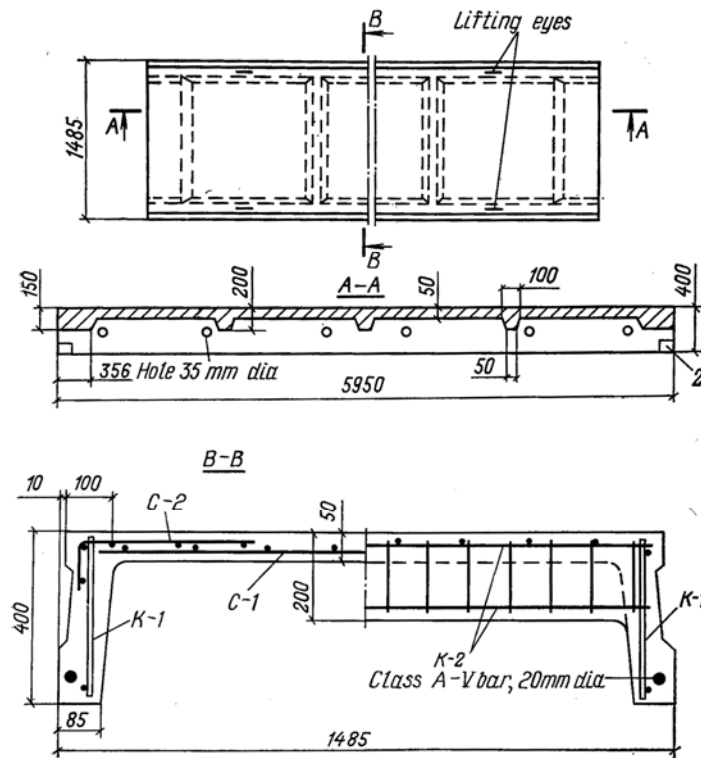


Fig. 14.9. Reinforcement of a ribbed floor slab

Slabs of all types are joined by welding together embedded steel parts and concreting the gaps between the slabs (Fig. 14.10, *a*). The longitudinal side faces of slabs have cavities intended (after the joints have been concreted) to form intermittent keys which enable the slabs to resist shear in the vertical and horizontal directions by composite action. Thus connected, a floor acts as a rigid horizontal diaphragm composite action.

If the live load applied to a floor exceeds  $10 \text{ N/m}^a$ , it is advisable to turn ribbed slabs into continuous slabs at the time of joint concreting. For this purpose, the joints between the ribbed slabs are reinforced with additional mats placed over the girders (Fig. 14.10, *b*). In terms of service loads, such slabs are analyzed and designed as continuous.

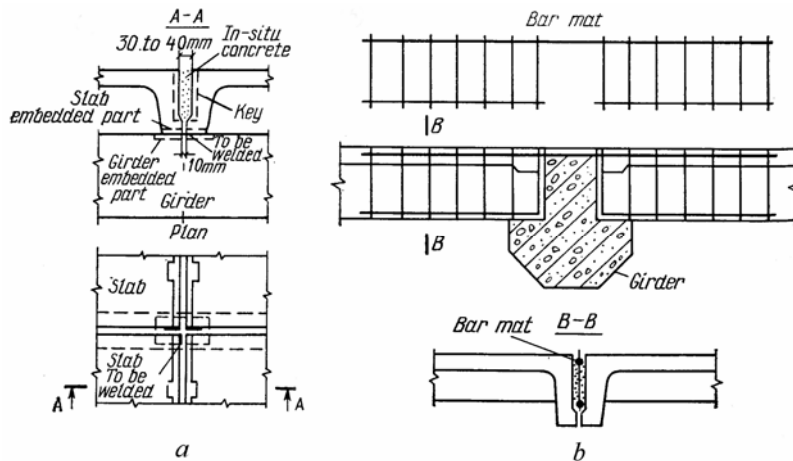


Fig. 14.10. Joints between slabs

#### 14.4. Girder Design

**Analysis of a Multispan Girder.** The girder of a multispan floor is a member of a frame. When it is simply supported and has equal spans, the girder may be regarded as a continuous beam. Then, it is possible to take into account the plastic strain which causes the distribution and equalization of bending moments between the individual sections.

Analysis of Statically Indeterminate Reinforced Concrete Structures with Allowance for the Distribution of Forces. Under a certain load, the stresses in the mild tensile steel reach the yield point. The plastic strain (yield) developing in a reinforced concrete structure

produces a portion, known as a plastic hinge, where the local strain is considerable.

In a statically determinate structure, say, a simply supported beam (Fig. 14.11, *a*), the plastic hinge permits the beam parts to deflect, and this results in a significant sag. This reduces the depth of the compression zone, the zone is stressed up to  $\sigma_c = R_c$  and the structure fails.

A statically indeterminate structure behaves differently (Fig. 14.11, *b*). Here, the deflection of the beam parts due to a plastic hinge, the accompanying sag and the build-up of stresses in the compression zone are opposed by redundant constraints produced by the fixation at the supports. This brings about Stage II-a of the stress-strain behavior where  $\sigma_s = \sigma_y$  but  $\sigma_c < R_c$ . So, as the load keeps increasing, the structure will not collapse at the plastic hinge until new plastic hinges are formed and the redundant constraints are cancelled. In a statically indeterminate system, the development of a plastic hinge is equivalent to eliminating a redundant constraint and reducing the degree of static indeterminacy by one. The first plastic hinge appearing in the above fixed beam turns it into a system statically indeterminate to the first degree; it will take three plastic hinges (at the supports and in the span) to destroy its rigidity completely.

In the general case, a system with  $n$  redundant constraints loses its rigidity when  $n + 1$  plastics-hinges are formed. Consider the manner in which the bending moment is distributed, using a fixed beam as an example. As a plastic hinge appears at one of the supports under the load  $F_0$  (Fig. 14.12, *a*), the beam becomes fixed at one end and simply supported at the other, (Fig. 14.12, *b*), and remains such as the load keeps increasing.

When the load increases by  $\Delta_1 F_0$ , a plastic hinge forms at the other support and the beam becomes simply supported (Fig. 14.12, *c*).

After a plastic hinge has appeared in a statically indeterminate structure, the increasing load causes the distribution of bending moments among the individual sections. As this takes place, the strain in the plastic hinge increases but the bending moment remains unchanged

$$M = R_c A_s z_c \quad (14.7)$$

After the plastic hinge has formed, the arm of the internal couple,  $z_c$ , rises insignificantly with increasing load and is assumed practically constant (Fig. 14.11, *c*).

Another increase in the load by  $\Delta_2 F_0$  causes the beam to lose its rigidity, that is, to fail.

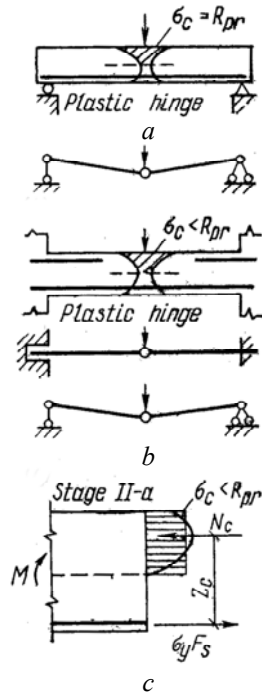


Fig. 14.11. Formation of a plastic hinge in reinforced concrete beams

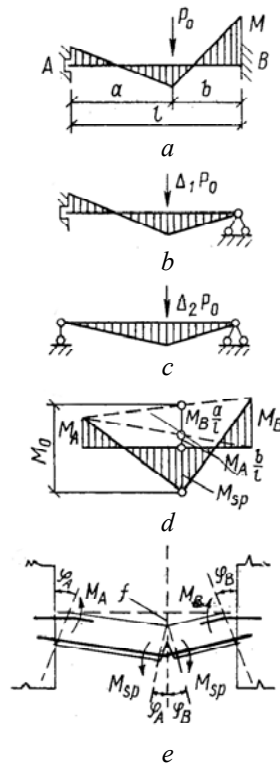


Fig. 14.12. Bending moment distribution diagrams for a statically indeterminate beam

Let us designate the ultimate design moments in the design sections (at the plastic hinges) as  $M_A$  (at support  $A$ ),  $M_B$  (at support  $B$ ) and  $M_I$  (in the span) (Fig. 14.12,  $d$ ). The ultimate design force is

$$F = F_0 + \Delta_1 F_0 + \Delta_2 F_0. \quad (14.8)$$

At the state of ultimate equilibrium, that is, immediately before failure, the bending moments may be found statically or cinematically. *Static approach.* The bending moment in the span is determined as

$$M_I = M_0 - M_A b/l - M_B a/l. \quad (14.9)$$

Hence, the equation for equilibrium can be written as

$$M_I + M_A b/l + M_B a/l = M_0 \quad (14.10)$$

where  $M_0 = Fab/l$  is the moment in a statically determinate simply supported beam.

Equation (14.5) suggests that the sum of the span moment and the fractions of the support moments corresponding to this section are equal to the moment in a simply supported beam,  $M_0$ . Also, it can be seen that the load-bearing capacity of a statically indeterminate structure does not depend on the relation between the supports and span moments, and the sequence of plastic hinge formation. This sequence may be any, provided the condition for equilibrium is satisfied. A change in the moment relation, however, affects the magnitude of load under which the first and the last hinges are formed, and also the crack width at the first plastic hinge.

The analysis and design of statically indeterminate reinforced concrete structures on the basis of equalized moments facilitates the assignment of reinforcement. This is especially important for joints between precast members at supports. This also makes it possible to standardize reinforcement and, if necessary, to use the same type of welded-wire fabric or bar mats in cases where the bending moments as determined by the elastic method differ. When a floor is exposed to live loads, the amount of steel found by the equalized-moment method may be reduced by 20 to 30 % of the figure found by the elastic method.

The crack width in the first plastic hinges is limited by keeping the equalized moment at not less than 70 % of that found by the elastic method.

In order to provide the conditions under which the ultimate-equilibrium method is applicable, that is, the formation of plastic hinges and the development of sufficient local strain in the state of ultimate equilibrium, the following measures should be taken:

- 1) the structure should be designed so as to prevent its failure due to shear in the compression zone or the crushing of the concrete caused by the principal compressive stresses;
- 2) the reinforcement of the sections where plastic hinges are to form should be limited so that the relative depth of the compression zone would be  $\xi \leq 0.35$ ;
- 3) use should be made of reinforcing steel with a definite yield or welded-wire fabric made of plain wire.

As far as dynamic loads (earthquake shocks, blast shocks, etc.) are concerned, it is good practice to design statically indeterminate reinforced concrete structures with allowance for plastic hinges as well.

In a structure reinforced with steel having no definite yield point, after one of the bending moments has reached the proof ultimate value,  $M_{0,2}$ , at the proof yield stress,  $\sigma_{0,2}$ , the moment will not cease growing, but will increase at a lower rate. In this case, the load-bearing capacity



of a structure is determined by the limit of elongation of the reinforcing steel or the ultimate strength of the concrete in the compression zone.

The redistribution of forces in a statically indeterminate reinforced concrete structure occurs already at an earlier loading stage due to changes in the rigidity of the sections at the supports and in the span produced by incipient cracking and crack opening in the tension zone. Although this has no pronounced effect on the distribution of forces in the state of ultimate equilibrium, that is, prior to the formation of plastic hinges, it markedly affects the behavior of the structure under service loads and, therefore, should be taken into account in design calculations.

For continuous beams, there is a simplified method for taking into consideration the distribution of forces. The bending moments at the supports are calculated as in the elastic method and multiplied by correction factors taking care of the difference in rigidity between the support and span sections. Then, using the above moments at the supports, the bending moments in the span are found. With distributed loads or several concentrated loads, the correction factors for the bending moments at the supports are

– for the middle supports of multispan beams

$$\lambda = 3/(2 + \beta); \quad (14.11)$$

– for the middle support of a two-span beam

$$\lambda = 1,5/(0,5 + \beta); \quad (14.12)$$

– for the first intermediate support of a multispan beam,  $\lambda$  is taken as the average of the above two values. Here,  $\beta = B/B_{\text{sup}}$  is the ratio of the rigidity of cracked sections in the span and at the supports.

In more detail, the analysis of statically indeterminate reinforced concrete structures with allowance for the distribution of forces is covered in appropriate manuals.

The analysis of a continuous girder as an elastic system involves the following distribution of bending moments. The effective span of the girder is taken equal to the centre-to-centre distance between the columns; the first effective span where one end of the girder is supported by a bearing wall is reckoned from the support axis at the wall to the axis of the column. The load imposed on the girder by the slabs may be either uniformly distributed (with hollow-core or solid slabs) or concentrated (with ribbed slabs). If there are more than four concentrated forces acting in the span of the girder, they may be replaced by an equivalent distributed load. For a trial determination of the self-weight of the girder, the dimensions of its cross section are taken as  $h = 1/10l$  to  $1/15l$  and  $b = 0,3h$  to  $0,4h$ .

If the spans of a multispan beam are equal or differ from one another by not more than 20 %, the bending moments and shearing forces are looked up in tables:

– for uniformly distributed loads

$$M = (\alpha g + \beta v)l/2; Q = (\gamma g + \delta v)l; \quad (14.13)$$

– for concentrated loads

$$M = (\alpha G + \beta V)l; Q = \gamma G + \delta V, \quad (14.14)$$

where  $\alpha$  and  $\beta$  are the tabular coefficients for  $M$  induced by dead and live loads, respectively;  $\gamma$  and  $\delta$  are the tabular coefficients for  $Q$  induced by dead and live loads, respectively.

When the live loads are applied at every other span, the bending moments are maximum in the spans being loaded; if the live loads are applied at two adjacent spans and then at a one-span interval, the bending moments are maximum (in absolute values) at the support (Fig. 14.13). In a multispan girder, it is advisable to reduce the amount of steel at the supports and simplify the joints. So, for the distribution of bending moments in the girder, the diagram for the moments induced by dead and some unfavorable combinations of live loads must be supplemented by triangular diagrams with ordinates at the supports arbitrary in sign and value (Fig. 14.14).

Here, the ordinates of the equalized moment diagram should be at least 70 % of those calculated by the elastic method. Various loading combinations are then used to construct the resultant diagrams for  $M$  and  $Q$ .

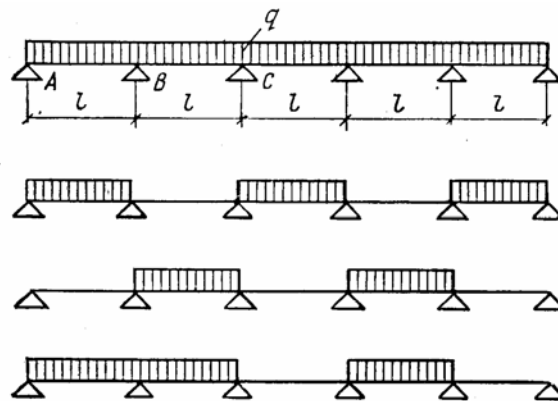


Fig. 14.13. Load configuration for a multispan beam

There exists another simplified method for the design of a multi-span girder based on equalized moments. Here, the design equalized moment diagram is the moment diagram for an elastic multispan beam, obtained for maximum bending moments in the span (that is, for the load applied at every other span).

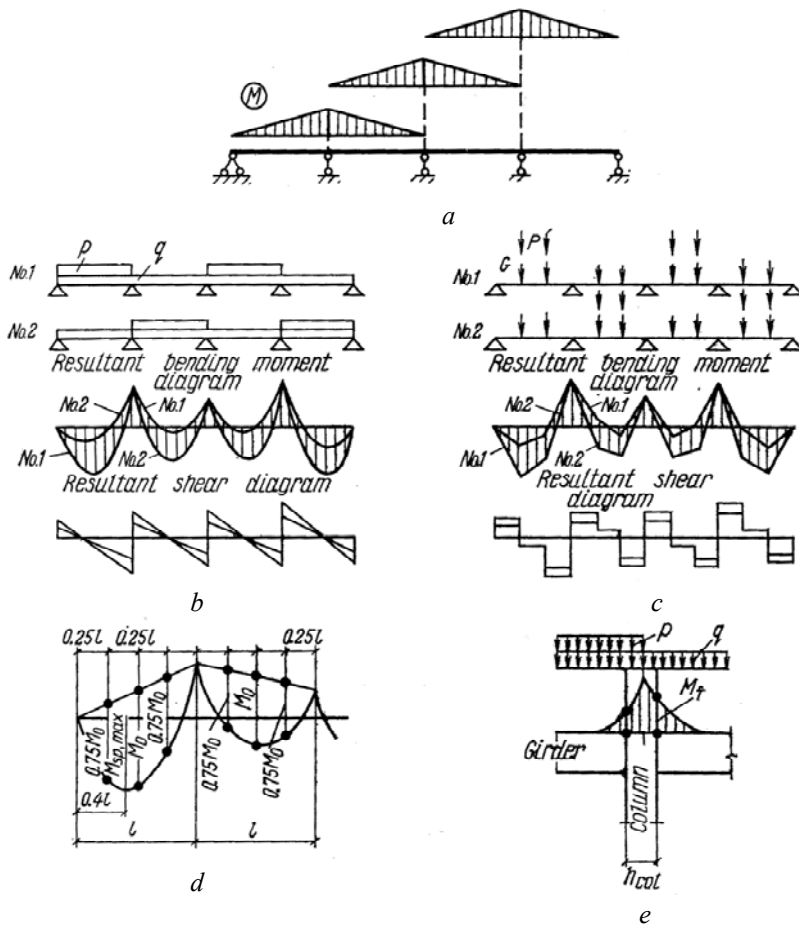


Fig. 14.14. A multispan design:  
*a* — supplementary bending moment diagrams; *b* — the construction of the bending moment diagram due to uniformly distributed load; *c* — the same, due to concentrated load; *d* — the construction of the bending moment diagram due to uniformly distributed load; *e* — determining the design moment in the girder at the column side

The design section of the girder at the support is that at the face of the column. Here, the bending moment is

$$M_1 = M - Qh/2. \quad (14.15)$$

In absolute value,  $M_1$  is greater on the span-side, where the girder is subjected only to the dead load; so, the value of  $Q$  in Eq. (14.13) should correspond to the loading of this span. The cross-sectional dimensions of the girder are then refined to fit the value of  $M_1$  and, for  $\xi = 0,35$ , we get

$$h_0 = 1.8(M_1/R_c b)^{1/2}. \quad (14.16)$$

The cross-sectional area of the longitudinal steel in the girder is chosen to fit the values of  $M$  in four normal sections, namely in the first and middle spans, at the first intermediate support and at the middle support. In determining the steel area necessary to give the required value of  $Q$ , three diagonal sections are of interest: those to the left and right of the first intermediate column and at the last support.

**Reinforcement and Layout of a Continuous Girder.** Girders can be rectangular, T-section with the flange facing up, and T-section with the flange facing down (Fig. 14.15). When floor slabs are supported by the bottom flange of a T-section girder, the overall depth of the floor is reduced.

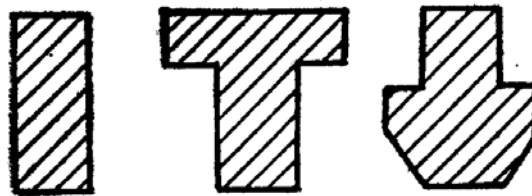


Fig. 14.15. Cross-sections of prefabricated girders

The joints in girders are usually designed to lie directly at the side faces of columns. The bending moment in a joint near a support causes tension in the top and compression in the bottom part of the girder (Fig. 14.16, *a*). At a joint, the girder may bear against a reinforced concrete cantilever protruding from the column or a removable steel table installed for the ease of erection.

At any joints, the tensile force in the top part of the girder is taken up by connection bars welded during erection to steel embedded parts or stick-outs.

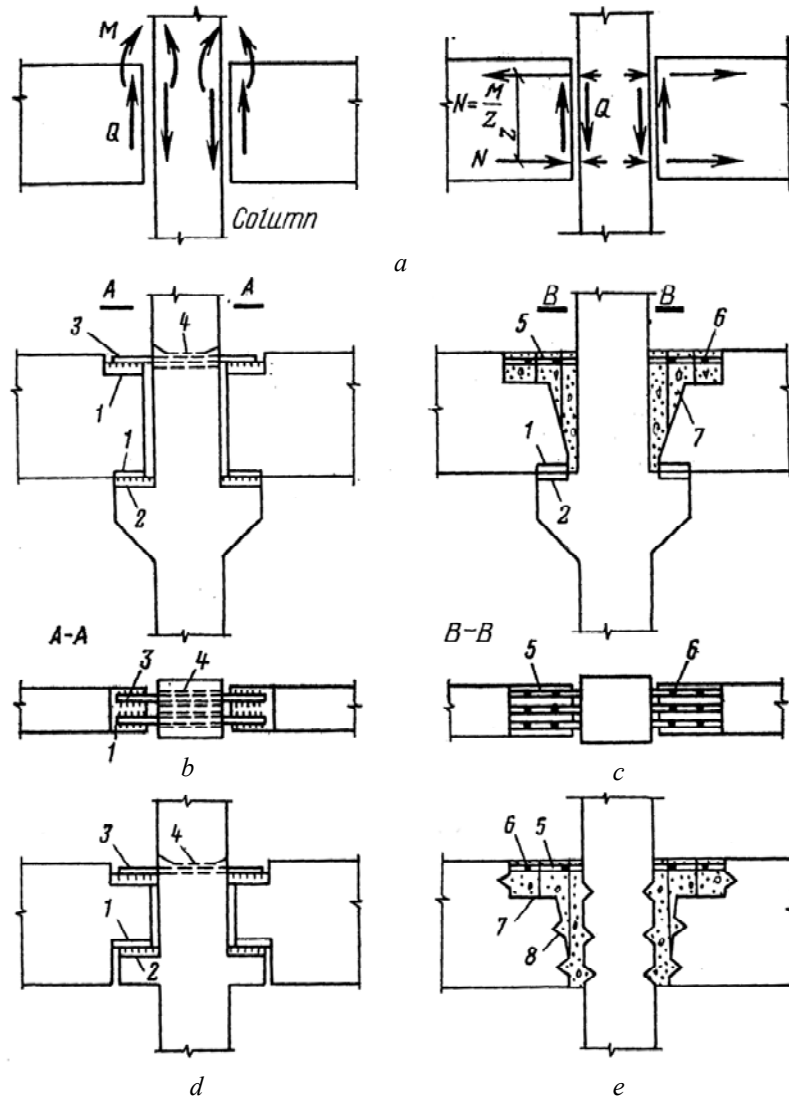


Fig. 14.16. Joint between a girder and a column:  
 1 — embedded steel part of the girder; 2 — embedded steel part of the column; 3 — connection bars; 4 — ducts in the column; 5 — additional bar; 6 — to be welded in a copper mould; 7 — in-situ concrete; 8 — prismatic cavity for concrete keys

During erection, the tensile connection bars may be placed in ducts left in the column (Fig. 14.16, *b* and *d*), or they may be concreted in the column during its manufacture. In the latter case, they should have

stick-outs 120mm long, which are to be welded at the site to the stick-outs in the girder (Fig. 14.16, *c* and *e*). If a joint carries a considerable bending moment at a support, and there are more than two bars to be welded, the alignment of the joint can be improved by inserting an additional bar welded to the column and girder stick-outs in a reusable copper mould (Fig. 14.16, *c*).

The compressive force in the bottom part of the girder may be transferred through the welded joints between the steel embedded parts of the girder and those of the column (Fig. 14.16, *b* and *d*) or through the concrete of the joint (Fig. 14.16, *c* and *e*).

Joints with cantilevers are capable of resisting considerable bending moments and shearing forces, so they are used where the floors are to carry heavy live loads, mainly in industrial buildings.

Concealed cantilever joints (with a part of the girder cut back) are difficult to manufacture because they require additional bar mats and embedded parts in the cantilevers, which increases the amount of steel and makes the manufacture more laborious. Besides, the load-bearing capacity and rigidity of the girder at the support are reduced (Fig. 14.16, *d*).

In non-cantilever joints (Fig. 14.16, *e*), as experiments have shown, shearing forces are taken up by the concrete of the joint and the concrete keys formed in the prismatic cavities in the side of the column and the end of the girder. For this type of joint, it is advisable to weld the stick-outs of the compressive steel in the girder to the angles of the supporting bed of the column (Fig. 14.16).

To make sure that a girder contains no excessive steel and also to check the girder cross-sections for strength, it is customary to construct what is known as the steel diagram (material diagram). The ordinates of the steel diagram are calculated as the moments due to the internal forces at a given cross-section of the girder

$$M_{\text{per}} = R_s A_s z_c \quad (14.17)$$

where  $A_s$  is cross-sectional area of the tensile steel in the section in question and  $z_c$  is the arm of the internal couple (Fig. 14.17).

At the theoretical cut-off points, the steel diagram appears as a staircase with vertical risers.

Where the steel diagram considerably departs from the bending moment diagram, the strength of the girder is excessive (that is, too much tensile steel is placed there); in contrast, where the stepped steel diagram crosses the bending moment diagram, the strength is insufficient.

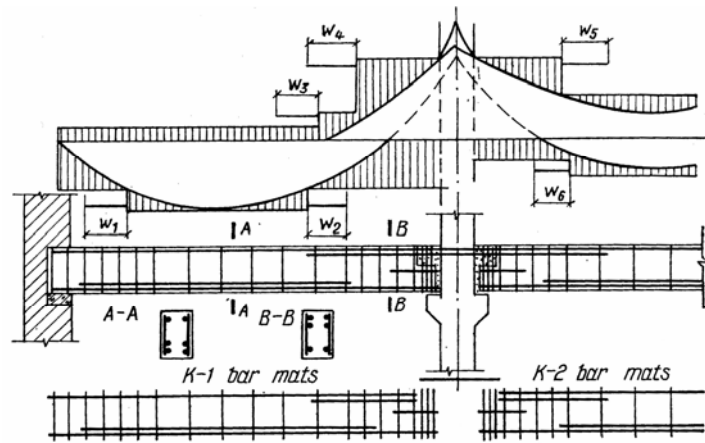


Fig. 14.17. Reinforcement and the steel diagram for a girder

#### ASSIGNMENTS TO CHAPTER 14

1. Speak on the classification reinforced-concrete overlapping? What side's relation makes plate work for a bend in two directions?
2. How is the constructive scheme of beam modular overlapping made up?
3. What forms of transverse cross-section do prefabricated plates of overlapping have?
4. How are modular plates (ribbed, continuous) reinforced?
5. What is the order of calculation of overlapping modular plates? How is preliminary plate section height determined?
6. How is calculation of a shelf of a modular ribbed plate for a local bend made?
7. What is the essence of calculation of statically indeterminate reinforced-concrete design in view of efforts redistribution? What is the advantage of this method?
8. What is the essence of static and kinematics calculation methods of the method of limiting balance?
9. What is the essence of constructive requirements for reinforcing statically indeterminate reinforced-concrete design which meet the calculation precondition by the method of limiting balance?
10. What is the order of calculation of a overlapping crossbar in the elastic scheme and possible schemes of loading with time loadings?
11. How is practical redistribution of the moments of continuous crossbar made out in calculation?
12. How is continuous crossbar reinforced and how is diagram of armature built?
13. What kind of joints of a modular crossbar with a column is used, what are their disadvantages and advantages?
14. How are column consoles reinforced and calculated for durability?

## 15. IN-SITU BEAM AND GIRDER CONSTRUCTION

### 15.1. Arrangement of Structural Members

A floor of the beam and girder construction consists essentially of girders, beams and a slab used in the short director (Fig. 15.1). All structural members are cast in-situ of class B15 concrete.

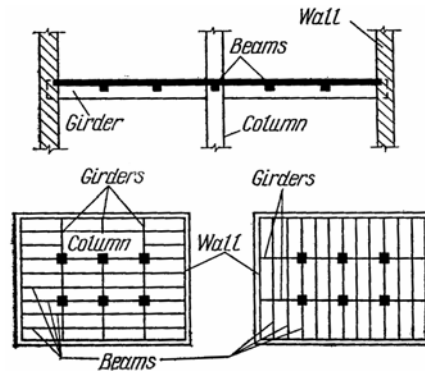


Fig. 15.1. Arrangement of girders, beams and slabs in the beam and girder floor

In this type of construction, the concrete is eliminated from the tension zone, leaving only ribs or joists carrying the tensile steel.

The topping of the ribs — the slab — is subjected to local bending along the span equal to the distance between the beams.

The beams are carried by, and monolithically connected with the girders, which, in turn, rest on columns and exterior bearing walls.

The girders may be placed along or across a building at every 6 or 8 m. The beams are distributed so that the axis of one of them passes through the axis of a column. The beam span may range between 5 and 7 m, and the slab span between 1,7 and 2,7 m.

From economic considerations, the slab should be as thin as possible. The minimum slab thickness is 60 mm for industrial buildings and 50 mm for civil buildings. If a floor is intended to carry heavy live loads, the slab may be made thicker. For example, at a live load of 10 to



15 kN/m<sup>2</sup> and a slab span of 2,2 to 2,7 m, the slab thickness is taken as 80 to 100 mm (depending on the amount of reinforcement).

As a rule, the beam depth is taken as  $1/12l$  to  $1/20l$  and that of girders as  $1/8l$  – to  $1/15l$ . The width of girder cross-section, designated may range between  $0,4h$  and  $0,5h$ .

### 15.2. Design of Slabs, Beams and Girders

The effective span of a slab,  $l_0$ , is taken equal to the clear distance between the beams. If one end of a slab is carried by a bearing wall, the effective slab span is assumed equal to the distance from the axis of the support on the wall to the rib side (Fig. 15.2). In plan, the design of a slab is carried out for a strip 1 m wide.

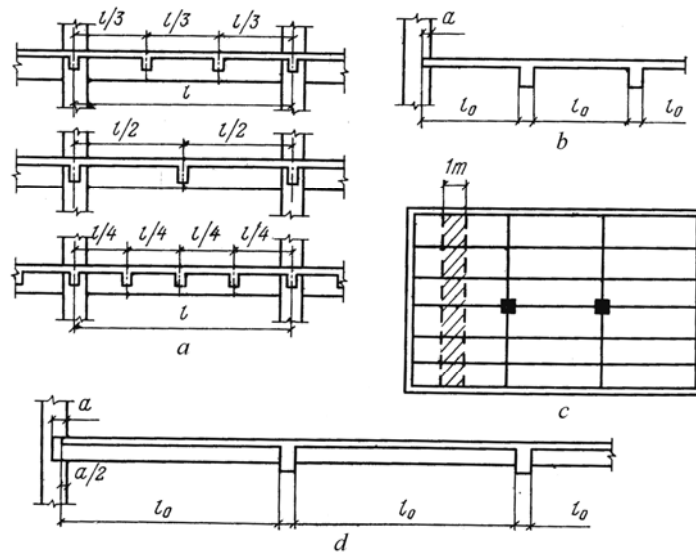


Fig. 15.2. Beams, girders and slabs of the beam and girder floor

The effective span of a beam, likewise designated  $l_0$ , is taken equal to the clear distance between the girders, or, if one end of a beam rests on a bearing wall, to the distance between the axis of the support on the wall to the girder side.

Bending moments in multispans slabs and beams having equal spans or spans differing by not more than 20 % from one another are determined with allowance for the distribution of moments, assuming that the moment in the span is equal to that at the nearest intermediate support. With a uniformly distributed load, the bending moments  $M_{sup}$ ,

at the middle supports of a multispan beam (Fig. 15.3) are equal to each other. Using Eq., which describes the equilibrium of forces in the section in the mid-span, we get

$$M_1 + M_{\text{sup}}/2 + M_{\text{sup}}/2 = (g + v)l^2/8. \quad (15.1)$$

Hence,

$$M = M_1 = M_{\text{sup}} = (g + v)l^2/16. \quad (15.2)$$

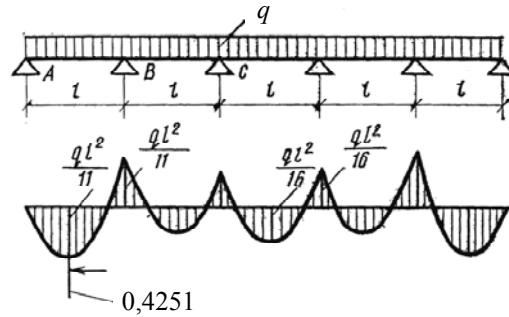


Fig. 15.3. Design of a multispan slab and beams

In the first span, the maximum bending moment is induced in the section lying within  $a = 0.425 l$  of the free support. In this case

$$M_0 = 0,123(g + v)l^2. \quad (15.3)$$

Using Eq. and recalling that  $M_A = 0$ , we obtain

$$M_1 = 0,123(g + v)l^2 - 0,425M_B. \quad (15.4)$$

If we assume the bending moment at the first intermediate support as

$$M_B = (g + v)l^2/14 \quad (15.5)$$

we shall get the bending moment in the first span:

$$M_1 = (g + v)l^2/11. \quad (15.6)$$

Making use of  $M = M_1 = M_B$  we may write

$$M = (g + v)l^2/11,6. \quad (15.7)$$

Rounding off the denominator so that the error in the bending moment is less than +5%, the bending moment at the first intermediate support and in the first span will be

$$M = (g + v)l^2/11. \quad (15.8)$$

For slabs supported around the periphery by monolithically connected beams and girders, the bending moments in the middle spans and at the middle supports (determined in the state of ultimate equilibrium without allowance for thrust) are reduced by 20 %. For beams, the resultant bending moment diagram is plotted for two loading systems (Fig. 15.4) namely:

- 1) the total load  $g + v$  in the odd spans and the assumed dead load  $g + 0,25v$  in the even spans;
- 2) the total load  $g + v$  in the even spans and the assumed dead load  $g + 0,25v$  in the odd spans.

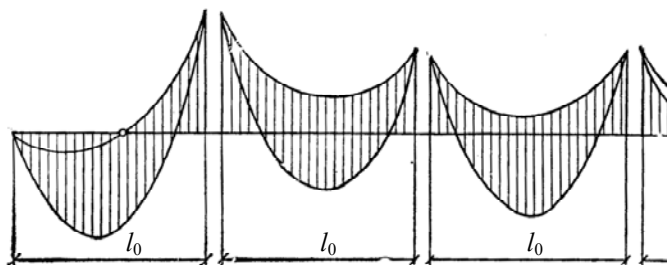


Fig. 15.4. Resultant bending moment diagram for a beam

The fictitious load is included into design calculations in order to determine the actual negative bending moments in the beam spans. The girders provide additional fixation, which prevents the beam supports from angular deflections, thereby reducing the effect that the live load applied to the loaded spans has on the unloaded spans.

The shearing forces in the beam are assumed as follows:

- at the first (last) free support

$$Q = 0,4ql; \quad (15.9)$$

- on the left of the first intermediate support

$$Q = 0,6 ql; \quad (15.10)$$

- on the right of the first intermediate support and at the remaining supports

$$Q = 0,5 ql. \quad (15.11)$$

When calculating the necessary cross-sectional area of the members, the first thing to do is to find the cross-sectional area of the beam required to resist the bending moment at the first intermediate support.

Since the design is carried out in terms of equalized moments,  $\xi = 0,35$ . At the support, the beam is subjected to a negative bending moment, the slab is in the tension zone, and the design should be carried out as for a rectangular section with the effective depth taken as

$$h_0 = 1,8(M/R_c b)^{1/2}. \quad (15.12)$$

Once the final standard cross-sectional dimensions,  $b$  and  $h$ , are determined, the next step is to decide on the amount of load-bearing reinforcement, for four design normal sections: in the first and the middle spans as for a T-section, and at the first intermediate and the middle supports as for a rectangular section. The amount of steel required to resist the negative bending moment in the middle span is determined as for a rectangular section.

The amount of transverse reinforcement is calculated for three diagonal sections: those to the left and right of the first intermediate support and at the first (or last) free support. The design procedures given for girders in the precast slab and girder construction hold for girders in the in-situ beam and girder construction.

The girder is subjected to the concentrated load from the beam support (which is determined on the assumption of a continuous beam only in the case of two-span beams). In addition, account should be taken of the self-weight of the girder.

At the points where the beams and girders intersect over a column, there are three layers of reinforcement, namely the top reinforcement of the slab, beam and girder. So at the support, the concrete cover of the girder is taken equal to  $a = 60$  to  $90$  mm, as a consequence, the effective depth of the girder is  $h_0 = h - (60 \text{ to } 90)$  mm, depending on the number of reinforcement rows.

When determining the cross-sectional dimensions of the girder on the basis of bending moments, it is necessary to keep in mind that it resists the positive moment in the span as a T-beam with a flange width of  $b'_f = 1/3$ , and the negative moment at the support as a rectangular beam of rib with  $b$ .

### **15.3. Reinforcement and Arrangement of Slabs, Beams and Girders**

According to the bending moment diagram, multispans slabs are reinforced with rolled welded-wire fabric having load-bearing wires running in the longitudinal direction; in the formwork, the fabric is

unrolled across the beams (Fig. 15.5). The fabric is bent up within  $0.25l$  of the axis of a support (where the bending moments are zero) and placed over the top bar mats of the beams. In the first span, the main fabric is covered by an additional piece which extends  $0.25l$  beyond the supports. If a stronger load-bearing reinforcement (6mm or more in diameter) is required, the slabs are separately reinforced in the span and at the supports with rolled welded-wire fabric with transverse load-bearing wires (Fig. 15.5).

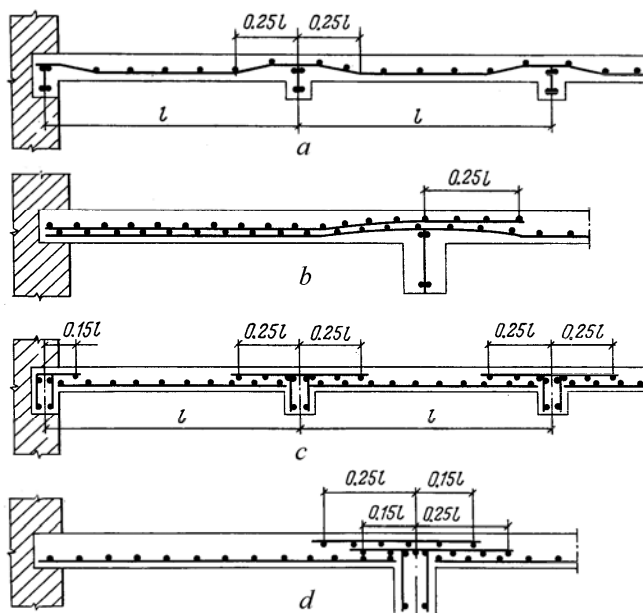


Fig. 15.5. Reinforcement of a multispan slab

In the span, the beams are reinforced with bar mats (as a rule, two bar mats are used), which, prior to installation in the formwork, are assembled into reinforcing cages by welding horizontal transverse bars to them. These cages extend as far as the girder faces where they are tied together at the bottom by joint bars (Fig. 15.5). At the supports, the beams are reinforced with welded-wire fabric with transverse load-bearing wires. For this purpose, rolled welded-wire fabric is placed over the girder, or flat fabric pieces are distributed over the entire length above the girders. Near the columns, the fabric is cut off to be replaced by additional bars or pieces of fabric having the same cross-sectional

area as the load-bearing wires within the column width. At the supports, the effective cross-sectional area of the beam tensile steel is taken equal to the total area of the load-bearing wires in the over-the-support fabric located between the axes of adjacent beams.

With large-span beams, the tensile steel at the supports may be provided by two partly overlapping fabric pieces. At the first (or last) support, the beams are reinforced with additional welded-wire fabric placed for structural reasons.

The cut-off points for the over-the-support fabric are located according to the negative moment diagram. Where  $v/g \leq 3$ , one of the fabric pieces is cut off within  $0,25l$  and the other within  $0,3l$  of the support axis. Past the cut-off points in the span, negative moments are taken up by the top reinforcement of the beam.

In the span, the girders are reinforced with two or three bar mats assembled into a reinforcing cage prior to installation in the form-work. Two bar mats extend as far as the column face, and the third (if any) is cut off according to the bending moment diagram. It is also possible to cut off some bars in the span. At the supports, the girder is reinforced with independent cages inserted through the reinforcing cages of the columns (Fig. 15.7, 15.8).

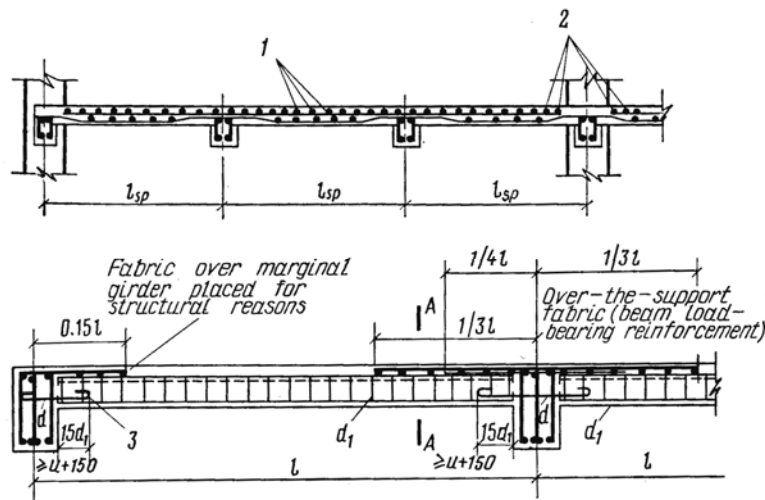


Fig. 15.6 Reinforcement of a beam

The cut-off points for the bar mats and separate bars are located by the reinforcement diagram.

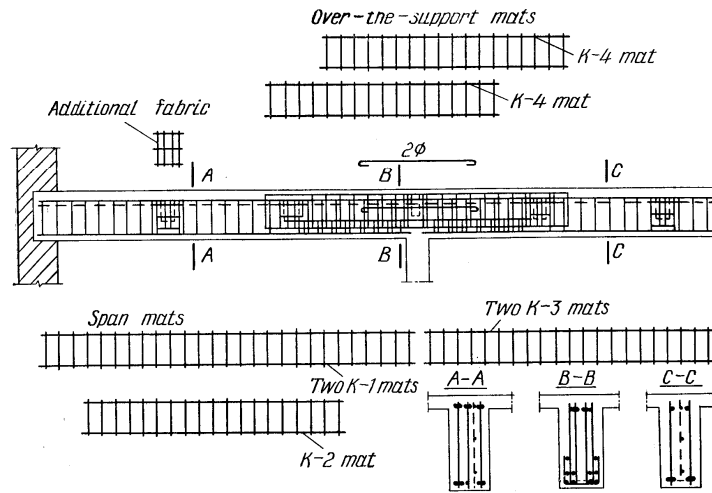


Fig. 15.7. Reinforcement of a girder

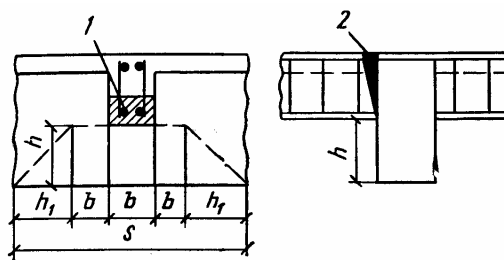


Fig. 15.8. Transmission of load to a girder:  
 1 — actual area of concentrated load transmission;  
 2 — crack in the tension zone

#### ASSIGNMENTS TO CHAPTER 15

1. Name all the main elements of multistoried frame building structures.
2. What methods provide spacious rigidity of multistoried frame buildings?
3. Name the main types of joints between columns and between girders, crossbars and columns.
4. Name constructive solutions of multistoried industrial buildings.
5. How are preliminary dimensions of cross-sections of frame elements chosen?
6. How are multistoried frames calculated for vertical and horizontal loads?

7. How are calculations of forces for girders and frame parts chosen?
8. What is the essence of configuration of the constructive scheme of ridge monolithic overlapping with beam plates?
9. How are beam plates reinforced and calculated? How is the influence of thrust in limiting balance taken into account?
10. What calculation scheme of a minor beam of monolithic overlapping is used and how is it reinforced?
11. How is the effort in a minor beam determined?
12. What is a specific feature of calculation of strength of normal sections of a minor beam of monolithic overlapping in span and on a support?
13. What is a specific feature of calculation and designing of the main beam of monolithic beam of ribbed overlapping?
14. What is a specific feature of the constructive scheme of ribbed monolithic overlapping with plates, which rest on a contour?
15. How are monolithic plates, which rest on a contour, reinforced?
16. How are plates, which rest on a contour, calculated with the method of limiting balance?
17. What is a specific feature of calculation of beams of monolithic overlapping with plates, which rest on a contour?
18. What is the constructive scheme of overlapping with plates which rest on three sides? What are economical advantages of this kind of overlapping?
19. How are plates, which rest on three sides, designed and calculated?
20. What is the essence of a prefabricated-monolithic design of beam overlapping?
21. What buildings is non-beam overlapping used for? What are the advantages and disadvantages?
22. What elements is the design of non-beam modular overlapping made up of?
23. How are elements of non-beam modular overlapping calculated?
24. How is non-beam monolithic overlapping reinforced?
25. How are the sizes of capitals of non-beam monolithic overlapping determined?
26. What order is non-beam monolithic overlapping calculated by the method of limiting balance?
27. What is a specific feature of a design of non-beam prefabricated-monolithic overlapping?
28. What reinforced-concrete plates of rib overlapping are referred to beam ones?
29. How can we receive optimum equal moment layout schemes of continuous plates and beams of the rib overlapping loaded with regularly distributed loading?



## 16. DESIGN OF ONE-STOREYED INDUSTRIAL BUILDINGS

### 16.1. Construction Types. Structural Elements

Buildings most commonly used in the metal-making, mechanical engineering, textile, food and other industries are of one-storeyed skeleton construction (Fig. 16.1, *a*). A major distinction of such buildings is that they are equipped with overhead cranes traveling on runways supported on columns or carried by roof members. The roof of a one-storeyed industrial building may be composed of linear beams and slabs, or space shells.

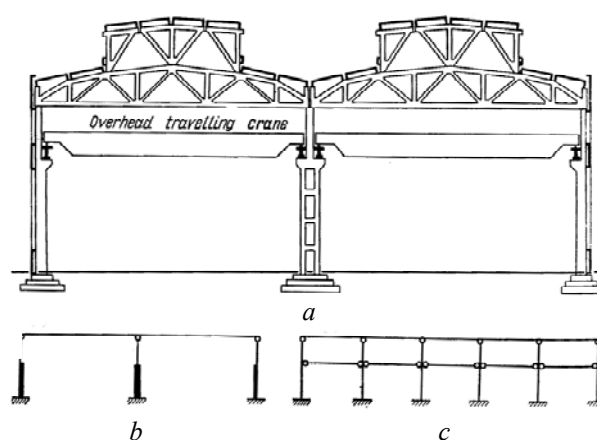


Fig. 16.1. One-storeyed crane-supporting industrial building:  
*a* — structural cross section; *b* — transverse frame; *c* — longitudinal frame

The structural elements of a one-storeyed skeleton-construction building with a beam-and-slab roof include columns (or posts) built into their footings, roof girders (such as beams, trusses, and arches) supported on the columns, roof slabs placed on the girders, crane beams, and monitors intended to secure better light or ventilation. The main structural element of the skeleton is the transverse frame formed by the columns and girders.

A one-storeyed skeleton-construction building owes its spatial rigidity and stability to the columns fixed in their footings, transverse frames, and longitudinal frames composed of the same columns, roof members, crane beams, and vertical braces (Fig. 16.1, *b* and *c*).

One-storeyed industrial buildings (without cranes or equipped with floor-type cranes) may have flat roofs with no monitors (Fig. 16.2).

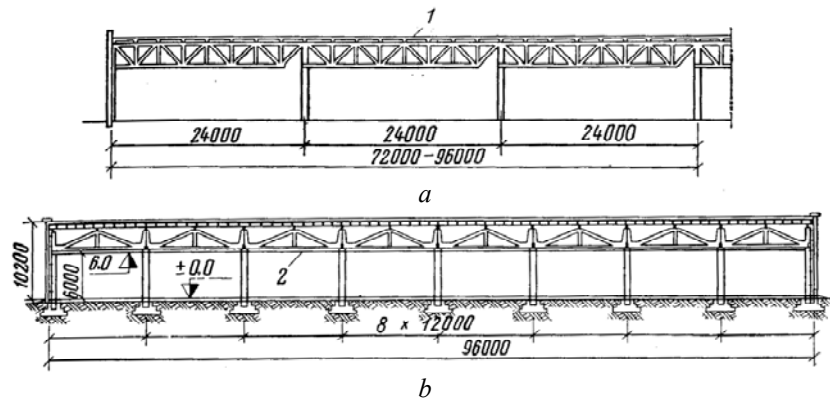


Fig. 16.2. Flat-roof one-storeyed industrial building:  
*a* — cross section; *b* — longitudinal section;  
*1* — flat roof; *2* — suspended ceiling

## 16.2. Overhead Travelling Cranes

An overhead traveling crane is a lifting plant carried on a horizontal girder spanning between rails above window level at each side of a workshop. It consists of a hoisting crab with a hook on a flexible or rigid suspension, which can itself travel from end to end of the girder (Fig. 16.3). The whole area of the workshop between the rails can thus be traversed by the crab. In the longitudinal direction (along the building), the crane is moved by the girder, which rides the rail track. For this purpose, the girder is equipped with wheels whose number for cranes with a lifting power of up to 50 tons is four (two wheels per rail).

Overhead traveling cranes are classed according to their duty and lifting power. Light-duty cranes are designed for irregular operation and slow motion (not more than 60 m/min); they are used in the machine rooms of thermal power stations, repair shops, etc. Medium-duty cranes are intended for active service and normal speed of motion (up to 100 m/min); they are used in machine shops, assembly departments, and molding shops of



where  $Q$  is the lifting power of the crane;  $Q_g$  is the weight of the girder; and  $G$  is the weight of the crab.

The basic horizontal load directed across the runways and induced by crab braking is taken as follows:

– for cranes with flexible load suspension

$$H_n = 0,05(Q + G); \quad (16.2)$$

– for cranes with rigid load suspension

$$H_n = 0,1(Q + G). \quad (16.3)$$

The load  $H_n$  may be directed inside or outside the span; it is transferred to one runway and is uniformly distributed between the two wheels.

The basic horizontal load applied along the rails and brought about by girder braking (by one brake wheel in cranes with a lifting power of up to 50 tons) is

$$H_{nmax} = 0,1 H_{max}. \quad (16.4)$$

In the design of structural elements for vertical and horizontal crane loads, the overload factor,  $\gamma_f$ , is taken equal to 1.1.

When a crane is moving on, the crane beams are subjected to dynamic effects caused by sudden application of loads and shocks due to the unevenness of the runways, especially at rail joints. So, in the bearing-capacity analysis of crane beams, the design vertical and horizontal loads imposed by the crane are multiplied by the dynamic factor,  $k_{dyn} = 1.1$ . The dynamic effect of an overhead traveling crane is assumed not to be shared by other structural elements.

### 16.3. Building Proportioning

According to the manufacturing process involved, the grid size adopted for one-storeyed skeleton-type buildings equipped with overhead traveling cranes may be 12×18 m, 12×24 m and 12×30 m, or 6×18 m, 6×24 m and 6×30 m. Most commonly, the bay length is 12 m. If, with such a bay, the wall panels are 6m long, intermediate (trellis-work) columns are additionally installed along the sides. When the bay of a building is 12 m, the roof trusses may be placed at 6m on centres, in which case, the necessary intermediate supports are provided by the secondary trusses (Fig. 16.4).

Experience shows that the best economical precast reinforced concrete roofs are those with a bay of 12 m and without secondary trusses.

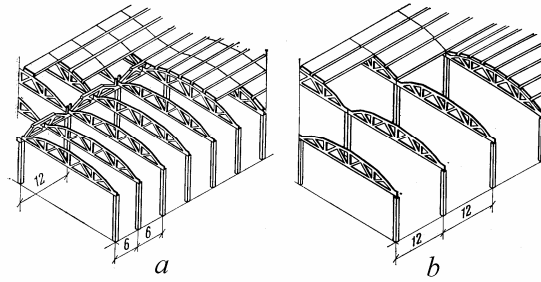


Fig. 16.4. Structural systems for a building:  
*a* — with a bay of 6 m, with secondary trusses;  
*b* — with a bay of 12 m, without secondary trusses

#### 16.4. Transverse Frames

The girders of transverse frames may be with or without holes in the body. Their connection with the struts may be rigid or hinged. The shape of a girder, its construction, and type of connection with struts depend on the space to be spanned, roof type, and the manufacturing process and the erection technique adopted.

Rigid connection of the girder and columns of a frame reduces the bending moments, but such a frame must be designed individually, because the load applied to the columns gives rise to bending moments in the girder, and the load applied to the girder induces bending moments in the columns (Fig. 16.5, *a*).

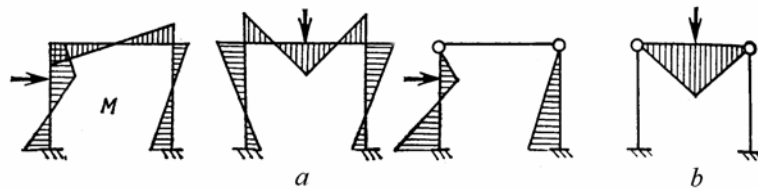


Fig. 16.5. To the selection of the most economical transverse frame structure; bending moment diagrams:  
*a* — with rigid joints between the girder and columns; *b* — with hinged joints

With hinged joints, the girder and columns may be standardized separately because the loads applied to one member do not induce any bending moment in another (Fig. 16.5, *b*). Hinged joints between girders

and columns are easy to make; also, the component members are simple in shape and can readily be adapted to streamlined manufacture. Because of this, one-storeyed frames with hinged joints are considered to be most economical and have been adopted as standard structures.

Physically, a hinge is produced by field-welding the supporting steel sheet of a girder to the embedded steel part at the end face of a column (Fig. 16.6).

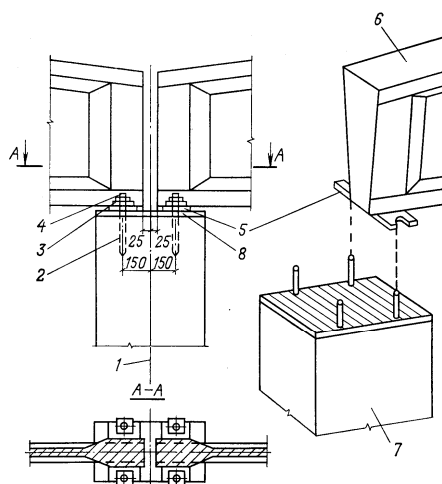


Fig. 16.6. Hinge between a girder and column produced by field welding and anchor bolts:

- 1 — row axis; 2 — anchors; 3 — washer; 4 — nut; 5 — steel plate,  $\delta = 12$  mm; 6 — girder; 7 — column; 8 — end-face steel plate

With spans of up to 18 m, girders are in the form of prestressed beams; with spans of 24 and 30 m, they are in the form of trusses.

The columns of a skeleton-construction building may be of the solid single-piece type rectangular in cross section, or of the openwork two-piece type (Fig. 16.7). When choosing a column type, it is necessary to take into account the lifting power of the overhead traveling crane to be used and the height of the building. Solid single piece columns are used if the lifting power of the overhead crane will not exceed 30 tonnes, and the building to be erected is relatively low. Open-work two-piece columns are intended for cranes with a lifting power of 30 tonnes or more, and for buildings more than 12 m high. The cross-sectional dimensions of a column above the crane beam it supports are assigned in view of the fact that the girders rest directly on the end face of the column without any special cantilevers. The depth of the column cross section is-taken as  $h_2 = 500$  or 600 mm for intermediate columns, and

$h_2 = 380$  or  $600$  mm for marginal columns. The width of intermediate and marginal columns is adopted as  $b = 400$  to  $600$  mm (the greater dimensions are specified for a bay of 12 m).

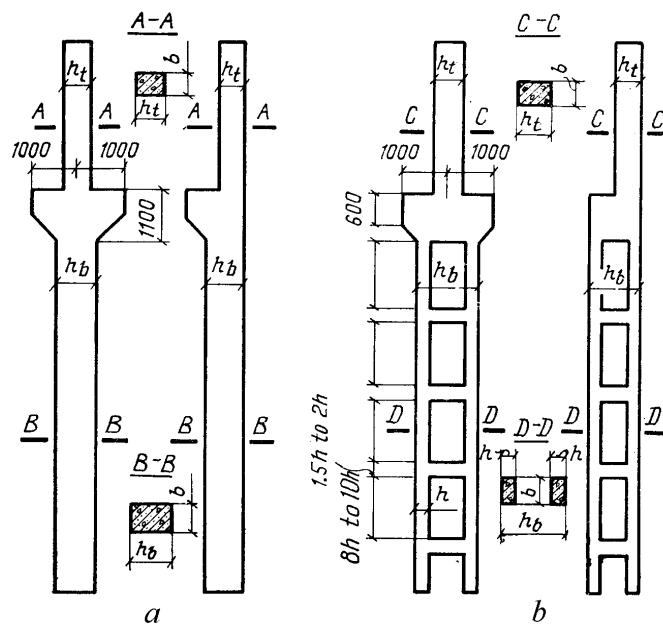


Fig. 16.7. Columns for one-storeyed buildings:  
*a* — solid single-piece columns rectangular in cross section;  
*b* — open-work two-piece columns

The cross-sectional dimensions of solid single-piece columns below the crane beams they support are mainly assigned to ensure the required bearing capacity and adequate rigidity so that horizontal deflections of the columns in the plane of a transverse frame would not jam the crane girder. Experience has shown that a column may be considered adequately rigid if the depth of its cross section is  $h_1 = 1/10H_1$  to  $1/14H_1$ .

Below the crane beams, open-work two-piece columns consist, as their name implies, of two pieces or side members joined by short spacers. In the bottom part of intermediate columns, the centre line of a side member may be set off from the axis of the crane beam. Here, the depth of the cross section for a two-piece column is taken as  $h_1 = 1200$  to  $1600$  mm. In outer columns,  $h_1 = 1000$  to  $1300$  mm. The depth of the cross-section for a side member is taken as  $h = 250$  or  $300$  mm, and the width as  $b = 500$  or  $600$  mm.

The distance in two-piece columns is set  $8h$  to  $10h$ . They are disposed so that the distance from floor level to the bottom of the first spacer should not be less than 1,8 m, and a convenient passage would be left between the side members. The bottom spacer is installed below floor level. The depth of the cross-section for a spacer is taken as  $1,5h$  to  $2h$ , and its width is assumed as that of a side member.

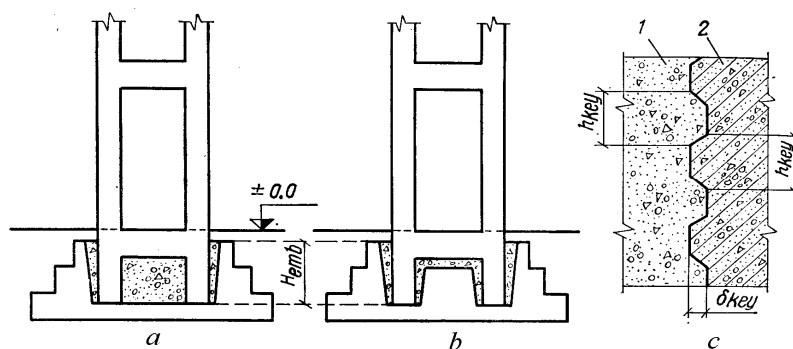


Fig. 16.8. Connection between a two-piece column and a footing:  
*a* — single socket footing; *b* — two independent footings; *c* — concrete keys;  
*1* — in-situ concrete; *2* — column

A two-piece column may be supported by a single socket footing, or by two independent footings. In the latter case, — the concrete to be placed in-situ is reduced (Fig. 16.8). The depth (in meters) to which a column should be embedded in its socket is taken as

$$H_{ah} = 0,5 + 0,33h_l \text{ or } H_{ah} = 1,5b,$$

whichever is the greater. In addition, the depth of embedment should be large enough to ensure the required anchorage length for the load-bearing reinforcement. If a tensile force is likely to arise in one of the side members, concrete keys should be provided between the column and the concrete cast in-situ.

In length, single- and two-piece columns are usually manufactured integral. Division into units to reduce the weight of the blocks to be handled is seldom done as this inevitably leads to problems in making the intermediate joints.

A likely reinforcement for single- and two-piece columns is shown in Fig. 16.9. Intermediate columns subjected to both positive and negative moments are reinforced symmetrically. The concrete class used for columns may vary from B15 to B30.



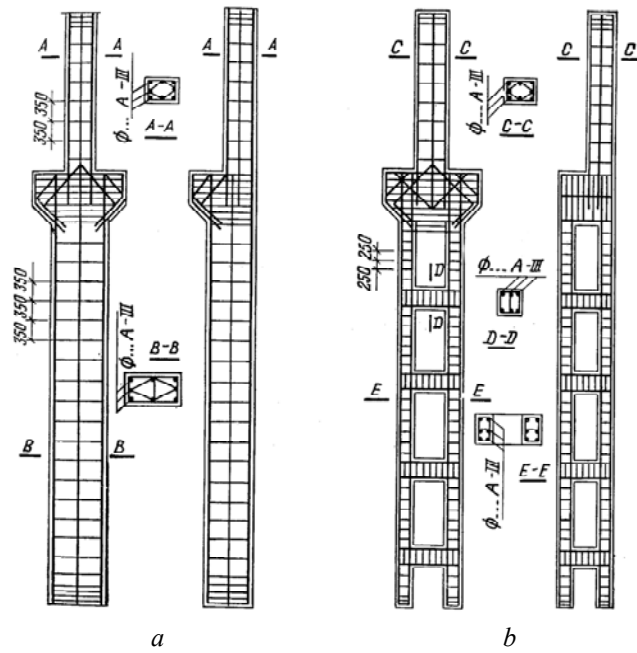


Fig. 16.9. Reinforcement of one-storeyed building columns:  
*a* — single-piece rectangular columns; *b* — open-work two-piece columns

### 16.5. Monitors

Monitors consist essentially of transverse trusses and struts carrying roof slabs, and supported on the girders of transverse frames. The space between the struts accommodates filler slabs. The width of a monitor and the height of a clerestory should be large enough to secure adequate illumination of the workshop. As a rule, this is achieved with monitors 0,3/ to 0,4/ wide. In order that standard structural members may be used, monitors for spans of up to 18m are taken to be 6m wide, and those for spans of 24 and 30 m, 12 m wide (Fig. 16.10). The load-bearing members of monitors are fastened to the girders of transverse frames by assembly bolts, and by welding together their embedded steel parts.

### 16.6. Bracing

The function of what we shall call vertical and lateral bracing is to: 1) ensure the overall rigidity of the roof; 2) enhance the stability of the compressive chords of transverse, frame girders; 3) resist wind

forces against the ends of a building; and 4) take up the lateral thrust of overhead traveling cranes. Bracing interacts with the main elements of the building skeleton and contributes to its spatial rigidity.

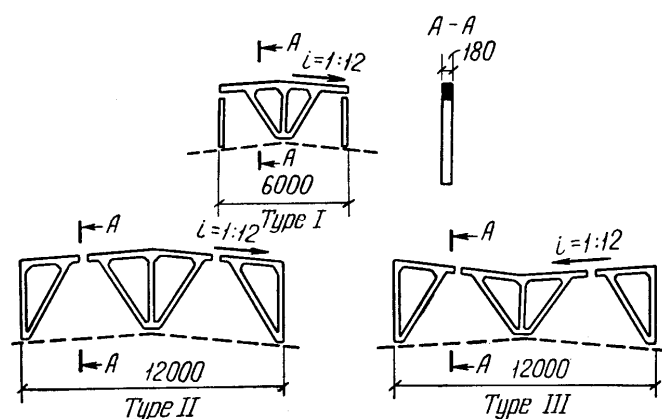


Fig. 16.10. Construction of monitor trusses

**Vertical Bracing.** The forces induced by longitudinal horizontal loads (such as wind against the ends of a building, thrust of cranes, and so on) are resisted by the longitudinal frame of the building, with the roof acting as its girder. The roof slabs are connected to the columns via beams or trusses whose rigidity at right angles to their plane is small. Therefore, if no bracing were provided, a horizontal force applied to the roof might cause the girders to deform considerably at right angles to their plane (Fig. 16.11, *a*), and a horizontal force applied to one of the columns and not shared by other columns might significantly deform it (Fig. 16.11, *b*). Vertical bracing in the plane of the building columns is provided to make the roof act as a rigid structure, which will not deform in the longitudinal direction.

Vertical lattice girders made of steel angles are installed between the columns forming the marginal sections of the building. They are combined by reinforced concrete or steel-angle ties at the top of the columns (Fig. 16.12). Lateral forces on the sides of the building are resisted by vertical cross bracing in the plane of the ends of the building. If the depth of the girder is small (under 800 mm), and the girder has a supporting rib able to take up a lateral force, it will be enough to brace the girders rigidly to the columns by knee braces. In this case, the embedded steel parts of the girders should be welded to those of the

columns which should be designed to resist the moment  $M - W_h$  and the vertical force  $F$  (Fig. 16.11). Vertical steel-angle braces between the columns are installed in each longitudinal row in the middle of each strain block. The braces are welded to the embedded steel parts of the columns.

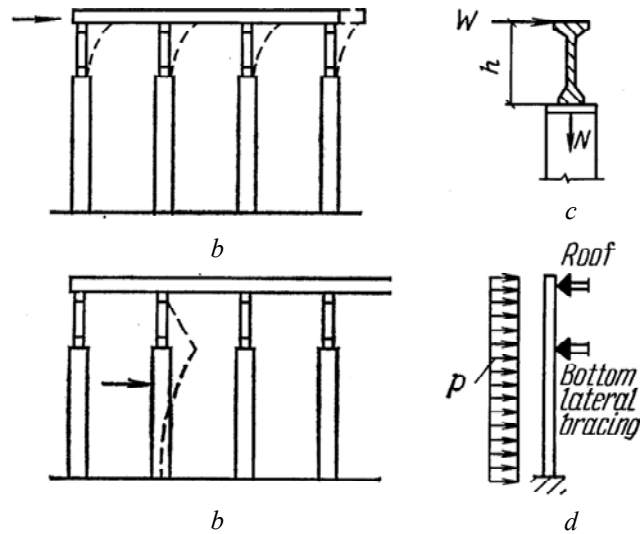


Fig. 16.11. Building skeleton deformations induced by lateral loads and loading systems

**Bottom Lateral Bracing.** Wind forces against the ends of a building cause the columns of the end walls to bend. To reduce the effective length of these columns, the roof is used as a lateral support (Fig. 16.11). In high and large-span buildings, it is advisable to make a lateral support for the end wall by providing cross bracing in the plane of the bottom chord of the girder (Fig. 16.12). An additional support for the end wall may also be furnished by cross bracing in the top plane of the crane beams. Bottom lateral braces are made of cross steel angles, which interact with the bottom chord of the marginal girder. The forces due to the lateral bottom bracing are transmitted through the vertical braces to all columns of the strain block.

**Top Lateral Bracing.** To hold the tops of the girders in position, bracing is also provided in the plane of the top chord. For this purpose, the roof slabs are connected to the girders by welding together their embedded steel parts. In monitor-type buildings, the effective length of the compressive chord of a girder, which is likely to lose its position, is made equal to the width of the monitor. To reduce the effective length

of the compressive chord of the girder, ties are installed along the axis of the monitor and fastened to the cross braces made of steel angles in the marginal bays of the strain block (Fig. 16.12). If the monitor does not extend the entire length of the strain block, no top lateral bracing is provided because, beyond the monitor, the reinforced concrete roof slabs themselves act as a rigid diaphragm. In this case, the ties are fastened to the roof members of the marginal bay.

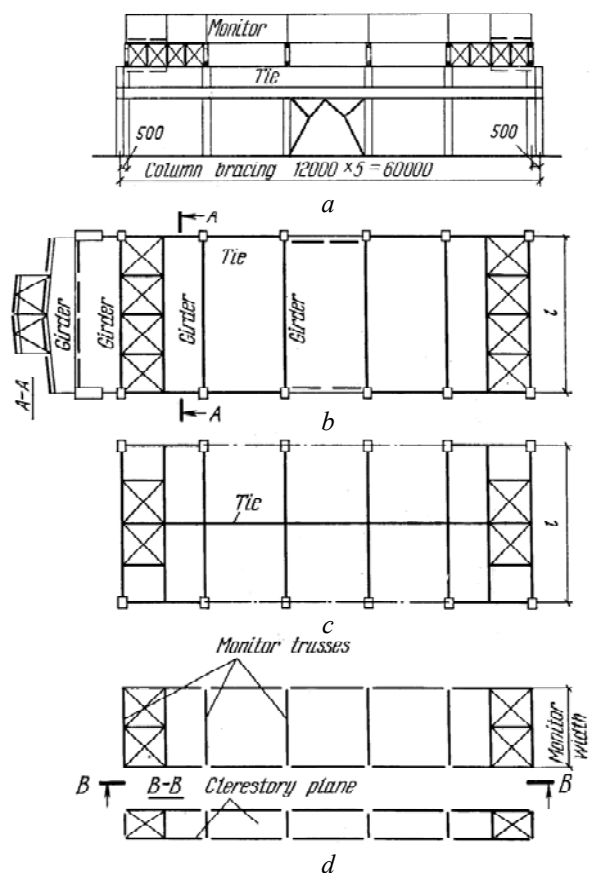


Fig. 16.12. Bracing:  
*a* — vertical bracing; *b* — bottom lateral bracing;  
*c* — top lateral bracing; *d* — monitor bracing

**Monitor Bracing.** Monitor trusses are combined in a rigid space block by steel braces – vertical in the plane of the clerestory and lateral in the plane of the monitor roof (Fig. 16.12).

## 16.7. Crane Beams

Since crane beams are subjected to dynamic loads imposed by overhead traveling cranes, it is recommended to confine prestressed reinforced concrete beams to light- and medium-duty cranes with a lifting power of not more than 30 tones. For heavy- and medium-duty cranes with a lifting power of 50 tones and more, steel crane beams are a better choice.

The best crane beams are those of I-section (Fig. 16.13). The wide top flange of the beam adds lateral stiffness, reduces the deflections in the case of lateral crane braking, and also improves the erection and service conditions for the runways and the crane.

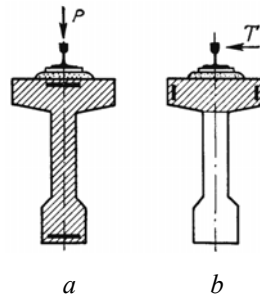


Fig. 16.13. Design sections of a crane beam:  
*a* — for vertical load; *b* — for lateral load

The bottom beam flange accommodates the prestressed reinforcement and provides additional strength when the prestress is transferred to the concrete. The design for vertical loads is carried out as for a T-section with its flange in compression, and that for horizontal loads is done as for a rectangular cross-section taken as the top flange of the I-beam in question.

The depth of crane beams is taken as  $h = 1/8l$  to  $1/10l$ , the depth of the top flange as  $h'_f = 1/7h$  to  $1/8h$ , and the width of the top flange as  $b'_f = 1/10l$  to  $1/20l$ . In order that runways may conveniently be fastened and straightened, the top flange should be not less than 500 to 650 mm wide. In standard 6-m crane beams are 1000 mm deep and 12-m beams are 1400 mm deep (Fig. 16.14).

To facilitate manufacture and erection, precast 6- and 12m crane beams are cut into submultiple lengths subsequently joined together at the columns.

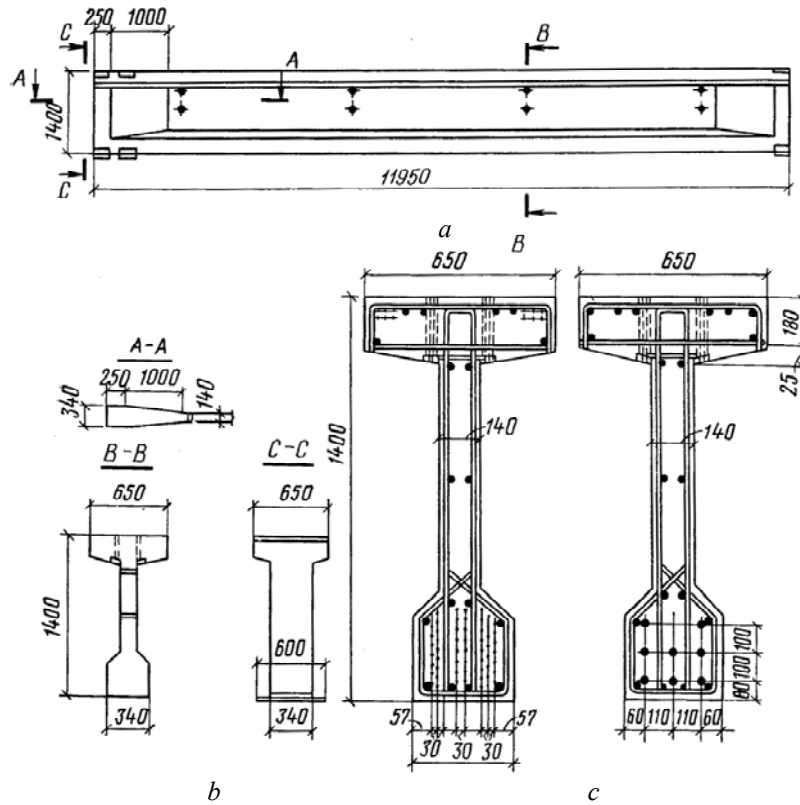


Fig. 16.14. 12-m prestressed crane beam:  
*a* — general view; *b* — prestressed wire reinforcement;  
*c* — prestressed bar reinforcement

In the strength analysis of crane beams, the design loads imposed by overhead traveling cranes are multiplied by the overload factor,  $\gamma_f = 1.2$  and the dynamic factor,  $k_{dyn} = 1.1$ . The design vertical load thus found is

$$F_{max} = \gamma_f \gamma_n F_{n\ max} \quad (16.5)$$

and the design horizontal load (per crane wheel) is

$$H_{max} = 0,5 \gamma_f \gamma_n F_{max}. \quad (16.6)$$

The horizontal load,  $H_{n\ max}$ , is actually applied at the head of the rail, but for simplicity and neglecting the small eccentricity, it is assumed to be applied at the middle of the flange depth.

Strength analysis is carried out for the design load due to two cranes of the same lifting power placed close to each other, and multiplied by a load combination factor of 0,85 (for light- and medium-duty cranes). The live load due to the cranes is distributed across the crane beam in such a way that  $M$  and  $Q$  would be maximal at several sections within the span. The distances between the four forces transmitted through the wheels are adopted according to the overall width and the wheel base of the cranes (Fig. 16.15).

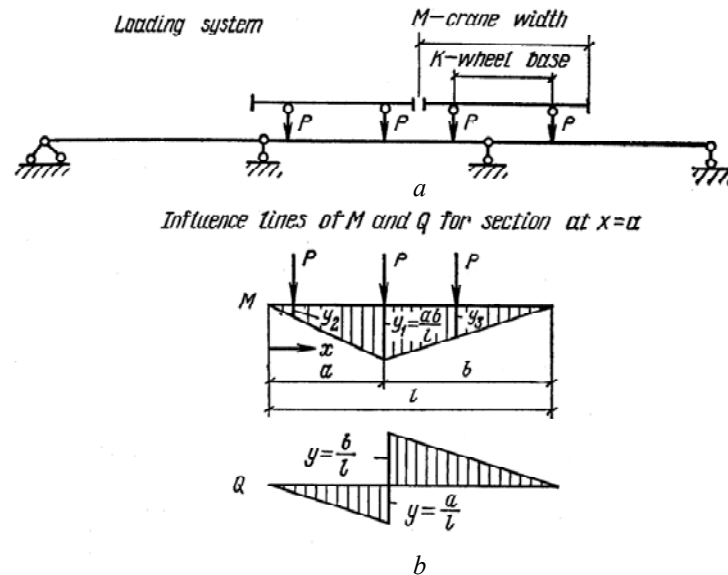


Fig. 16.15. To the design of a crane beam

The design is carried out by plotting the influence lines of  $M$  and  $Q$  with one of the forces applied at the apex (Fig. 16.15). The maximum forces are determined by adding together the products of the forces and the corresponding ordinates.

For example, the maximum bending moment at a given section is

$$M = F_1 y_1 + F_2 y_2 + \dots = \Sigma F y. \quad (16.7)$$

Once the forces are found, the resultant diagrams for  $M$  and  $Q$  can readily be plotted. The ordinates of the resultant diagrams may be looked up in relevant tables given in handbooks.

Fatigue analysis is done for the design vertical load due to one overhead traveling crane. This load is found by multiplying the basic

load by 0.5. The dynamic factor is not included in this analysis. The stresses in the concrete and the reinforcing steel are determined on the assumption that the transformed section behaves elastically under the external load and the prestress,  $N_p$ , taken with allowance for all losses. Here, the following conditions should be satisfied

$$\sigma_{c,\max} \leq m_{c2}R_c \text{ and } \sigma_{s,\max} \leq m_{c1}R_s.$$

If the beam is to carry an alternating load, no tensile stresses are allowed in the concrete of the zone, which is assumed to be in compression. The fatigue analysis of inclined-sections is carried out on the assumption that the principal tensile forces acting at the centroid of the transformed section should be taken up by the stirrups in which the stresses are equal: to the design strength,  $R_s$ , multiplied by the service factors,  $m_{s1}$  and  $m_{s2}$ .

Analysis for normal and diagonal incipient cracking in the beam is based on the following condition

$$\sigma_{ct\max} \leq R_{ct}m_{c2}.$$

The conditions for  $\sigma_{\text{prin.,ten}}$  and  $\sigma_{\text{prin.,com}}$  should be written to include the stresses,  $m_{c2}R_{ct}$  and  $m_{c2}R_c$ .

The sag of the beam is determined with allowance for long- and short-time live loads taken with an overload factor equal to unity. The value of the sag should be  $f \leq 1/400$ .

Prestressed crane beams are reinforced with high-strength wire, bars, or class K-7 wire strands. Since crane beams are designed to carry dynamic loads, their reinforcing cages should be tied, rather than welded. At the supports, crane beams are strengthened with ribs (by expanding the ends) and additional reinforcing bars, stirrups or wire fabric which contribute to the strength and crack-resistance of the beam ends when the prestress is transferred to the concrete. In crane beams are made of B30-B50 concrete, and the weight of a 12-m beam usually ranges between 10 and 12 tones.

Crane beams are fastened to the columns by welding together their embedded steel parts (Fig. 16.16). To transmit horizontal braking forces, the joints are equipped with rib plates welded to the top embedded parts of the beams and special embedded parts in the columns. The impacts and shocks imposed on the beam by the traveling crane are damped by an elastic pad made of rubberized cloth 8 to 10 mm thick, which is inserted between the crane beam and the rail. The pad also reduces wear of the rails.

When placing the pad, it should be remembered that prestressed beams are cambered, whereas the crane rails should be horizontal. After



straightening, the rail is fastened to the beam by bolts and steel parts as shown in Fig. 16.16, *b*.

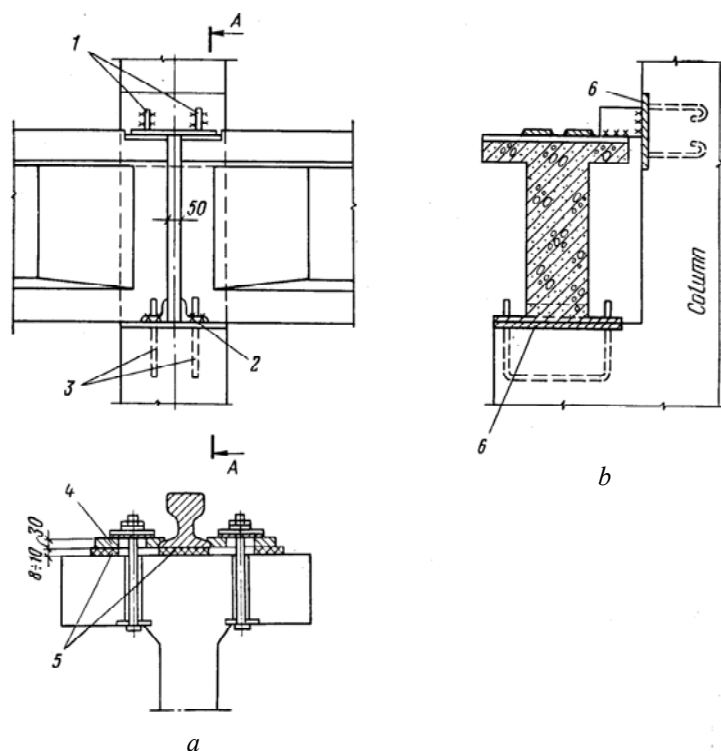


Fig. 16.16. Fastening of a crane beam to a column (*a*);  
fastening of a rail to a crane beam (*b*):  
1 — 100×12 mm rib plates; 2 — embedded part of the crane beam;  
3 — anchors extending from the column; 4 — clamp; 5 — elastic pad;  
6 — embedded parts of the column,  $\delta = 8$  mm

### 16.8. Design of Transverse Frame. Loads and Their Disposition

The transverse frame of a one-storeyed building of skeleton construction is subjected to the dead load due to the weight of the roof and various live loads due to snow, vertical and lateral thrust of overhead traveling cranes, positive and negative wind forces (Fig. 16.17).

For design purposes, the connection between the girder and a column is regarded as a hinge, and that between a column and the footing is considered to be rigid. The length of a column is taken equal to the distance

from the top of the footing to the bottom of the girder. In designing the transverse frame, the objective is to determine the forces in the columns and to choose their cross-sectional area. The girder of the frame is independently designed as a single-span beam, truss or arch.

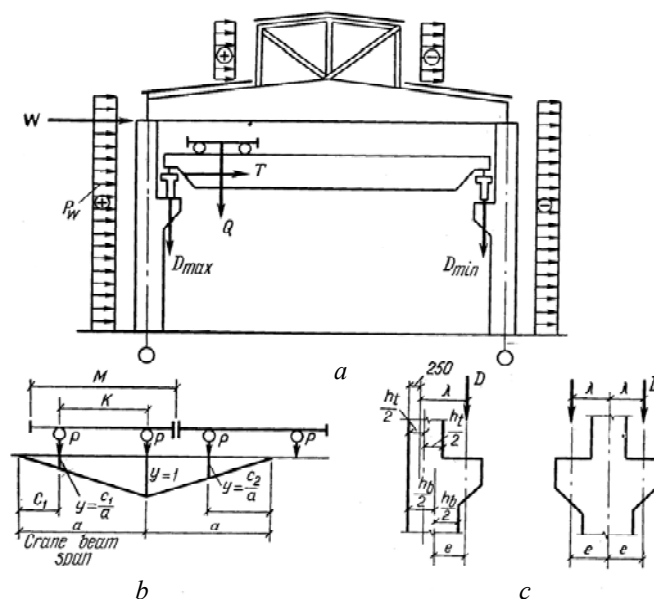


Fig. 16.17. Structural and loading system of a crane-supporting transverse frame

**Dead Load due to Roof Weight.** This is transferred to the column as a vertical force,  $N$ , induced by the girder. Its magnitude is calculated according to the loaded area of the roof. The vertical load is applied along the girder support axis, and is transmitted to the column, provided that the offset between the external face of the column and the layout axis is 250mm, with the eccentricity taken as

$e = 0,25/2 = 0,125$  m above the crane beam (in the case of a zero offset,  $e = 0$ );

$e = (h_1 - h_2)/2 - 0,125$  below the crane beam (in the case of a zero offset,  $e = (h_1 - h_2)/2$ ).

The bending moments due to the eccentricity are equal to  $M = Ne$ .

**Snow Load.** The magnitude of this load is taken according to the climatic region and the roof configuration. The snow load is likewise transmitted to the column as a vertical force,  $N$ , and is calculated using the same loaded area as in the case of the roof load.

**Live Load due to Overhead Traveling Cranes.** The magnitude of the crane load is found for two cranes operating close to each other. The design vertical and horizontal crane loads are multiplied by the overload factor  $\gamma_f = 1,1$ .

The vertical load imposed on a column is determined using the influence line of the support reaction for the crane beam. The concentrated force exerted by one crane wheel is placed directly at the support, and the remaining forces are disposed according to the standard distance between the crane wheels (Fig. 16.17, *b*). The maximum force applied to the column is

$$D_{\max} = F_{\max} \Sigma y. \quad (16.8)$$

Under the circumstances, the force applied to the opposite column of the frame is

$$D_{\min} = F_{\min} \Sigma y. \quad (16.9)$$

The vertical force,  $D$ , is applied by the cranes through the crane beam with an eccentricity  $e = 0,25 + \lambda - 0,5h_2$  to a marginal column in the case of a zero offset, the eccentricity is  $e = \lambda - 0,5h_2$  and  $e = \lambda$  to an intermediate column (Fig. 16.17, *c*).

The respective moments induced by the crane load are as follows

$$M_{\max} = D_{\max} e; \text{ and } M_{\min} = D_{\min} e.$$

The horizontal load applied to the column due to the braking of two cranes operating close to each other is transmitted through the crane beam in the same manner as the vertical force. Accordingly the lines of influence for this load are the same, and the load itself is found as

$$H = H_{\max} \Sigma y. \quad (16.10)$$

**Wind Load.** The wind pressure per square meter of the wall and monitor surface is taken according to the locality and the height of the building. The wind pressure is considered to be positive from the weather side, and negative from the other side. The wind load is transmitted through the wall panels to the columns as a distributed load with an intensity of  $p = wa$  where  $a$  is the centre-to-centre distance between the columns. Since the wind load is not uniformly distributed over the height of the building, it is customary to simplify the matter by replacing it with a uniformly distributed load which produces the same bending moment at the bottom of the column.

The wind pressure applied to the monitor and the wall above the columns is replaced by a concentrated force,  $W$ , in the design loading system.

### 16.9. Determining Load-Induced Forces in Columns

A transverse frame can most conveniently be designed to resist various loads by the slope-deflection method, which involves one unknown,  $\Delta$ . This unknown is the horizontal deflection (the sway) of a plane-loaded frame. Introducing an imaginary restraint in the direction of the unknown deflection, we get the basic load system for the frame (Fig. 16.18, *a*). The basic system is subjected to the unit action of the unknown which gives rise to reactions,  $R_{\Delta}$ , and bending moments in the columns (Fig. 16.18, *b*). Then, the system is successively loaded by dead and live loads,  $F$ ,  $M$ ,  $H$  and  $p$ , which induce the respective reactions and bending moments in the columns (Fig. 16.18, *c* through *e*). The reactions,  $R$ , in stepped columns of variable cross-section and fixed at the top may be found using equations. In the equation

$$C_{dim}r_{11}\Delta + R_{1p} = 0. \tag{16.11}$$

$r_{11} = \Sigma R_{\Delta}$  is the reaction of the transverse frame against the unit deflection, and  $R_{1p} = \Sigma R$  is the sum of the load-induced reactions at the top of the columns considered to be positive if they point in the direction of the unknown deflection.

For all types of loads, except crane load,  $c_{dim}$  is unity.

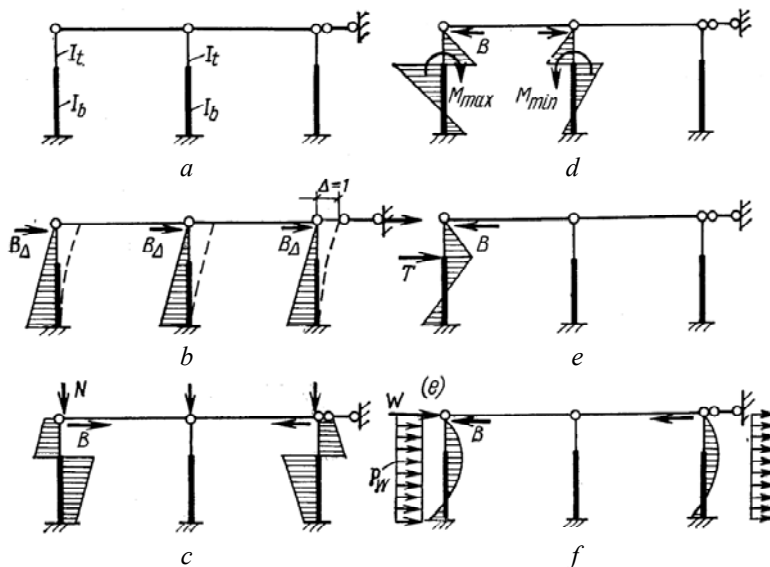


Fig. 16.18. Basic system for a transverse frame and bending moment diagram

We solve Eq. (16.12) for the unknown,  $\Delta$ , and find the elastic reaction

$$R_e = R + \Delta R_\Delta. \quad (16.12)$$

In three-bay or wider frames, the top supports of the columns subjected to crane load are regarded as fixed, with  $\Delta = 0$ .

For frames in which the girders are located in two levels and the stiffness of the interior columns,  $B_1$ , is five times or more as great as that of the exterior columns,  $B_2$ , the middle part may, for design purposes, be treated as an independent single-bay frame (Fig. 16.18). This frame is likewise designed with allowance for the spatial interaction of the skeleton.

Bending moments and shearing forces in the cross sections of the columns are determined as in a cantilever beam subjected to an external load and reaction,  $B_e$ . As a rule, design calculations are done for three sections, namely section 1–0 above the crane cantilever, section 1–2 below the crane cantilever, and section 2–1 at the bottom of the column.

As a first step, moment and diagrams are plotted for each type of load imposed on the frame. Table is compiled for  $M$ ,  $N$  and  $Q$ , and the design force or load combinations are chosen for the design sections of the column. Dead load is included in all combinations and live loads in the most unfavorable ones.

According to relevant standards, live loads (such as snow, wind, and crane loads) imposed on a transverse frame are treated as short-time loads. When a transverse frame is designed for the basic load combinations including one short-time live load, this load is taken without any reduction in magnitude. If the basic load combinations include two or three short-time live loads, the design magnitudes of these loads or respective forces are reduced, that is, multiplied by the combination factor 0,9. In such a case, one short-time live load is assumed to be the load imposed by two cranes on one runway, multiplied by 0,85 for light- and medium-duty cranes, or the load imposed by four cranes operating in different spans lying in the same plane, multiplied by 0,7.

The cross-sections of transverse frame columns are designed with allowance for the effect that buckling has on the eccentricity of the longitudinal force (Fig. 16.19). The columns are analyzed for buckling as compression members. In addition, they are designed to resist loads arising in transit and during erection.

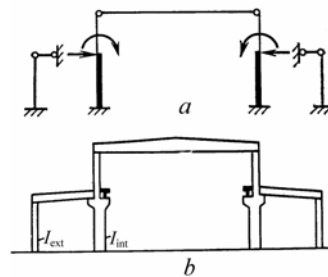


Fig. 16.19. To the design of a two-level transverse frame:  
*a* — structural system;  
*b* — loading system

The effective length,  $l_0$ , of the top and bottom parts of prefabricated columns for buildings using overhead traveling cranes is different in and at right angles to the plane of the transverse frame, and is assigned according to Table 16.1.

Table 16.1

**Effective Length  $l_0$  of Prefabricated Reinforced Concrete Columns for Buildings Using Overhead Traveling Cranes**

Load combination	Column part being designed	In plane of transverse frame	At right angles to transverse frame	
			with vertical braces	without vertical braces
Including crane load	Below crane beam	$1,5H_1$	$0,8H_1$	$1,2H_1$
	Above crane beam	$2H_2$	$1,5H_2$	$2H_2$
Excluding crane load	Below crane beam in:			
	single-span buildings	$1,5H$	$0,8H_1$	$1,2H$
	multi-span buildings	$1,2H$	$0,8H_1$	$1,2H$
	Above crane beam	$2,5H_2$	$1,5H_2$	$2H_2$

The effective length of prefabricated reinforced concrete columns for buildings using no overhead traveling cranes is  $l_0 = 1,5H$  for single-span buildings, and  $l_0 = 1,2H$  for multi-span buildings.

The short crane-beam supporting cantilevers of columns (Fig. 16.20) are designed to resist shear subject to the condition

$$Q \leq k_3 k_4 R_{ct} b h_0^2 / a. \quad (16.13)$$

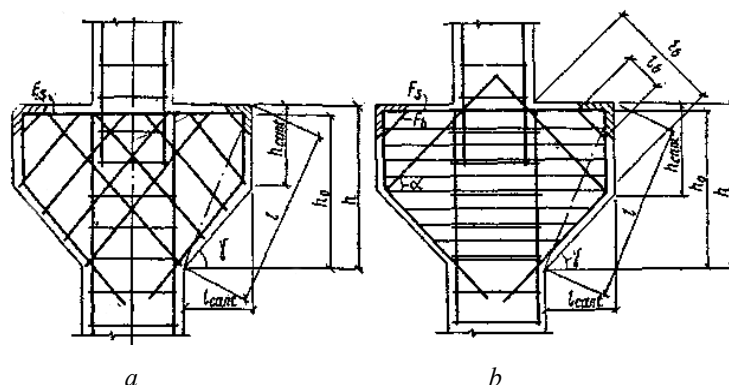


Fig. 16.20. Reinforcement of column cantilevers:  
*a* — with inclined stirrups; *b* — with horizontal stirrups and bent-up bars

Here, the right-hand side of the expression should be not more than  $2,5R_{ct}bh_0$ ;  $k_3$  is taken as 1,2 for heavy concretes and 0,8 for porous-aggregate concretes;  $k_4$  is taken as 1 for light- and medium-duty cranes, 0,75 for heavy-duty cranes, and 0,5 for super heavy-duty cranes;  $h_0$  is the effective depth of the cantilever at the support section; and  $a$  is the distance from the point of application of the shearing force,  $Q$ , to the bottom face of the column.

A cantilever is considered short if its arm is  $l_{cant} \leq 0,9h_0$ . The angle made by the compressive face of a cantilever with the horizontal should be  $\gamma \leq 45^\circ$ , and the depth of the section at the free end should be  $h_{cant} \geq h/2$  (where  $h$  is the depth of the support section).

When  $h \leq 2,5a$ , cantilevers are reinforced with diagonal stirrups; when  $h > 2,5a$ , with horizontal stirrups and bent-up bars. If  $h > 3,5a$  and  $Q \leq R_{ct}bh_0$ , then bent-up bars may be omitted. In all cases, the distance between stirrups should not be more than 150 mm, nor more than  $h/4$ ; the diameter of bent-up bars should be not more than 1/15 of the length of the diagonal part, nor more than 25mm. The total cross-sectional area of the bent-up bars and diagonal stirrups crossing the top half of the line connecting the extreme points within the cantilever arm (see Fig. 16.20) should be not less than  $0,002bh_0$  or

$$A_{ct} + A_s = (Q - Q_c)/R_s \sin \alpha. \quad (16.14)$$

Whichever is the smaller. Here, determining  $Q_c$ , take  $c = a + h_0/3$ . The cross-sectional area,  $F_s$ , of the longitudinal reinforcement in the cantilever is taken to carry the bending moment arising at the point where the cantilever joins the column, increased by 25 %. The longitudinal reinforcing steel has anchors in the form of welded-on washers or angles.

#### ASSIGNMENTS TO CHAPTER 16

1. Enumerate main structures of one-storeyed frame buildings.
2. Draw a short console of a column. Provide a scheme of reinforcement.
3. What loads are industrial building walls calculated for?
4. What joints exist in one-storeyed frame buildings?
5. Name the main types of slabs used for covering and methods of their calculation.
6. Enumerate main types of covering slabs.
7. Name the peculiarities of calculation and design of stepped trusses and arches. When does the necessity in under step beams and trusses appear? Provide their calculation scheme.
8. What modular elements are the skeleton of a building, a transverse and longitudinal frame made up of?

9. How is spatial rigidity of one-storey building — in a transverse direction, in a longitudinal direction- provided?
10. What constructive scheme of a building with plates of a covering on span is applied?
11. What groups of operating modes of bridge cranes are established and what loading to a skeleton of a building do they transfer?
12. What is the essence of designs of a building? What constructive schemes ensure the best technical and economic parameters?
13. What is the choice of a rational unit connection of a crossbar with a column of a transverse frame on conditions of elements type-design based?
14. What types of columns of one-storey industrial building exist? What is the choice of columns design based on?
15. What is the design of crane beams? What is the essence of the method of loading calculation from the bridge crane?
16. How is the design scheme of a transverse frame with loading affecting it represented?
17. How can loading onto a transverse frame from bridge cranes be determined?
18. How is spatial work of a building skeleton taken into account at crane loading? How is the factor of spatial work determined?
19. What design procedure of a transverse frame and determination of efforts in columns are applied?
20. What is a specific feature of determination of efforts in double branch columns?
21. How can we determine a design building frame of columns with bridge cranes due to a bend in frame plane, from frame plane?
22. What designs of covering plates are applied and what are their technical and economic estimations?
23. What spans of RC beams, frames, arches are expedient in covering one-storeyed industrial buildings?



## 17. ROOF CONSTRUCTION

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### 17.1. Roof Slabs

Non-purling roofs use large ribbed slabs  $3 \times 12$  m and  $3 \times 6$  m in plan. They are directly supported on the girders of the transverse frames. Smaller slabs  $1,5 \times 12$  m and  $1,5 \times 6$  m in plan are used as supplementary members. These are installed near monitors to carry increased snow load, and also near the folds in the roof configuration. Roofs built with purling use much smaller slabs ( $3 \times 0,5$  m and  $1,5 \times 0,5$  m in plan). In this case, slabs are supported by reinforced concrete purling which, in turn, is carried by the girders of the transverse frames. Non-purling roofs are better adapted to the preassembly of members into larger elements, which helps to reduce the number of units to be handled during erection. Because of this, this type of roof is most commonly used for one-storeyed buildings of skeleton construction.

Ribbed slabs  $3 \times 12$  m in plan, which have been adopted in the USSR as standard, have longitudinal ribs  $100 \times 450$  mm in cross-section, transverse ribs  $40 \times 150$  mm in cross section, a flange 25 mm deep and angle brackets which strengthen the slabs against repetitive lateral crane thrust (Fig. 17.1).

Longitudinal ribs are reinforced with prestressed bars or wire strands; transverse ribs and flanges are reinforced welded-bar mats and welded-wire fabric. Slabs are made of class B30, B40 concrete. Ribbed slabs  $3 \times 6$  m in plan, which have also been adopted as standard products, have longitudinal and transverse ribs and are reinforced with prestressed steel. Double-T slabs  $3 \times 12$  m and  $3 \times 6$  m in plan have longitudinal ribs spaced 1,5 m apart and overhanging flanges (Fig. 17.2, *a* and *b*). Since bending moments in such slabs are small, no transverse ribs are provided, which makes the slabs easier to manufacture. In  $3 \times 12$  m slabs, the prestressed longitudinal ribs are fabricated first, and the flanges are concreted afterwards. The ribs are connected to the flange by reinforcement stick-outs and monolithic concrete. Because the components of double-T slabs are fabricated separately, the concrete brand number for the flanges may be reduced to B15. Slabs  $3 \times 6$  m in plan may be assembled from precast components or fabricated as one-piece units.

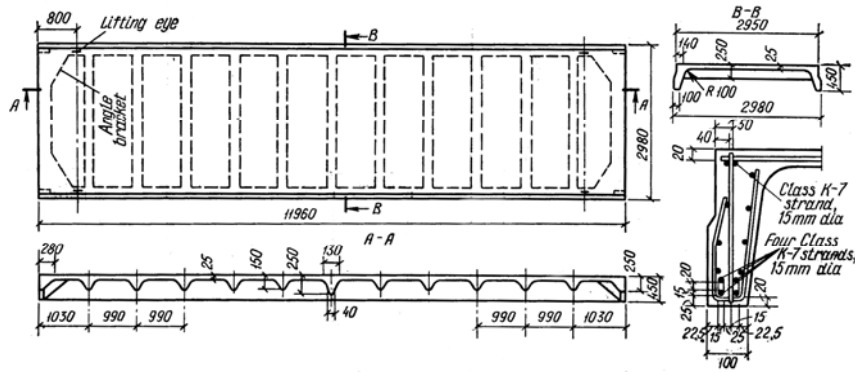


Fig. 17.1. Ribbed Slab 3×12 m

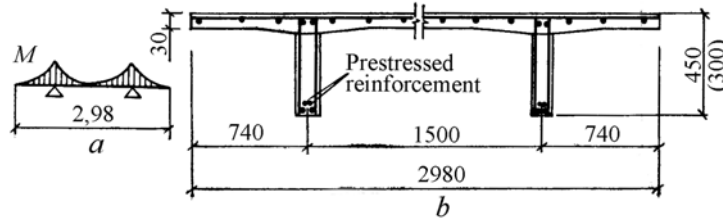


Fig. 17.2. 2T type roof slab

The main parameters of roof slabs are given in Table 17.1.

Table 17.1

**Roof Slab Particulars**

Slab type	Weight tonnes	Concrete class number	Reduced concrete depth, cm	Amount of steel per slab. Longitudinal ribs reinforced with	
				bars	wire stands or high-strength wire
Ribbed, 3×12 m	6,8	B30, B40	7,65	265–391	205–288
Same, 3×6 m	2,38	B25, B30	5,3	70–101	56–70
Double – T, 3×12 m	6,8	B40	7,65	330	237
Same, 3×6 m	2,38	B25	5,3	85	63
Ribbed, lowpitched, 3×18 m	12,2	B40	8,98	—	581
Barrel, 3×18 m	10,9	B40	8,03	—	431
Ridge, 3×18 m	15,1	B40	11,2	—	382

Large-size slabs  $3 \times 18$  m and  $3 \times 24$  m in plan, supported on girders 6 or 12 m long are used in ridge and low-pitched roofs (Fig. 17.3). Here, the double-T slabs consist of longitudinal ribs with a slope of 1:12 in the top chord and a flange varying from 25 to 60 mm in depth.

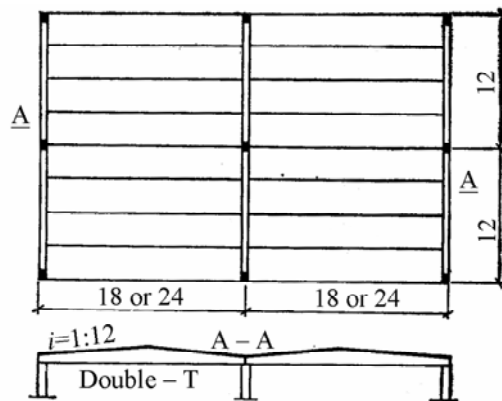


Fig. 17.3. Roof using ridged double-T slabs  $3 \times 18$  m in plan

Large-size reinforced concrete barrel roof slabs have curved longitudinal ribs widened at the top and bottom, and a smooth flange 40 to 50 mm deep in the midspan and 140 to 160 mm deep at the supports (Fig. 17.4).

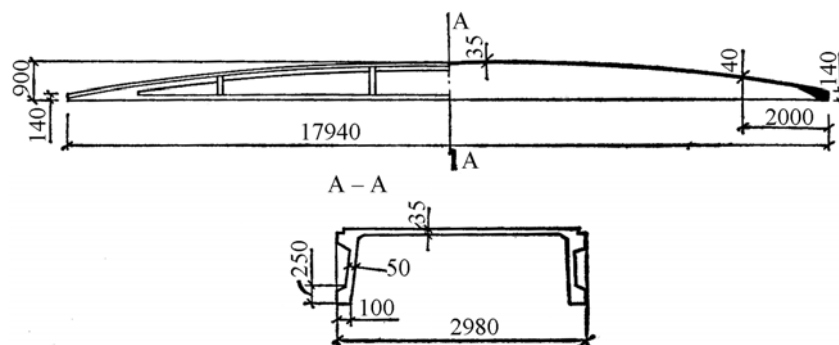


Fig. 17.4. Example of a large-size reinforced concrete barrel slab  $3 \times 18$  m in plan

Ribbed slabs intended for low-pitched roofs consist of trapezoidal longitudinal ribs with the upper chord sloping at 1:20 or 1:30, transverse ribs spaced 1000 mm apart, and a flange 25 mm deep (Fig. 17.5).

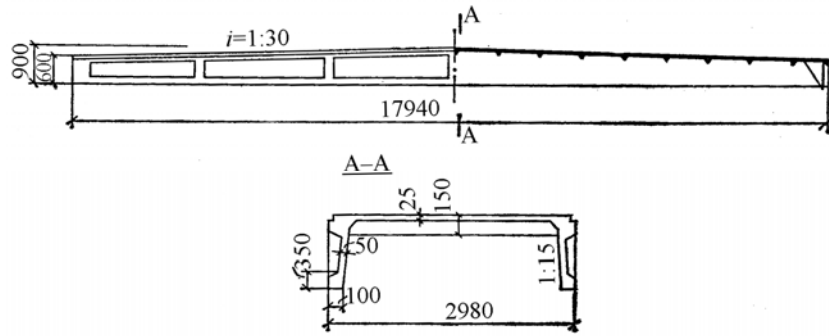


Fig. 17.5. Example of a low-pitched

Ribbed low-pitched slabs are somewhat inferior to large-size barrel slabs. Their advantage, however, is that the installation of low-pitched roofs can be mechanized, which is not the case with curved barrel roofs.

## 17.2. Roof Beams

The standard spans of roof beams are 12 and 18 m, although in some structures the figure may reach 24m. In ridge roofs, the top chord of the beams may be constant-slope trapezoidal, broken or curved (Fig. 17.6, *a* through *c*). The beams used in shed roofs have parallel chords or broken bottom chords, those used in flat roofs have parallel chords (Fig. 17.6, *d* through *f*). Roof beams are available for bays of 6 or 12 m.

The most economical cross-section of roof beams is the I-section. They are made with a web 60 to 100 mm wide. This width is chosen because it provides enough space for reinforcing cages and ensures adequate strength and crack resistance. At the supports, the web gradually widens to form stiffening ribs. In the midspan where shearing forces are small, the web may have round or polygonal holes made to reduce the amount of concrete, and to provide room for the wiring and other services.

In the midspan, the beam depth is taken as  $1/10l$  to  $1/15l$ . Depth of a trapezoidal ridge beam in the midspan governs the slope of the top chord (1:12) and the standard depth at the support (800 or 900 mm). Owing to a somewhat greater slope of the chord in the marginal quarter of the span, the depth of beams with a polygonal top chord is larger, with the depth at the support remaining standard. The configuration of roof beams with the broken top chord is close to that of the bending moment diagram; theoretically, such beams are more advantageous because they use less material. This, however, is offset by the increased cost of manufacture due to the more complex shape.

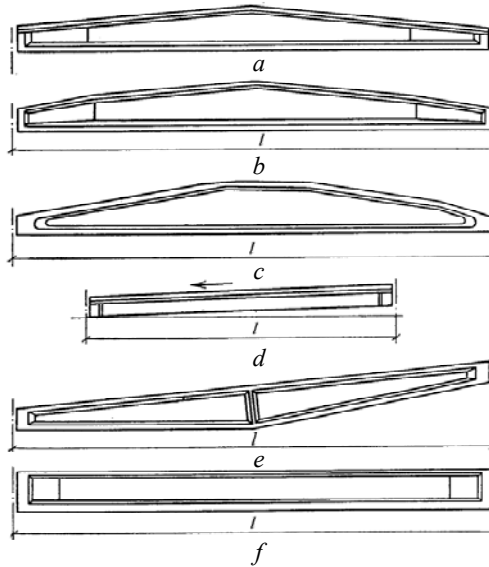


Fig. 17.6. Types of roof beams

To provide adequate stability in transit and during erection width of the top compression flange of a beam is taken as  $1/50$ – $1/60$ . The width of the bottom flange is taken as 250 to 300 mm for the installation of the longitudinal tensile steel.

Ridge beams are made of B25 to B40 concrete and reinforced prestressed wire, bars or wire strands (Fig. 17.7).

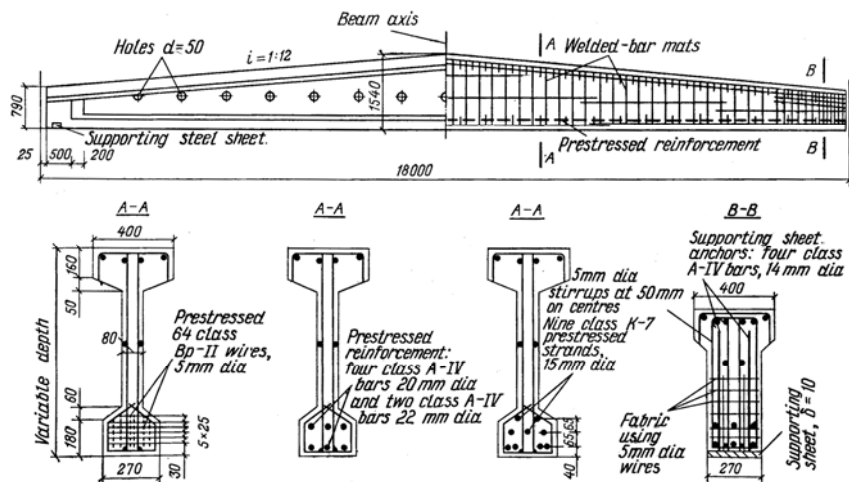


Fig. 17.7. 18-m I-section ridge roof beam

When high-strength wire is used as reinforcement, it is distributed in groups of two wires along the depth of the beam, so that it is convenient to concrete the beam in a vertical position. The web reinforcement consists of welded-bar mats, the longitudinal bars of which serve as distribution reinforcement, whereas the transverse bars are intended to take up shearing forces. To prevent them from longitudinal cracking (or to limit the width of these cracks) when the prestress is transferred to the concrete, the beam portions at the supports are additionally reinforced with transverse bars welded to embedded steel parts. Crack resistance can also be enhanced by biaxial prestressing, in which case, the transverse bars are prestressed, too.

To limit the width of cracks occurring in the top zone of ridge I-beams when the prestress is transferred to the concrete, additional prestressed steel may be provided at the top of the support sections (Fig. 17.8). This reduces the eccentricity of the prestressing force and the preliminary tensile stresses in the concrete of the compression zone.

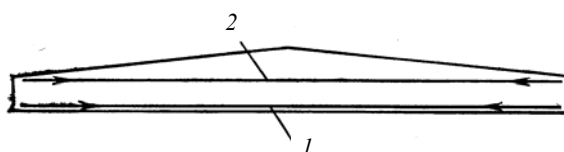


Fig. 17.8. Disposition of prestressed reinforcement in a ridge beam:  
1 — bottom reinforcement; 2 — top reinforcement

Rectangular ridge beams with closely spaced holes may arbitrarily be called lattice beams (Fig. 17.9). According to the design load to be carried, the width of rectangular lattice beams may be taken equal to 200, 240 or 280 mm. Beams of all types have steel parts embedded in their top chords for connection to roof slabs.

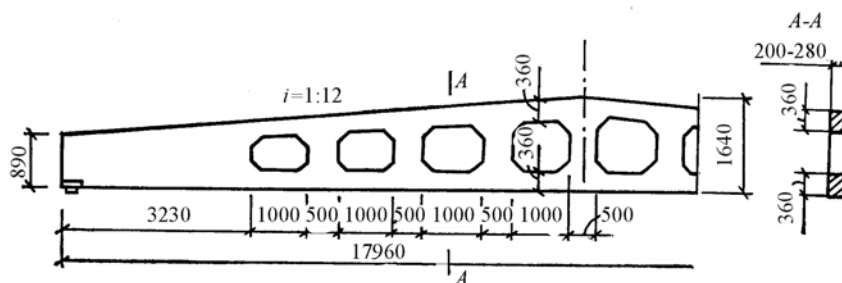


Fig. 17.9. 18-m rectangular lattice ridge beam

For design purposes, roof beams are treated as simply supported beams, with the load from the roof slabs transmitted through their ribs. If the number of concentrated forces is five or more, the load is assumed to be uniformly distributed over the beam. In ridge beams, the design section is that lying within a distance,  $x$ , of the support. For example, with the top chord sloping at 1:12 and a beam depth of  $h = 1:12$  in the midspan, the beam depth at the support is  $h_{op} = 1:24$ , and that within a distance,  $x$ , of the support is

$$h_x = (l + 2x)/24. \quad (17.1)$$

Assume that the effective depth of the beam is  $h_0 = \beta h_x$  and the bending moment due to the uniformly distributed load is

$$M_x = qx(l - x)/2. \quad (17.2)$$

Then, the required cross-sectional area of the longitudinal reinforcement will be

$$A_{sx} = M_x / R_s \xi h_0 = 12qx(l - x) / R_s \xi \beta (l + 2x). \quad (17.3)$$

The design section for this beam will be that in which  $A_{sx}$  is a maximum. To find this section, we equate the derivative of  $F_{yx}$  to zero

$$dA_{sx}/dx = 0. \quad (17.4)$$

Differentiation of the above equation at  $\xi\beta = \text{const}$  gives  $2x^2 + 2x l - l^2 = 0$ . In general, the distance from the support to the design section is  $x = 0,35l$  to  $0,4l$ .

In a monitor-type building, the design section may be chosen beneath a monitor strut.

The amount of transverse reinforcement to be used is found by inclined-section analysis which is then followed by analysis for cracking resistance and sagging, and also for strength and cracking resistance in transit and during erection. When determining sags in trapezoidal beams, it should be kept in mind that their stiffness varies with length.

At present, the optimum designs of roof beams are found by computers; the particulars of ridge roof beams according to their cross-section and type of prestressed steel are given in Table 17.2.

Referring to the table, in I-beams, the amount of reinforcement is by about 15 %, and the amount of concrete is by about 13 % less than in lattice beams.

If roof beams are to carry overhead cranes and loads, their amount of steel should be increased by 20 to 30 %.

Table 17.2

**Particulars of 18m Ridge Roof Beams Spaced 6m Apart and Subjected to Design Load of 3.5 to 5.5 kN/m<sup>2</sup>**

Beam type	Weight, tones	Concrete class number	Volume of concrete, m <sup>3</sup>	Total amount of steel per beam, kg
I-section with: prestressed bars:	9,1	B25, B40	3,64	568–738
prestressed bars	9,1	B30, B40	3,64	360–565
prestressed wire	9,1	B25, B40	3,64	359–552
Lattice with: prestressed bars:	8,5–12,1	B30, B40	3,4–4,84	530–875
prestressed bars	8,5–12,1	B30, B40	3,4–4,84	418–662
prestressed wire	8,5–12,1	B30, B40	3,4–4,84	397–644

### 17.3. Trusses

Reinforced concrete trusses are used for spans of 18, 24 and 30 m, and for bays of 6 or 12m. In contrast to steel trusses, the amount of steel required by reinforced concrete trusses is almost halved, but the labor consumption and the cost of manufacture are somewhat higher. For spans of 36m and more, use is usually made of steel trusses. In practice, however, reinforced concrete trusses may be used for spans of about 60 m and more.

Ridge, low-pitched and flat roofs use reinforced concrete trusses differing in chord and lattice configuration. The main types of trusses are the camel-back truss with a polygonal top chord and linear portions between the joints (Fig. 17.10, *a*); the bowstring truss with widely spaced web members and a curved top chord (Fig. 17.10, *b*); the bowstring truss without diagonal web members, with rigid joints between the chords and the struts, and a curved top chord (Fig. 17.10, *c*); the warren truss (Fig. 17.10, *d*); the quadrangular truss (Fig. 17.10, *d*); and the polygonal truss with a polygonal bottom chord (Fig. 17.10, *e*).

Whatever the type of truss, the depth at the midspan is usually taken as 1/7 to 1/9 of their span. The top chord panels of all trusses, except the bowstring type with diagonal web members, are designed to be 3 m long in order that all of the roof slab load can be transmitted directly to the joints, thereby preventing the chord members from buckling. The bottom tensile chords of all trusses and the tensile diagonals of some trusses are prestressed, using the pretension technique.



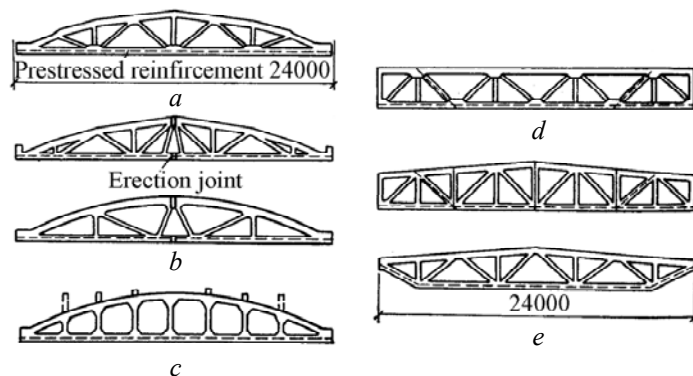


Fig. 17.10. Types of reinforced concrete trusses

As far as their static behavior is concerned, the camel-back and bowstring types are best because the shape of their top chord is almost a mirror-image of the bending moment diagram.

The web members of such trusses carry insignificant loads, and the trusses are relatively shallow at the supports, which serves to reduce their weight and the height of the exterior walls. In bowstring trusses with web diagonals, the bending moments induced by the load applied to the top chord between the joints are reduced because of the eccentricity of the longitudinal force which gives rise to a reverse moment. Owing to this, the distance between the joints in the top chord may be increased, and the web members may be spaced wider apart (Fig. 17.11). In bowstring trusses without web diagonals, the bending moments arising in the struts and chord members are rather large. Because of this, they require additional reinforcement to enhance their strength and crack-resistance. Nevertheless, such trusses are easier to fabricate and more convenient to install in buildings with low-pitched or flat roofs. Also, if additional struts are made above the top chord, they provide more room for services. Polygonal trusses with a broken bottom chord are more stable during erection and do not require additional bracing because their centre of gravity lies below the support level.

The cross-sectional area and the weight of all trusses may be reduced by increasing the concrete brand number B30 through B50 and the percentage of chord reinforcement.

Trusses should preferably be fabricated in one piece, because their fabrication as half-trusses to be assembled later during erection increases the cost of construction. Trusses for a span of 18m are fabricated as

one piece, those for a span of 24 m may be made either in one piece or as two half-trusses, and those for a span of 30 m are manufactured as two half-trusses only. The web of a half-truss should be designed so that the bottom chord members could be connected between the joints (see Fig. 17.10, *a*), as this makes the erection easier. To strengthen the bottom chord connection during erection, use is made of additional diagonals, which are not taken into account in design calculations.



Fig. 17.11. Bending moment diagrams for the top chord of a bowstring truss

Trusses can be manufactured in two ways. The first method consists in that web members having reinforcement stick-outs are fabricated in advance and then embedded for a distance of 30 to 50mm into the joints, and the chords are concreted. The second approach consists in that the chords and the web members are concreted simultaneously. The latter technique is used on a wider scale because in the former case the web members must be narrower than the chords; in the latter case, the chords and the web members may be of the same width.

For simplicity of manufacture, the top and bottom chords of a truss are made of the same width. The chord width is taken as 200 to 250 mm in trusses for a bay of 6m, and 300 to 350 mm for a bay of 12 m.

The bottom tensile chord should be reinforced so as to provide enough space between the prestressed bars, wire strands and paired wires for the concrete mix to be poured and compacted. All tensile steel should be embraced by closed stirrups spaced 500 mm apart and installed according structural reasons.

The top compression chord and the web members are reinforced with non-prestressed welded-bar mats. If they are to resist great forces, the tensile web members are prestressed.

For the forces to be transmitted from one member to another the joints of reinforced concrete trusses are angle bracketed, as this enhances the disposition and anchorage of the web reinforcement (Fig. 17.12).

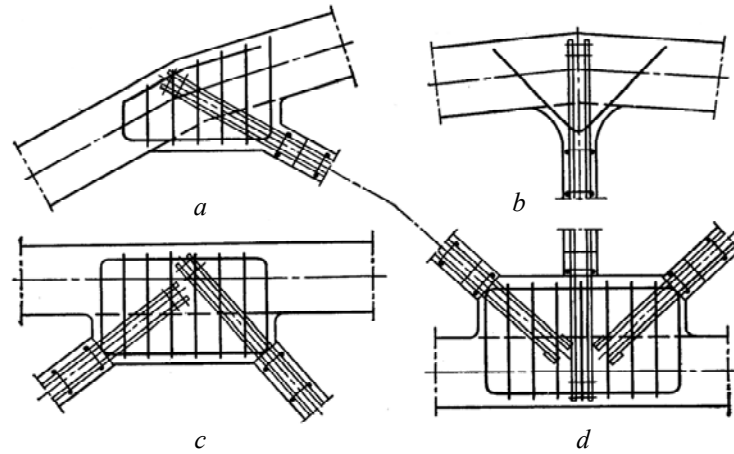


Fig. 17.12. Reinforcement of truss joints:  
*a, b, c* — top chord; *d* — bottom chord

The truss joints are reinforced with closed bent bars 10 to 18 mm in diameter and vertical transverse bars 6 to 10 mm in diameter spaced 100 mm apart and combined into welded mats.

The web reinforcement is extended into the joints, and the tensile bars are anchored by short bars, bookstand buttons or rivet heads. The reinforcing steel is checked for anchorage security by special analysis. The joints at the supports are additionally reinforced with non-prestressed longitudinal steel and transverse bars which contribute to the anchorage of the tensile steel in the bottom chord and to the shear strength of the joint. In addition, in order to prevent the chord from longitudinal cracking when the prestress is transferred to the concrete, use is made of transverse bars and welded-wire fabric welded to the steel parts embedded at the supports.

A similar reinforcement for a 24-m camel back truss is given in Fig. 17.13. Here, the prestressed reinforcement of the bottom chord consists of class K-7 wire strands, class A-IV bars and class Bp-II high-strength wire. Pretensioning is used throughout.

The stirrups in the bottom chord are made of U-shaped fabric pieces embracing the prestressed steel. The joint at the support contains additional 12 mm diameter, non prestressed longitudinal bars extending into the adjacent bottom chord panel, and transverse bars 10 mm in diameter.

Roof trusses are designed to resist dead loads (roof weight, truss weight and crane weight) and various live loads. The roof weight is assumed to be applied at the joints of the top chord, and the loads due

to overhead traveling cranes — at the joints of the bottom chord. When designing a truss, it should be taken into account that snow load is non-uniformly distributed over the roof and near the monitors. Design should also be carried out for the case when one half of a truss is subjected to snow and crane loads, which is unfavorable for the web members (Table 17.3).

In the stress analysis of a truss, its chords and web members are assumed to be pin-connected at the joints. The rigidity of the joints has little effect on the forces in the truss members and may be neglected. When determining the bending moments due to a load applied between the joints, the top chord is treated as a continuous beam supported at the joints.

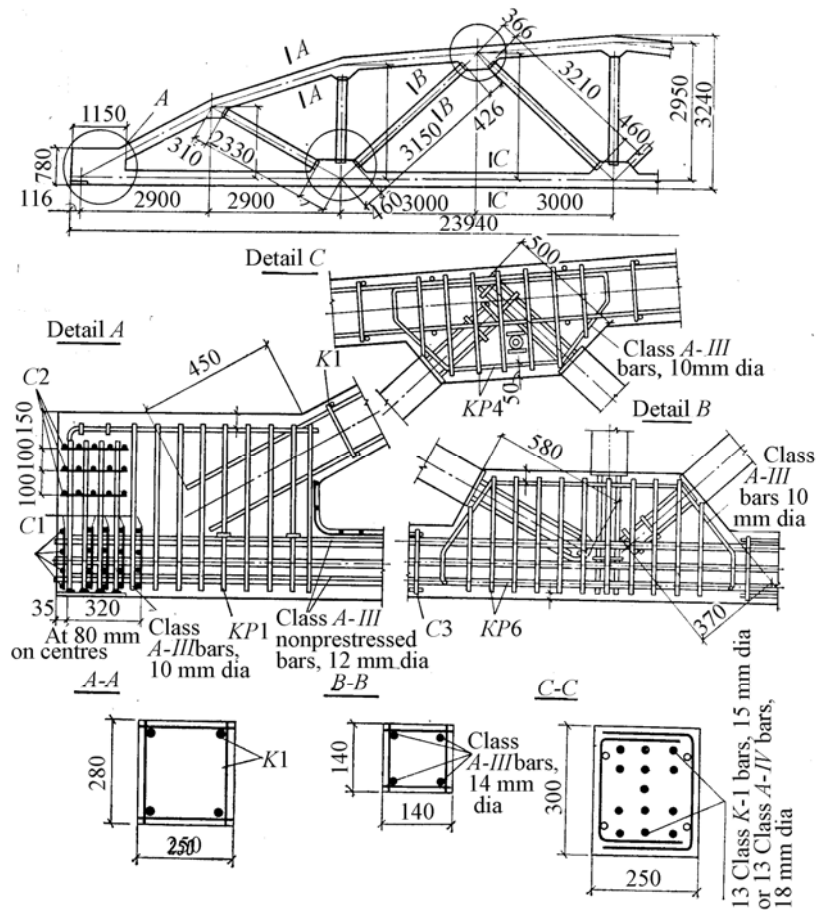


Fig. 17.13. 24-m camel back reinforced concrete truss

Table 17.3

**Main Particulars of Roof Trusses Subjected to Design Load  
of 3.5 to 5.5 kN/m<sup>2</sup>**

Type of truss	Truss weight, tones	Concrete class number	Volumes of concrete, m <sup>3</sup>	Amount of steel per truss (kg) with tensile chord reinforcement		
				bars	wire strands	high-strength wire
Camel-back, with diagonals, 18-m span: 6-m by 12-m bay	4,5–6 7,8–9,4	B30, B40 B30, B40	1,8–2,42 3,11–3,75	289–468 550–736	238–391 439–591	223–372 408–547
Bowstring, without diagonals, 18-m span: 6-m by 12-m bay	6,5 9,2–10,5	B30, B40 B30, B40	2,7 3,7–4,2	390–486 570–720	330–450 463–586	319–436 450–562
Camel-back, with diagonals, 24-m span: 6-m by 12-m bay	9,2 14,9–18,6	B30, B40 B30, B40	3,68 5,94–7,42	690–768 1096–1539	557–625 853–1204	510–595 787–1128
Bowstring, without diagonals, 24-m span: 6-m by 12-m bay	9,2–10,5 14,2–18,2	B30, B40 B30, B40	3,7–4,2 5,7–7,8	756–862 1281–1489	654–715 1020–1201	623–697 988–1128
Based on the table, camel-back trusses with diagonals take about 10 % less steel and 12 % less concrete than bowstring without diagonals. Trusses carrying overhead cranes have a 20–30 % increase of steel used for their manufacture						

The strength analysis of cross-sections for the chords and web members is carried out using the equations for compression and tension members. It should be kept in mind that the effective length of compression members in and across the truss plane is different (Table 17.4). Note. In the table,  $l$  is the centre-to-centre distance between adjacent joints;  $e_0$  is the eccentricity of the longitudinal force;  $h$  is the cross-sectional depth of the top chord;  $b$  is the cross-sectional width of the top chord; and  $bd$  is the cross-sectional width of the strut.

Experiments have shown that the reinforcement for a joint at a support may be designed using the loading system of Fig. 17.14, *a*.

It is assumed that the decrease in the design bearing capacity of the prestressed steel due to inadequate anchorage in the joint is compensated by the additional longitudinal non-prestressed reinforcement and

transverse bars working in tension. The cross-sectional area of the longitudinal non prestressed steel is given by

$$A_s = 0,2N/R_s \quad (17.5)$$

where  $N$  is the design force in the panel adjacent to the support. Let us resolve the total force,  $N_x$ , acting in the transverse bars normal to the axis within portion  $l_2$  (from the face of the support to the internal face of the, support joint) into a horizontal ( $N_x \text{ ctg}\alpha$ ) and a diagonal component (here,  $\alpha$  is the angle which the horizontal makes with the line AB joining point A at the face of the support to point B where the bottom face of the compressed diagonal borders on the joint).

Table 17.4

Effective length Member

Member	Effective length
Compression top chord in the truss plane: with $e_0 < (1/8)h$ with $e_0 \geq (1/8)h$	0,9 <i>l</i> 0,8 <i>l</i>
Compression top chord perpendicular to the plane of truss: for portion under monitor 12m or wider in other cases	0,8 <i>l</i> 0,9 <i>l</i>
Compression diagonals and struts in and perpendicular to the plane of truss: with $b/b_d < 1,5$ with $b/b_d \geq 1,5$	0,9 <i>l</i> 0,8 <i>l</i>

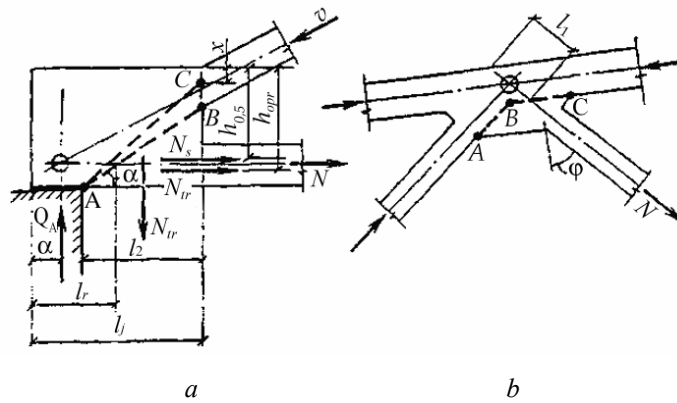


Fig. 17.14. Design of truss joints:  
*a* — support joint; *b* — intermediate joint

The condition for strength at the inclined section AB may be written as

$$N \leq N_p + N_s + N_{sw} \text{ctg} \alpha. \quad (17.6)$$

Hence, the force is determined for longitudinal reinforcement as:

$$N_s = A_{sp} R_{sp} l_{an}^0 / l_{an} \quad (17.7)$$

and the force of transverse bars is

$$N_{sw} = (N - N_{sp} - N_c) / \text{ctg} \alpha \quad (17.8)$$

The cross-sectional area of one stirrup is

$$A_{sw} = N_{sw} / n R_{sw} \quad (17.9)$$

$n$  is the number of transverse bars crossed by line AB (minus the bars placed within less than 10cm of point A);  $l_p^0$  and  $l_{an}^0$  are the anchorage lengths in the support joint past line AB of the prestressed and non-prestressed reinforcement, respectively; and  $l_p$  and  $l_{an}$  are the anchorage lengths at which the longitudinal prestressed and non-prestressed reinforcement acts at full strength, respectively.

For heavy concrete (B30 and higher),  $l_p$  is taken as 1500 mm for K-7 wire strands, 1000 mm for Bp-II high-strength wire 5mm in diameter, and  $35d$  for class A-IV reinforcing bars. For class A-III bars,  $l_{an}$  is taken equal to  $35d$ .

The bending strength of the support joint is checked at the diagonal section AC (where AC is the line joining point A at the face of the support to point C at the bottom of the compression zone on the internal face of the joint) on the assumption that the bending moment due to external forces does not exceed the moment due to internal forces:

$$Q_A(l - a) \leq N_w(l_2 - 10)/2 + N_s(h_{0s} - x/2) + N_{sp}(h_{0p} - x/2) \quad (17.10)$$

where  $Q_A$  is the support reaction of the truss;  $l$  is the length of the support joint; and  $a$  is the distance from the face to the centre of the support joint.

The depth of the compression zone in the inclined section is

$$x = (N_{sp} + N_s) / R_c b. \quad (17.11)$$

The required cross-sectional area of reinforcement in an intermediate joint is determined using the loading system of Fig. 17.14, *b*. Here, we likewise assume that the decrease in the design force of the reinforcing steel in the tensile diagonal along the anchorage length is compensated for by the tensile action of the transverse bars. The condition for strength in section ABC may be written as

$$N_{sw} \cos \varphi \leq N(k_2 l_1 + a) / k_1 l_2. \quad (17.12)$$

Using the above equation, we find  $N_{sw}$  and the cross-sectional area of one transverse bar

$$A_{sw} = N_{sw}/nR_{sw} \quad (17.13)$$

where  $N$  is the design force in the tensile diagonal;  $\varphi$  is the angle made by the transverse bars and the tensile diagonal;  $n$  is the number of transverse bars crossed by ABC (minus the bars lying within less than 100 mm of points A and C, and those anchored for a length less than  $30d$  including the hooked portions of the transverse reinforcement) within the angle brackets;  $l_1$  is the anchorage length of the steel of the tensile diagonal past line ABC;  $k_2$  is the factor taking care of the behaviour of the joint combining the tensile and compressive diagonals: for top chord joints,  $k_2 = 1$ , for bottom chord joints,  $k_2 = 1,1$  if one of the members in the joint should satisfy the second category of cracking-resistance requirements, and the joint contains struts, or compression diagonals making an angle of more than  $40^\circ$  with the horizontal; in all other cases,  $k_2 = 1,05$ ;  $a$  is the increase in the anchorage length of reinforcing steel, depending on the type of anchorage:  $a = 5d$  for two short bars,  $a = 3d$  for one short bar and a hook, and  $a = 2d$  for a button or rivet head;  $l_{an}$  is the anchorage length of the tensile diagonal at which the reinforcing steel acts at its full strength; for heavy concrete (class B30 and higher) and class A-III reinforcing steel,  $k_1 = \sigma_s/R_s$ , where  $\sigma_s$  is the stress in the reinforcement of the tensile diagonal due to the design load.

The transverse bars of an intermediate joint combining two tensile web members are designed successively for each web member, using Eq. (17.9) and assuming that the adjacent members are in compression.

The crack-resistance analysis of the tensile chord of a truss with diagonals should be carried out taking into account bending moments arising due to the joint rigidity. In trusses where the forces resisted by the web members are small, these moments may quite accurately be determined by treating the bottom chord of the truss as a multi-span beam for specified settlements of the supports. The settlements are found using the deflection diagram of the truss members.

In addition, trusses should be designed to resist forces which are likely to arise in manufacture, in transit and during erection.

In the strength and crack-resistance analysis of trusses without diagonals, the chords and struts are assumed to be rigidly connected at joints. The forces,  $M$ ,  $Q$  and  $N$ , are determined as for a closed-contour statically indeterminate system. Here, use may be made of both exact and approximate design methods.

Programs have been compiled for finding an economical design of a truss by computers.



## 17.4. Secondary Roof Structures

Secondary roof beams or trusses are used in one-storey industrial buildings with a roof truss bay of 6 m and a column bay of 12 m (Fig. 17.15). They are made of B30, B40 prestressed concrete, and are reinforced with pretensioned wire strands, bars or high-strength wire.

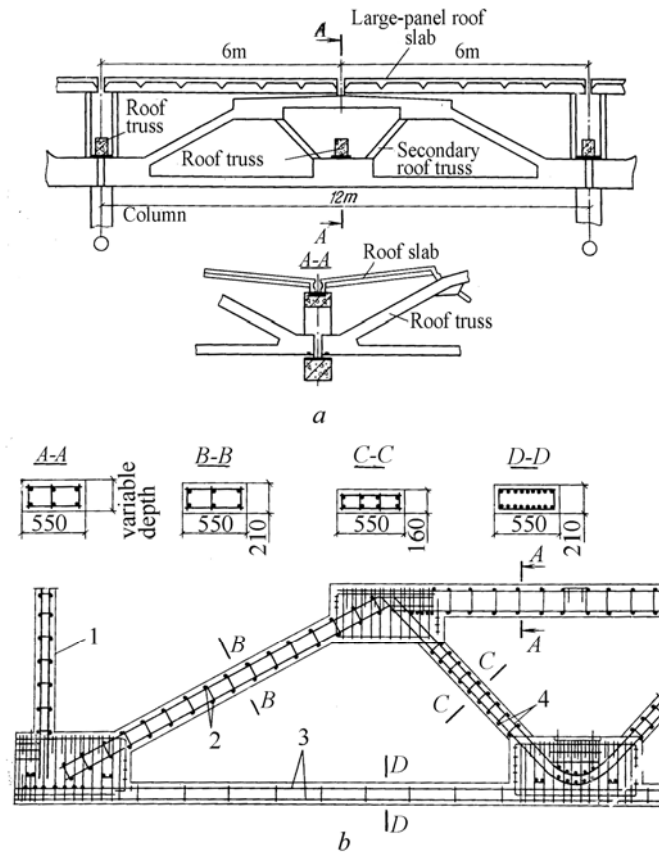


Fig. 17.15. Secondary Roof Structures:  
*a* — construction and *b* — reinforcement of a secondary truss:  
 1 — strut supporting a roof slab; 2 — reinforcing steel in a compressive web diagonal; 3 — prestressed steel in the bottom chord; 4 — prestressed steel in a tensile web diagonal

Roof trusses are welded to secondary structures during erection. A likely construction of a secondary roof truss is shown in Fig. 17.15.

The bottom chord of the truss is reinforced with prestressed wire strands, bars and high-strength wire. The amount of non-prestressed

reinforcement in the tensile diagonals is determined by strength and crack-opening analysis.

#### ASSIGNMENTS TO CHAPTER 17

1. What precast members do a building skeleton, transverse and longitudinal frames consist of?
2. How is spatial rigidity of a one-storey building ensured — in transverse direction, in longitudinal direction?
3. What construction scheme of a building with purlin roof beams is used?
4. What groups of operation modes of overhead traveling cranes have been established and what loads do they impart to a building skeleton?
5. What does the assembly of the building skeleton construction consist in? What construction schemes give the best technical-economic indicators?
6. What is the choice of an efficient joint unit of a girder with the column of a transverse frame based on according to the requirements of elements typization?
7. What types of one-storey industrial building columns are used? What is the choice of column construction based on?
8. What is the significance of the system of bonds? How do horizontal and vertical bonds work?
9. What is the construction of crane girders? What do the methods of calculating loads due to overhead traveling cranes consist in?
10. How is the design scheme of a transverse frame subjected to loads represented?
11. What spans of RC beams, frames, arches are expedient in covering one-storey industrial buildings?
12. What designs of covering beams are applied? How their calculation is made and how is calculated section placement determined at double-pitch delineating top of belt?
13. What constructive schemes of frames R of covering are applied? How do belts and grids work under loading?
14. What is the technical and economic estimation of different types of covering frames?
15. What is the essence of the principle of designing and calculation of covering frames?
16. What design procedure is applied for units of covering frames?
17. What kinds of sub step designs are used?
18. What constructive schemes of RC arches of covering are applied? What is the principle of their calculation?
19. What constructive schemes of monolithic RC frames of one-storey frame buildings are applied and how are their units designed?

20. What normative document do the design institutes use for the choice of construction material and constructive form (a beam, a frame, an arch) of beam elements of covering?
21. What construction material and shape are optimums for bearing elements of hangars covering of airports of civil aviation with spans of more than 30 meters?
22. What constructive material and what constructional shape are applied for bearing elements of covering with spans up to 18 meters?
23. What purpose is the system of connections in skeletons of one-storey industrial buildings arranged?
24. What are the main advantages of monolithic reinforced-concrete transverse frames of one-storey skeleton constructions in comparison to modular (precast) frames?
25. Why are step frames rather than beams more economically efficient for covering more than 18m spans?
26. What is the basic difference of girder and non-girder frames?

## 18. THIN SHELL ROOFS

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### 18.1. General

As their name implies, shell roofs consist essentially of thin shells (thin slabs) and edge structures (such as ring-beam supports and diaphragms in the shape of beams, trusses, arches, etc.). Shells are given the shape of a curve or a polyhedron.

Thin shell roofs are many and diverse (Fig. 18.1, *a* through *g*). They may use barrel (or cylindrical) shells and prismatic folded plates, shells of revolutions with vertical axes of symmetry (such as domes), double-curved shells of positive and negative Gaussian curvatures usually rectangular in plan, and composite shells formed by intersecting curvilinear surface.

A special position is held by corrugated roofs, that is, roofs made of corrugated arches or multi-folded plates with a small corrugation-width-to-span ratio (Fig. 18.1, *h*), and also suspend roofs (held by guys) of various space and plane configurations (Fig. 18.1, *i* and *j*).

In practice, use is also made of many other thin shell roofs. Thin shell roofs are especially advantageous in industrial and civil buildings where large unobstructed areas (about 30 x 30m or more) are to be roofed over without intermediate supports. In fact, they are also widely used for smaller spans.

Owing to their action in two directions in plane, shell roofs are economical of material and have a smaller self-weight as compared with roofs made of plane structures (roof slabs, trusses, beams, arches, and secondary roof trusses). Shell roofs are very expressive architecturally.

Outside the CIS, thin shell roofs are commonly cast in-situ, which requires false work and formwork. In the CIS, shell roofs are made of prefabricated members adapted to industrialized manufacture.

The first thin shell roofs were erected in the 1920 s. In the USSR, the early cylindrical shells were used to roof over a water tank (1925), a post-office building (1928), a motor depot (1929), and a machine-building plant (1931), and theater buildings (1934 and 1939).

With progress in technology and experience, thin shell roofs have continually been improved.

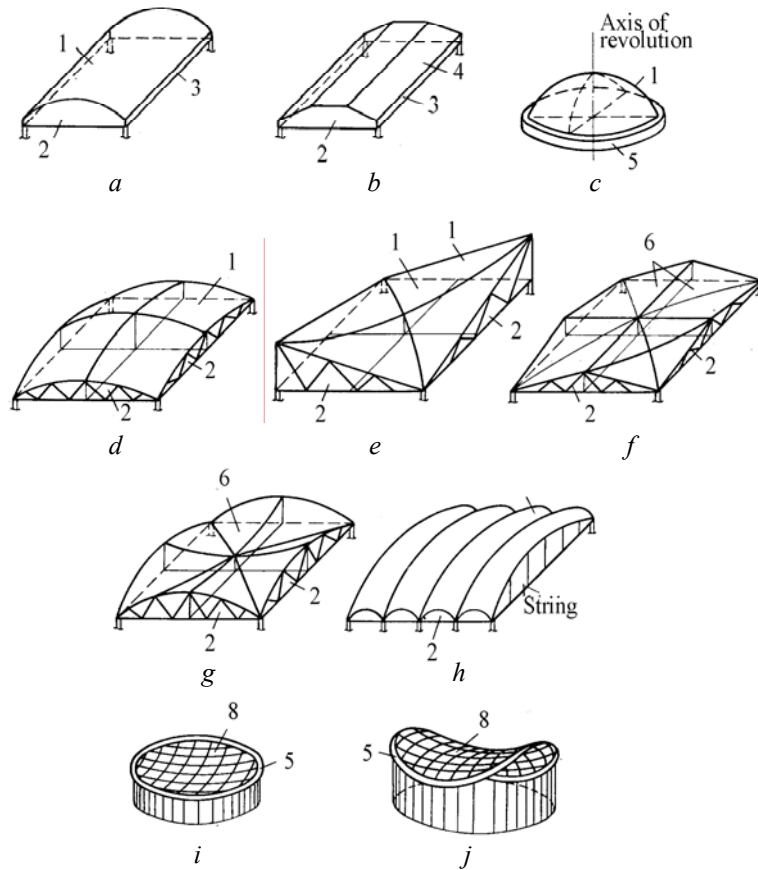


Fig. 18.1. Most common thin shell roofs:  
*a* — cylindrical (barrel) shell; *b* — prismatic folder plate;  
*c* — shell of revolution with a vertical axis (dome);  
*d* — double-curved rectangular-plan shell of positive Gaussian curvature;  
*e* — double-curved rectangular-plan shell of negative Gaussian curvature;  
*f* — composite shell assembled from rectangular members; *g* — composite shell assembled from triangular members; *h* — corrugated vault;  
*i* — positive-curvature suspended roof;  
*j* — negative-curvature suspended roof;  
*1* — shell; *2* — diaphragm;  
*3* — edge member; *4* — face of a folder plate; *5* — ring-beam support;  
*6* — shell member; *7* — individual corrugation; *8* — suspended shell

Of late, a number of unique shell roofs made of precast members have been installed in some Soviet cities.

Nowadays, it has become standard practice to prestress contour structures and the corner zones of shells; ever wider use is made of lightweight concrete, whole-span shell-slabs (such as large-size reinforced concrete barrel slabs, hyperbolic shells, and the like), shells made of fine-mesh wire-fabric reinforced concrete, and also reinforced concrete shells combined with steel diaphragms.

Double-curved surfaces may be formed in two ways; a surface is generated by revolving a plane curve (generatrix) about an axis lying in the same plane as the generatrix (Fig. 18.1, *c*). The latter technique consists in transfer a plane generatrix along parallel guides (Fig. 18.1, *d*). A double curved surface may also be formed by the translation of a plane curve (or a straight line, as a special case) along two nonparallel, nonintersecting guides (Fig. 18.1, *e*).

Most commonly, shell roofs use shallow shells with a rise of not more than 1/5 or 1/6 of any base side.

In a surface of positive Gaussian curvature, the centers of curvature at any point of the surface lie on the same side of the surface. If the centers of curvature are located on either side of a surface, the surface is said to have a negative Gaussian curvature.

As has been proved experimentally, thin shell roofs, as well as other reinforced concrete structures, behave elastically in the initial loading stage (that is, prior to cracking in the concrete of the tension zone). After the onset of cracking, the rising loads and stresses in the concrete and reinforced steel bring about plastic strains, which grow until the materials attain the state of ultimate equilibrium. The elastic behavior of shells has been investigated fairly well; whereas their behavior in plastic state and the state of ultimate equilibrium has yet to be studied. The plastic behavior of shell is expected to contribute to the stability of structures and to cut down material consumption.

In the general case, the normal sections of shells are subjected to normal forces,  $N_x$  and  $N_y$ , shearing forces,  $N_{xy}$  and  $N_{yx}$ , bending moments,  $M_x$  and  $M_y$ , shearing forces,  $Q_x$  and  $Q_y$ , and torques,  $H_x$  and  $H_y$  (Fig. 18.2). The above forces are given per unit length of the section.

Thin shells have a low flexuous rigidity as compared with the rigidity against the forces acting in the middle surface of the shell. As a result, external loads mostly give rise to the internal forces,  $N_x$ ,  $N_y$  and  $N_{xy}$ . Because of this, in most shells subjected to loads uniformly distributed over their surface (such as self-weight and snow load), a moment-free state of stress exists over nearly all the area of the shell, whereas a state of total stress is developed in some limited zones where the middle surface experiences an appreciable bending. Such bending shows up

where the shell joins the edge structures, in regions of sharp or sudden change in the load magnitude, in regions of sharp or sudden change in the surface curvature, and also where local forces (that is, those concentrated in small areas) are applied.

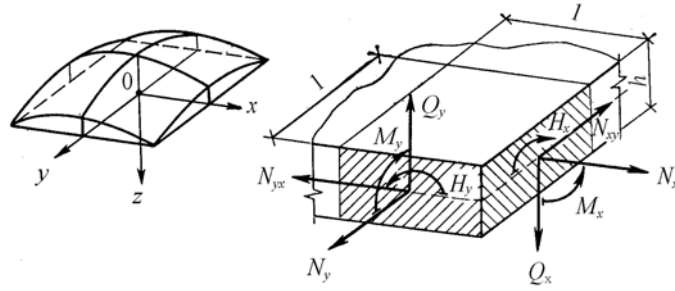


Fig. 18.2. Forces acting in the shell

The moment-free state of stress in thin shallow shells (Fig.18.2) is described by the following equation

$$k_y \frac{\partial^2 \varphi}{\partial x^2} + k_x \frac{\partial^2 \varphi}{\partial y^2} - 2k_{xy} \frac{\partial^2 \varphi}{\partial x \partial y} = -q, \quad (18.1)$$

where  $q$  is the load continuously distributed over the shell and normal for its surface. The  $\varphi(x, y)$  stress function is related to the internal forces of the shell as

$$N_x = \frac{\partial^2 \varphi}{\partial y^2}, \quad N_y = \frac{\partial^2 \varphi}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y}. \quad (18.2)$$

The surface curvatures,  $k_x$  and  $k_y$ , in the direction of the  $ox$  and  $oy$  axes and the torsion curvature of the surface,  $k_{xy}$ , are determined as

$$k_x = \frac{\partial^2 z}{\partial x^2}, \quad k_y = \frac{\partial^2 z}{\partial y^2}, \quad k_{xy} = -\frac{\partial^2 z}{\partial x \partial y}. \quad (18.3)$$

In most cases, the sag,  $w$ , of the middle surface in areas of local bending may be determined by giving only one coordinate, say, along the  $ox$ -axis. Then, the state of total stress can approximately be described by the following equation

$$-D \frac{d^4 w}{dx^4} + k_x N_x + k_y N_y + 2k_{xy} N_{xy} = -q. \quad (18.4)$$

Here,  $D$  is the cylindrical flexural rigidity of the shell, determined as.

$$D = \frac{EJ}{1-\nu^2} \approx E \frac{h^3}{12}, \quad (18.5)$$

where  $h$  is the thickness of the shell, and  $\nu$  is Poisson's ratio which is 1/6 for concrete.

## 18.2. Constructional Features of Thin Shell Roofs

The structure of a thin shell roof is chosen according to the purpose of the building, its architectural proportions, size, and erection method. Roofs made of pre-fabricated members should be designed so as to minimize labor consumption in the manufacture of pre-cast members, to keep to a minimum the number of member types and sizes used, to simplify erection and jointing, and to minimize the number of reusable supporting fixtures. In-situ r.c. roofs should be adapted to the use of traveling or portable reusable formwork.

To ensure the necessary strength and rigidity of pre-cast members in manufacture, in transit and erection, they are usually strengthened around the periphery. The result is ribbed shells. Instead of ribs, use can be made of removable steel spreaders installed at the edges of members for transit and erection, and left in place until the concrete in the joints hardens.

The type of joint between pre-cast shell members is chosen according to the nature and magnitude of the forces likely to occur in the joints.

In all cases, joints must be concreted. For better concreting, width of a joint should be not less than 30mm if the depth of the members at the joint does not exceed 100 mm or less than 50mm if the depth of the member exceeds 100 mm.

If a joint is designed to transmit axial or eccentric compressive force (in the latter case, the eccentricity should not exceed the kern size) and small shearing forces, it may be reinforced with overlapping reinforcement stick-out to suit structural considerations.

If joints are spaced not more than 1.5 m apart, tensile and shearing forces transmitted through a joint may be taken up by the reinforcement provided in transverse joints. In the joints, the reinforcement stick-outs of adjacent members are welded to one another.

The reinforcement of pre-cast shell members may also be connected by embedded steel parts welded to it. During erection, the connection between the embedded steel parts is provided by welded fishplates. The



cross-sectional area of the fishplates and the length of the welds are found by calculation.

It is good practice to use pre-stressed edge structures in shell roofs. For one thing, this contributes to the cracking-resistance of tension areas; for another, pre-stressed edge structures help to combine the pre-cast roof members in a single entity.

Shell areas subjected to biaxial compression should be checked for stability.

In the stage of ultimate equilibrium, the load-bearing capacity of structures is practically independent of intermediate states of stress. Consequently, the strength-analysis procedure for precast, in-situ, pre-cast/in-situ, pre-stressed and non-pre-stressed shell roofs is the same. In the design in terms of other limit states, special attention should be given to the forces due to the self-weight of pre-cast members, arising prior to joint concreting.

In addition, prestressed shell roofs should be checked for cracking-resistance under service loads, and also for strength in the course of pre-stressing.

Pre-fabricated members should be checked for strength under loads likely to occur in manufacture and transit.

The amount of reinforcement and proportions of thin shell roofs are found according to the internal normal and shearing forces and bending moments. The maximum principle compressive stresses should not exceed  $R_c$ .

In the compression zones and zones where the principle tensile stresses are smaller than  $R_{bt}$ , the reinforcement is distributed for structural reasons, with the cross-sectional area of reinforcing steel being not less than 0,2 % of the concrete, and with the bars being spaced 20 to 25 cm apart. For shell slabs more than 8 cm thick, double fabric is recommended.

Where the principle tensile stresses exceed  $R_{ten}$ , all of the internal forces should be taken up by the reinforcing steel in the form of bars placed so as to match closely the direction of the principle tensile stresses, or in the form of fabric made up of longitudinal and transverse bars. The amount of reinforcement carrying bending moments in smooth shell is determined as that in slabs. The reinforcing steel is disposed in the tension zone to fit the bending moment diagram, with the minimum concrete cover.

At the joints between edge members and diaphragms, shells are reinforced with double mats made of bars 6 to 10mm in diameter and spaced not wider than 20 cm apart.

In ribbed structures, the amount of rib load-bearing reinforcement is designed according to the bending moments arising in manufacture and service. Ribs are reinforced with welded bar mats in which transverse bars 5 to 6mm in diameter are spaced 20 to 25 cm apart.

Edge structures are designed according to the general rules of structural mechanics to resist the forces transferred by the shells, and erection loads.

Small openings and holes in shells are hemmed with flanges. The cross-sectional area of the flanges in the compression zones of shells is taken equal to that of the removed concrete. Oblong openings are strengthened with intermediate spacers. The flanges of openings cut out in the tension zones of shells are reinforced so that the steel carries all the forces that would be applied to the removed part.

### 18.3. Roofs Made of Cylindrical Shells and Prismatic Folded Plates

Cylindrical shell roofs (see Fig. 18.1, *a*) consist essentially of thin cylindrical slabs, edge members and face diaphragms. The whole structure is supported on columns placed at the corners.

The main parameters of a shell (Fig. 18.3, *a*) are the longitudinal span,  $l_1$  (that is, the center to center distance between diaphragms), the arch span,  $l_2$  (that is, the distance between edge members) the rise  $f$ .

Shell may be rounding, elliptic or parabolic in cross section. Owing to the ease of manufacture, the first type is used most frequently.

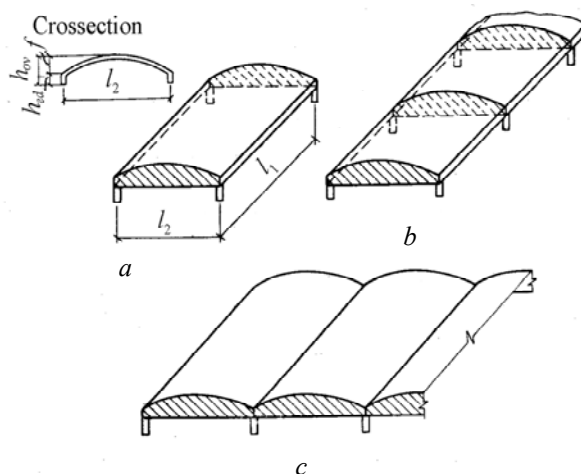


Fig. 18.3. Types of cylindrical shells:  
*a* — single-span shell; *b* — multi-span shell; *c* — multi-barrel shell

According to their construction, cylindrical shells (Fig. 18.3) may be classed into single span shells supported on two diaphragms placed

along their generatrices, multi span shells supported on more than two diaphragms, single barrel shells, multi-barrel shells consisting of several single barrel shells, plain shells and ribbed shells.

The stress-strain state of a shell depends on the relative magnitudes of  $l_1$  and  $l_2$ . Shells with  $l_1/l_2 > 1$  or 2 are rated as long, and with  $l_1/l_2 < 1$ , as short.

The overall thickness,  $h_1$ , of a non-prestressed shell including the thickness of its edge member is usually not less than  $l_1/15$  to  $l_1/10$ . The thickness of prestressed shells may be smaller. Also, the value of  $h_1$  should not be less than  $l_2/6$ . The thickness,  $h$ , of an in situ shell is taken as  $l_2/200$  to  $l_2/300$  but not less than 5 or 6 cm. Prefabricated ribbed shells should be at least 3 cm deep (according to manufacture conditions).

**Long Shells.** Under certain conditions, a long cylindrical shell resisting the self weight and snow load deforms as a beam of span,  $l_1$ , width,  $l_2$ , and of an odd shaped cross section of depth,  $h_1$ , including the edge members (Fig. 18.3). The bottom part of the shell cross section is in tension, and the top part in compression.

The function of the edge members is to contribute to the strength and rigidity of the roof cross section, to accommodate the tensile load bearing reinforcement, and to strengthen the straight line edges of the shell against local loads. The shape and size of the edge members depend on the construction of the shell and to design calculation.

In situ r.c. shells are usually made plain. If it is to carry suspended concentrated loads, a shell is made ribbed. Prefabricated shells generally have longitudinal and transverse ribs intended to provide additional strength in manufacture, in transit and during erection.

The most popular diaphragms are solid beams, trusses, bowstring arches and frames. To secure better light and ventilation, cylindrical shells may be made of the shed type (Fig. 18.4, *a*), or have skylight opening at the apex (Fig. 18.4, *b*). The openings are hemmed with longitudinal ribs; long openings are strengthened with spacers.

The internal forces in shells may be determined by accurate and approximate design methods. The latter are preferable for the structural analysis, and the former for the design of a shell; although, in most cases, approximate methods yield quite satisfactory results.

When  $l_1/l_2 > 3$  (or  $l_1/l_2 > 2$  for intermediate barrels), single and multi-barrel symmetrical shells with the edge members unsupported in the longitudinal direction may be designed as tray-like beams for strength, rigidity and crack-resistance against vertical uniformly distributed symmetrical loads imposed by the self-weight (Fig. 18.3). The strength of shells is determined for design loads in the state of ultimate

equilibrium, and rigidity and crack-resistance for basic loads. For design purpose, a unilateral uniformly distributed snow load which does not exceed  $1/4$  of the total snow load may be replaced by a symmetrical load of the same magnitude. Light live loads imposed by telfers with a lifting power of up to 1 ton, suspended to the edge members and may be regarded as a symmetrical load applied simultaneously to both edge members.

All ribbed long cylindrical shells having at least three transverse ribs not less than  $l_2/25$  in depth may be treated as reinforced concrete beams.

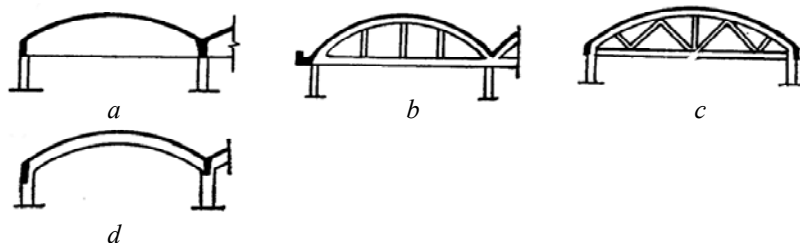


Fig. 18.4. Types of diagrams:  
*a* — solid beam; *b, c* — bowstring arch; *d* — truss frame

In other cases, long shells are analyzed as elastic systems deformable in space.

The butterfly edge of a multi-barrel shell with the edge members unsupported in the longitudinal direction may approximately be designed as a symmetrical single-barrel shell, and the intermediate barrels as single-barrel shells but with allowance for the fixation of the longitudinal edges against displacement in the lateral direction (Fig. 18.5).

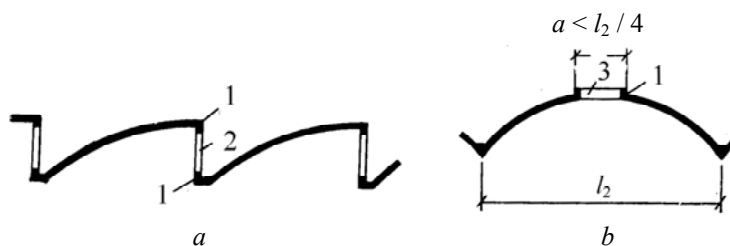


Fig. 18.5. Shells with skylight openings:  
*a* — shed type shell; *b* — barrel shell;  
*1* — longitudinal rib; *2* — strut in the plane of the clerestory; *3* — spacer

In the case of multi-barrel shell subjected to a uniformly distributed load (Fig. 18.6), it is sufficient to limit the analysis to a single-barrel

shell on hinged immovable supports, of span,  $l_0$ , equal to the distance from the edge member to the zero-point on the bending moment diagram plotted or the corresponding multi-span beam and then assume that the internal forces in the shell vary in the same manner as the ordinates of the bending moment diagram of the multi-span beam.

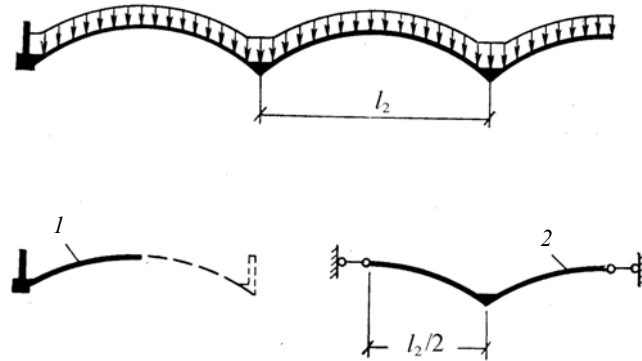


Fig. 18.6. Loading systems for a multi-barrel shell butterfly edge:  
 1 — side semi-barrel; 2 — intermediate barrel

Let us analyze a long cylindrical shell of symmetrical circular cross section subjected to a vertical symmetrical load in the state of ultimate equilibrium as a reinforced concrete beam. Fig. 18.7 shows a diagram representing the state of stress in the shell cross section.

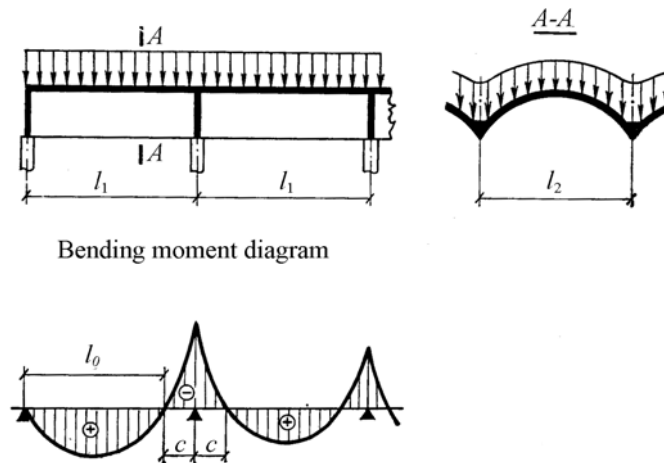


Fig. 18.7. Loading system for a multi-barrel shell

Here,  $A_s$  is total cross-section area of the tensile steel;  $\theta_1$  is half of the shell arch central angle;  $\theta_p$  is one half of the central angle of the compression zone arch;  $c$  is the distance from the resultant of the forces in the tensile steel to the center of curvature of the circular part of the shell;  $t$  and  $R_y$  are the depth and the radius of the cylindrical part of the shell, respectively; and  $d_1$  is the distance from the resultant of the forces in the tensile steel to the top of the edge member.

The condition for strength in terms of the internal force moments taken about the center of the circular part of the shell is

$$M \leq 0,8(2R_c t R_y^2 \sin\theta_c - c_o R_s A_s) \quad (18.6)$$

where  $M$  is the moment due to external forces, calculated about the same point as in a beam, and 0,8 is an empirical service factor.

The position of the neutral axis may be found from the following equation

$$2R_c \theta_c R_y t = R_s A_s \quad (18.7)$$

when checking the shell for strength, the value of  $\theta_c$  found from Eq. (18.6) is substituted into Eq. (18.7). When calculating the amount of reinforcement required, both expressions are combined as

$$\sin\theta_c - \frac{c}{R_y} \theta_c - \frac{M}{2R_c t R_y^2} = 0. \quad (18.8)$$

The above equation may be solved by successive approximation by assuming  $\sin\theta_c = \theta_c$  and finding  $A_s$  from Eq. (18.7).

The tangential forces in the shell reach their maximum at the support; they can be computed by the strength of materials expressions

$$\tau t = \frac{QS}{J}, \quad (18.9)$$

where  $Q$  is the shearing force at the support, calculated as for a beam;  $\tau$  is the shell tangential stress (Fig. 18.8).

In order to determine bending moments,  $M_y$ , acting across the barrel, it is necessary to consider a transverse strip of unit length (Fig. 18.9, *a*). It is subjected to an external vertical distributed load,  $q$ , applied to its surface, and tangential forces,  $T$  and  $T + \Delta T$ , acting in the planes of the sections bounding the strip. In the strip, the load,  $q$ , the differential force,  $\Delta T$ , and the forces,  $M_y$ ,  $Q_y$  and  $N_y$ , are in equilibrium (Fig. 18.9, *b*). Consequently, the bending moment,  $M_y$ , in any section of the strip is determined as the

sum of the moments due to  $q$  and  $\Delta T$  about the axis,  $a-a$ , in the section in question (Fig. 18.9,  $b$ ). The diagram for  $M_y$  is shown in Fig. 18.9,  $c$ . The internal forces in a long cylindrical shell (Fig. 18.9) regarded as an elastic space system may approximately be found from the moment-free state of stress.

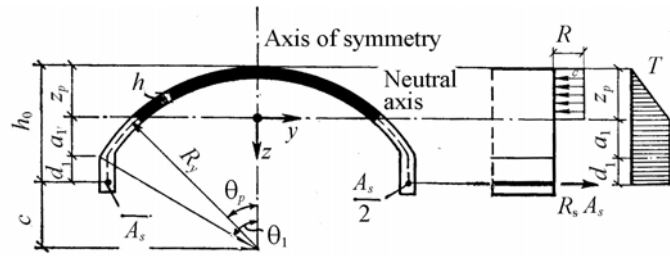


Fig. 18.8. Design of a cylindrical shell in the state of ultimate Equilibrium as an r.c. beam

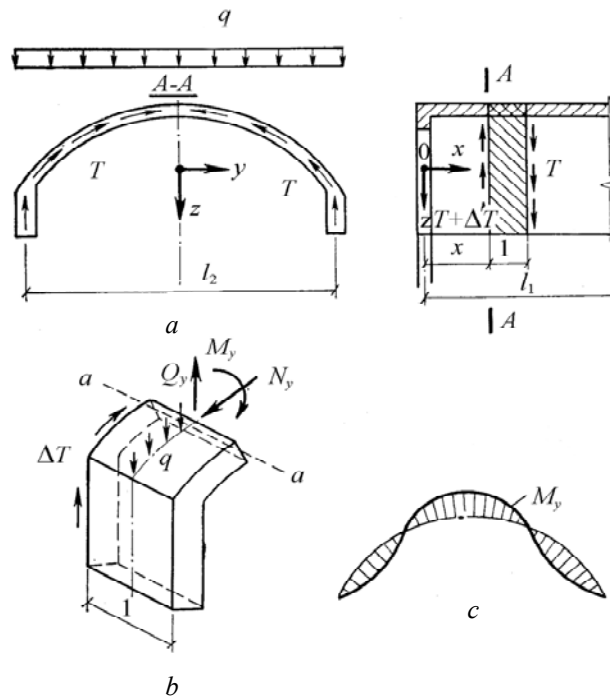


Fig. 18.9. Design of a long cylindrical shell across the barrel:  
 $a$  — barrel strip of unit length with the tangential forces;  
 $b$  — part of the barrel strip with the load and internal forces;  
 $c$  — diagram for the bending moments acting across the barrel

For a shallow shell, we may use Eq. (18.1) where  $k_x$  is assumed to be zero. Hence,

$$k_y = \frac{1}{R_y}; k_{xy} = 0; D = 0; \frac{\partial^2 \phi}{\partial x^2} = -qR_y. \quad (18.10)$$

The results of the static analysis are used to determine the cross-sectional area of the shell reinforcement (Fig. 18.10). The cross-sectional area of the longitudinal tensile steel of type 1 (if the shell is analyzed as a space system) is computed by the following formula

$$A_s = z_{\max}/R_s. \quad (18.11)$$

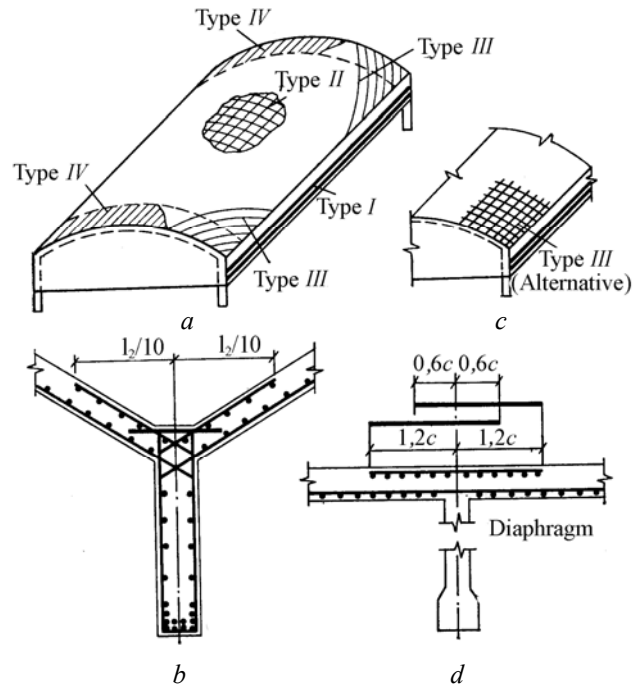


Fig. 18.10. Reinforcement of a long shell:  
*a* — shell; *b* — shell reinforcement near an intermediate edge member;  
*c* — same, above an intermediate diaphragm

Here,  $z_{\max}$  is the amount of tensile forces from the diagram for  $N_x$ .

In roofs using vertical edge members, 80 % of  $A_s$  is disposed within the edge member, 60 % of which is concentrated at the bottom. In shells with other types of edge member, 40 % to 60 % of  $A_s$  is distributed over



the bottom part of the edge member, and the rest in the tension zone of the shell where the steel is disposed linearly. The cross-sectional area of steel in the tension zone should be at least 0,2 % of that of the concrete.

Along the barrel the cross-sectional area of the longitudinal reinforcement of type 1 may be reduced in proportion to  $N_x$ , but at least 30 % of the reinforcement should extend as far as the support. The cross-sectional area of the longitudinal reinforcement is reduced by using bars of a smaller diameter and by welding at the joints, rather than by cutting off the bars.

In the longitudinal direction, the compression zone of the shell is reinforced for structural reasons with bars 5 to 6mm in diameter spaced 20 to 25 cm apart. The cross-sectional area of the reinforcement is found as for a slab in proportion to the maximum ordinates of the  $M_y$  diagram (see Fig. 18.9, *c*). The reinforcing bars are placed according to the sign of the moment diagram across the barrel.

The above bars are combined into type 2 fabrics, which are distributed over the entire shell (Fig. 18.10, *a*). Near diaphragms, the normal forces,  $N_x$ , are approximately equal to zero, whereas the shearing forces,  $N_{xy}$ , reach their maximum and give rise to the principal tensile forces equal to  $N_{xy}$  in magnitude and making an angle of  $45^\circ$  with the horizontal generatrix. In the region where the principal tensile stresses exceed  $R_{ct}$ , they are taken up solely by the reinforcement. If the cross-sectional area of type 2 fabric proves to be insufficient, use is made of additional type 3 reinforcement (diagonal bars or square-mesh fabric) anchored in the edge members or diaphragms.

The regions where the shell adjoins the diaphragm type 4 reinforcement is used which is designed according to the diagram for  $M_x$ .

In multi-barrel shells, the regions near intermediate edge members are additionally reinforced with transverse bars 6 to 10 mm in diameter and spaced 10 to 20 cm apart (Fig. 18.10, *b*), which are intended to resist the moments,  $M_y$ , at the supports.

Within the regions of length  $c$  near supports in multi-barrel shells (see Fig. 18.7), the forces  $N_x$  are assumed to be varying longitudinally in proportion to the bending moments, and the forces,  $N_{xy}$ , in proportion to the shearing forces of a similar multi-span beam. Within region  $c$ , the bending moments,  $M_y$ , are assumed to be distributed uniformly.

The cross-sectional area of the tensile steel above the intermediate diaphragms is also found by Equation (18.10). In the middle half of the barrel arch, the tensile steel may be distributed uniformly, whereas in side quarters of the barrel arch, it should be disposed according to a triangular diagram. Along the shell, all of the steel is extended on either

side of the diaphragms for at least  $0,6 c$  (Fig. 18.10), and half of the steel is extended as far as  $1,2 c$  reckoning from the intermediate diaphragm. Here,  $c$  is the distance from the support to the zero-moment section (see Fig. 18.7).

The diaphragms resist the shearing forces acting in the middle of its surface and transmitted from the shell (Fig. 18.11). The static analysis of diaphragms consists in determining the internal forces,  $M$ ,  $N$  and  $Q$ , caused by  $N_{xy}$ , taking into account the structural features of the diaphragms.

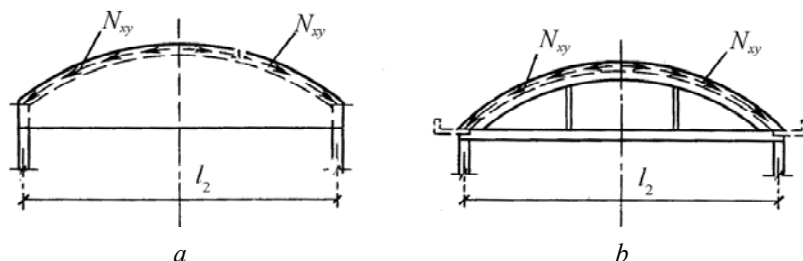


Fig. 18.11. Transfer of forces from the shell to the diaphragm:  
*a* — beam diaphragm; *b* — arch diaphragm

In truss diaphragms, the forces,  $N_{xy}$ , are transferred from the middle surface of the shell to the top chord axis with an appropriate bending moment applied, and are applied as concentrated loads at the truss joints. The longitudinal forces in the truss members are found and the members are proportioned in compliance with the general rules.

Bowstring arch diaphragms (Fig. 18.11, *b*) are similar to ordinary arches in construction. Under the action of  $N_{xy}$ , the middle part of an arch is in eccentric tension, the regions at the supports are in eccentric compression, and the string is in tension.

In practice, the component members of long pre-cast cylindrical shell roofs can be fabricated with or without edge members. In the former case (Fig. 18.12, *a*), all prefabricated members are assembled into a single structure by means of pre-stressed reinforcement passed through longitudinal ducts. Such structures, however, are very complex in shape, require very accurate alignment of reinforcement ducts, and are very expensive to erect because they need false work.

In the latter case (Fig. 18.12, *b*), prefabricated members have a simpler shape and can be erected without false work (shell panels are laid on edge members which are additionally supported during erection).

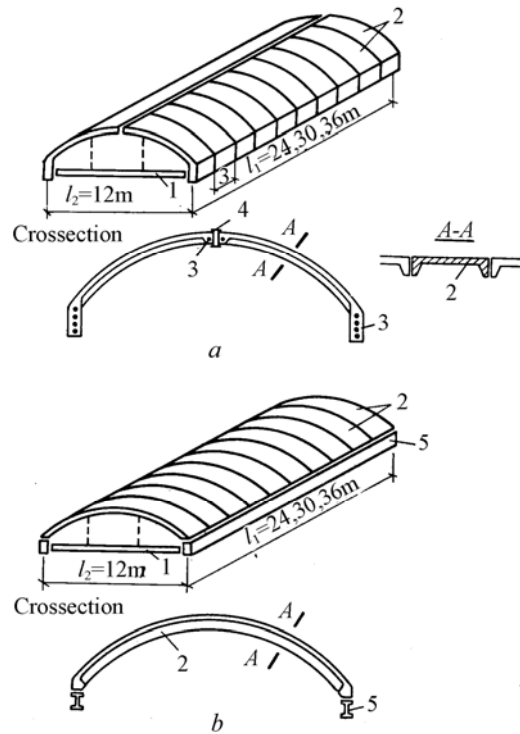


Fig. 18.12. In-situ r.c. roofs using long cylindrical shells:  
*a* — shell monolithically connected to edge members;  
*b* — shell detached from edge members;  
 1 — diaphragm string; 2 — precast shell; 3 — prestressed reinforcement;  
 4 — fishplate; 5 — edge member

The joints between shell panels and edge members, however, are rather complex because concrete keys are required for better transmission of shearing forces. In view of this, the quality of such joints is rather difficult to control.

There also exist other ways of dividing cylindrical shells into precast component members. For example, good designs have been developed for shallow shell roofs. Abroad, roofs built of long cylindrical shell are very popular. They are mostly cast in-situ as this meets the established local reinforced concrete practice.

### Short Shells

As already mentioned, shells are rated as short if  $l_1/l_2 < 1$  (Fig. 18.13).

The practical recommendations for the design of in-situ r.c. short shell of  $l_2 = 12$  to 30 m,  $l_1 = 6$  to 12 m and  $f > l_2/7$  are based on large-

scale experiments. For structural reasons, the thickness of shell is taken as 5 or 6 cm for  $l_1 = 6$  m, and 7 or 8 cm for  $l_2 = 9$  to 12 m. The depth of edge members is assumed as  $h_2 = l_1/10$  to  $l_1/15$ , and the width as  $d = 0,2h_2$  to  $0,4h_2$ . Shells are reinforced for structural considerations with fabric made of bars 5 to 6 mm in diameter and spaced 10 to 20 cm apart.

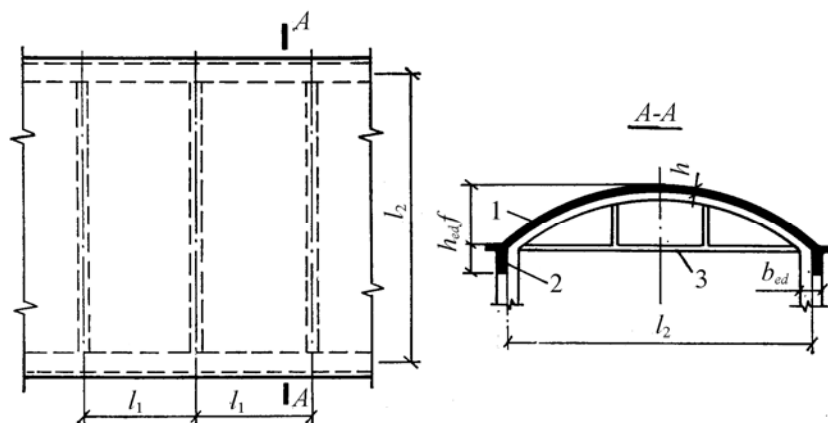


Fig. 18.13. In-situ r.c. short cylindrical shell:  
1 — barrel slab; 2 — edge member; 3 — diaphragm

In the direction of  $l_1$ , a shell is analyzed as a beam. At the mid-span of a single-span single-barrel shell, the bending moment is found as

$$M = \frac{(ql_2)l_1^2}{8}. \quad (18.12)$$

The required cross-sectional area of the tensile steel is determined as

$$A_s = \frac{M}{R_s z} = \frac{ql_2 l_1^2}{4,5R_s (f + h_2)}, \quad (18.13)$$

where  $z$  is the arm of the internal forces couple; according to calculations and tests,  $z = 0,55(f + h_2)$ .

The tensile steel thus determined is placed in the edge members. If we have a single-span multi-barrel shell, the cross-sectional area of the reinforcement in intermediate edge members is taken equal to  $A_s$ , whereas in the marginal members it is taken as  $A_s/2$ . The cross-sectional area of the steel in the middle spans of a multi-span shell is taken as one half of that in the marginal spans.

The longitudinal bars of the edge member reinforcement are combined into welded mats, in which the transverse bars are disposed to

suit structural requirements.  $N_{\text{ear}}$  the edge members, the shell is reinforced with additional fabric pieces (Fig. 18.14, *a*). Also, additional reinforcement is provided above the diaphragms extending for  $0,1l_1$  on either side of the diaphragm (Fig. 18.14, *b*). In either case, additional reinforcement is of the same type as the main fabric.

In the direction of  $l_2$ , the diaphragm is analyzed in interaction with the shell slab (Fig. 18.15).

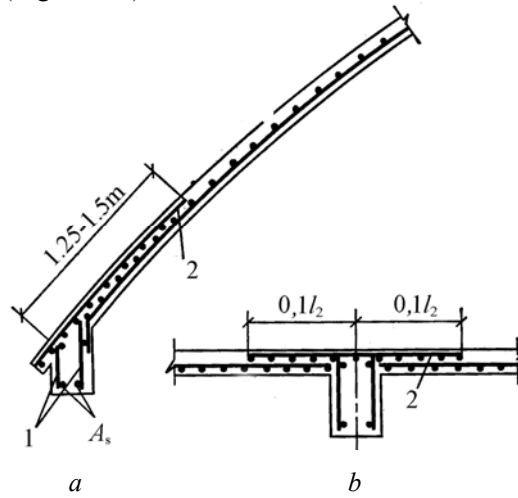


Fig. 18.14. Reinforcement of in-situ r.c. short shells:  
*a*—at the edge members; *b* — above intermediate diaphragms;  
 1 — welded-bar mats; 2 — additional welded-bar mats

Experiments and calculations have shown that, under load, a statically determinate diaphragm (an arch with a cut string) is in compression, and the maximum compressive force  $N_{y,\text{max}} = -qR_y l_1$  (where  $R_y$  is the radius of the shell curvature) is at the apex. Along the arch, the compressive force varies as a quadratic parabola:

$$N_y = -\frac{4qR_y l_1}{l_2^2} y(l_2 - y), \quad (18.14)$$

from the condition for equilibrium, we may conclude that the diaphragm resists the forces of the same magnitude but opposite in direction. So the internal forces in statically determinate diaphragms should be determined as

$$\left. \begin{aligned} M_y &= M_y^0 - N_y a; Q = Q_y^0 \\ N_y &= N_y^0 + N_y \end{aligned} \right\}, \quad (18.15)$$

where  $M_y^0$ ,  $Q_y^0$  and  $N_y^0$  — are the moment and the forces calculated about the diaphragm axis as in a statically determinate system under total vertical load; and  $a$  is the distance from the diaphragm axis to the middle surface of the shell.

Prefabricated roofs using short cylindrical shells consist of diaphragms, roof channel slabs, and edge members (Fig. 18.16, *a*). The joints between the prefabricated slabs should be properly concreted and crossed by anchors. The joints between the slabs and diaphragms are strengthened by concrete keys.

Among the advantages of prefabricated roofs are the ease of member manufacture and erection, and the increased overall rigidity of the system. This, however, is somewhat hampered by the complex construction of the joints between the roof slabs and trusses.

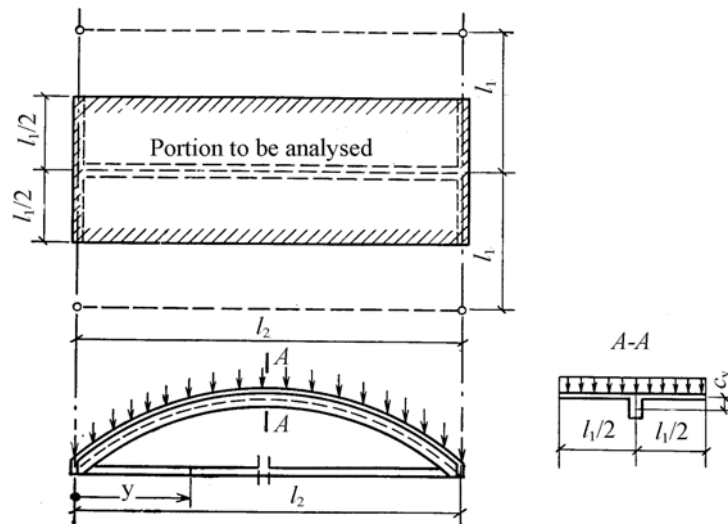


Fig. 18.15. Loading system for the diaphragm of a short shell

Another type of short cylindrical shell roofs uses large-size reinforced concrete barrel slabs 3m wide roofing over areas 12 to 24 m in span (Fig. 18.16, *b*). A large-size reinforced concrete barrel slab is a shallow thin cylindrical shell curving in the longitudinal direction. It is strengthened with two longitudinal variable-section diaphragms serving as ribs, and transverse edge members. The main prestressed load-bearing reinforcement is disposed in the ribs. Large-size r.c. barrel slabs have found wide application in building construction.

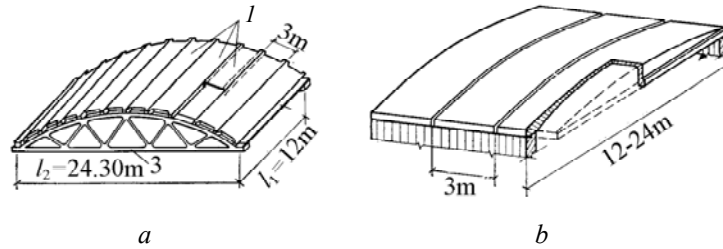


Fig. 18.16. Precast roofs using short cylindrical shells:  
*a* — trusses and roof slabs; *b* — large-size r.c. barrel slabs;  
 1 — precast slabs; 2 — edge member; 3 — truss diaphragm

### Prismatic Folded-Plate System

A folded-plate roof consists essentially of flat slabs (monolithically connected to one another at their longer sides), edge members, and diaphragms (Fig. 18.17, *a*).

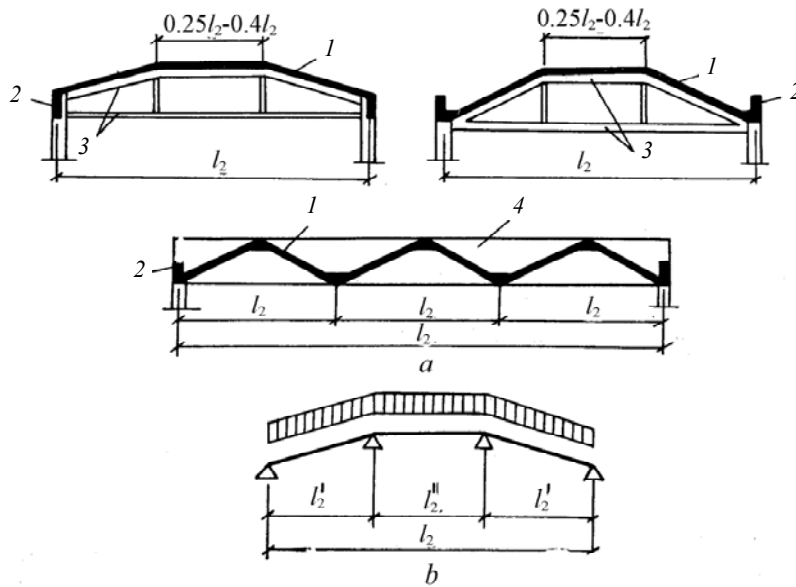


Fig. 18.17. Prismatic folded-plate roofs:  
*a* — types of cross sections and diaphragms; *b* — loading system  
 for a folded plate in the transverse direction;  
 1 — folded plate; 2 — edge member; 3 — queenpost diaphragm;  
 4 — beam diaphragm

Folded-plate roofs may bridge one or several longitudinal and transverse spans. In the longitudinal direction, the design of folded plates is based on the same assumptions as that of long cylindrical shells.

In the transverse direction, folded-plate roofs behave in bending as multi-span beam slabs with a broken axis (the ribs are treated as supports) (Fig. 18.17, *b*). The width of the side faces usually ranges between 3 and 3,5 m. The transverse span  $l_2$  of three-face folded plates is from 9 to 12 m. The longitudinal span  $l_1$  usually exceeds  $l_2$ , and the thickness of folded plates is taken as  $1/7 \dots 1/10 l_1$ .

In the transverse direction, folded plates are reinforced as multi-span slabs, to fit the bending moment diagram. The longitudinal compressive reinforcement (in the direction of  $l_1$ ) is placed for structural reasons; it consists of bars 5 to 8 mm in diameters spaced 20 to 25 cm apart. The amount of longitudinal tensile reinforcement in the direction of  $l_1$  is found by calculations; this reinforcement is placed in the edge members.

In other respects, folded-plate roofs are analyzed and designed as roofs built of long cylindrical shells.

#### ASSIGNMENTS TO CHAPTER 18

1. What typical schemes of thin spatial roofs are most often used in construction?
2. What components characterize moment-free and bending states of thin shells?
3. What equation is used to determine the moment-free state components of thin shells?
4. What is the general characteristic of a stressed state of thin shells, rectangular in plan, exposed to a uniformly distributed continuous load?
5. What types of cylindrical shells are used in construction practice?
6. What is the design scheme of a long cylindrical shell in its limit state?
7. What are the schemes for reinforcement of long cylindrical shells?
8. What are construction schemes of precast roofs with long cylindrical shells?
9. What are characteristic schemes of thin-walled spatial designs of coverings most often applied in construction?
10. What components are characteristic of non-moment and bending conditions of thin-walled shells?
11. What equation is used for determination of components of non-moment conditions of thin-walled shells?
12. What is a general characteristic of an intense condition of thin-walled shells, rectangular in the plan, loaded with uniformly distributed continuous loading?
13. What kinds of cylindrical shells are used in construction?
14. What is the calculation scheme of a long cylindrical shell in its limiting state?



15. What delineating has diagram forces  $N_x$ ,  $N_y$  and the moments in transverse section of cylindrical shells?
16. What are the schemes of reinforcing of long cylindrical shells?
17. What are constructive schemes of modular covering with long cylindrical shells?
18. What attributes are characteristic of short cylindrical shells?
19. What are constructive decisions on modular and monolithic short cylindrical shells?
20. How are characteristic diagrams of internal forces  $N_x$  and  $N_{xy}$ ,  $N_{m1}$ ,  $N_{m2}$  in flat shells of double curvature (with the square plan) represented?
21. What scheme are flat shells of double curvature, rectangular in the plan reinforced by?
22. What can constructive schemes of modular coverings rectangular in the plan with flat shells of double curvature, according to carrying, layer, revolution be like?
23. What is the calculation scheme of a dome shell like?
24. What are calculation scheme of a dome, elastic fixed on a contour in a basic ring like?
25. How are schemes of reinforcing of domes shell represented?
26. What is the essence of calculation of a basic ring and a dome shell?
27. What is the constructive scheme of modular domes?
28. What constructive schemes of modular domes are used in practice?
29. Which schemes of hanging coverings with surfaces of double unequivocal curvature are used? What systems of rope-arrangement are applied?
30. How are schemes of hanging coverings with surfaces of double two-value curvature represented?
31. What schemes are applied to hanging covering with double system of stay ropes?
32. What are the principles of calculation of hanging covering with radial and orthogonal arrangement of stay ropes?

## **19. DESIGN OF MULTISTORIED BUILDINGS OF SKELETON AND BEARING-WALL CONSTRUCTION**

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### **19.1. Design of multi-storeyed Industrial Buildings. Construction Types**

Multistoried buildings are widely used to accommodate plant and processes in the mechanical engineering, instrument-making, chemical, electrical engineering, radio engineering, textile, food, and some other industries, and also warehouses, cold-storage houses, garages, and the like. As a rule, such buildings are of the skeleton construction with filler wall panels.

According to the industrial process to be accommodated, industrial buildings are generally from 3 to 7 storeys high (with a total height of up to 40 m). In some cases, buildings intended for lightweight equipment may be 12 to 14 storeys high. The width of industrial buildings may be from 18 to 36 m or even more. The height per storey and the column grid size are assigned so that the component members and the overall dimensions of the assembly units can be standardized. The height per storey is taken as a multiple of 1,2 m, being 3,6, 4,8 and 6 m; the ground storey may sometimes be as high as 7,2 m. The most widely used column grid sizes are 6×6 m, 9×6 m and, though seldom, 12×6 m. The small column grid sizes are chosen in anticipation of heavy live loads imposed on the floors, which may reach 15 kN/m<sup>2</sup> or, in some cases, 25 kN/m<sup>2</sup> and more.

For industrial purposes, the most convenient buildings are of the multistoried skeleton-type without any vertical diaphragms that might take up some of the area necessary for process equipment and services. The main load-bearing elements of a multistoried skeleton-type building are reinforced concrete frames and floors combining the frames into a single structure. In the transverse direction, a building derives its resistance to lateral loading from the rigidity of the connections between the columns and girders of multistoried frames, so that the building behaves as a frame-type structure. In the longitudinal direction, the building owes its space rigidity to vertical steel braces or reinforced concrete shear walls installed in the column rows and in the plane of the exterior walls; in this case, the building acts as a shear wall structure (Fig. 19.1).

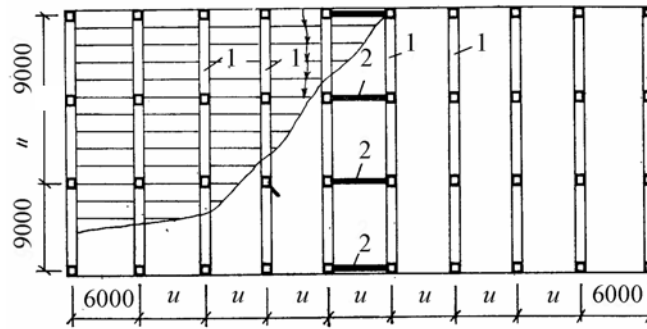


Fig. 19.1. Layout of a skeleton-type multistoried industrial building:  
 1 — transverse frames; 2 — longitudinal shear walls; 3 — floor slabs

If, for some reasons, braces or shear walls cannot be installed in a building, they must be replaced by longitudinal girders. In this case, the building will behave as a frame-type structure in both transverse and longitudinal directions.

A likely design of a multistoried industrial building with beam and girder floors is given in Fig. 19.2. If overhead traveling cranes will be installed (say, in chemical industry buildings), the upper storey is assembled from columns, girders and crane beams similar to those used in one-storey industrial buildings.

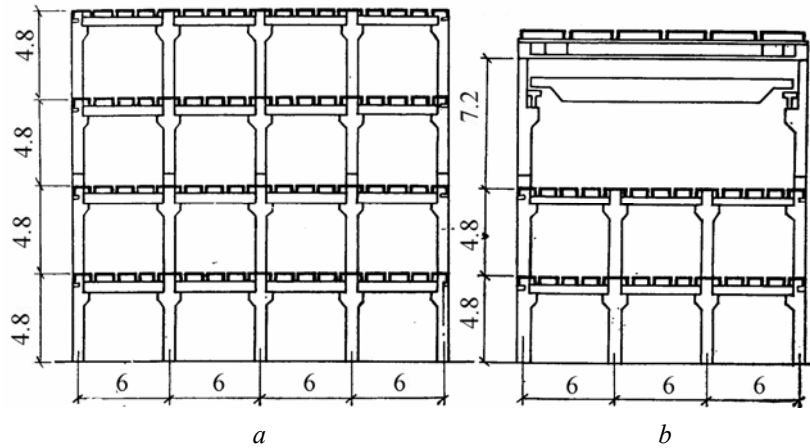


Fig. 19.2. Multistoried industrial buildings:  
 a — regular; b — with overhead travelling cranes in the upper storey

The girders are supported on the column (strut) cantilevers; the reinforcement stick-outs are welded in a pool of molten metal within a

reusable mould, and the joint is concreted during erection (Fig. 19.3). The floors are made of ribbed slabs 1500 mm wide.

The floor slabs installed along the column rows serve as girders (spreaders) which contribute to the rigidity of the building in the longitudinal direction during erection. In such buildings, floor slabs may be supported either on the flanges of T-section girders (when the floors are intended to carry machine tools, in which case the load may roughly be regarded as uniformly distributed), or on the top of rectangular

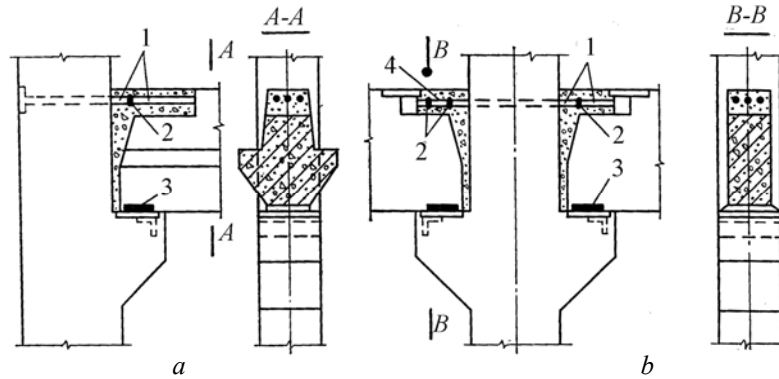


Fig. 19.3. Connection between the girders and columns:

*a* — at an outer joint; *b* — at an intermediate joint;

1 — reinforcement stick-outs; 2 — field-welded reinforcement stick-outs;

3 — field-welded embedded parts; 4 — reinforcement inserts

girders (this mostly applies to chemical industry buildings where the process equipment extends from floor to floor, and a large concentrated load is transferred to an individual column). In both cases, type-design girders for spans of 6 or 9 m are made with the same depth (800 mm) and rib width (300 mm). Type-design multistoried industrial buildings with beam and girder floors have been developed for column grid sizes of 6×6 and 9×6 m, and for live loads ranging between 5 and 25 kN/m<sup>2</sup>. Fig. 19.4 shows a likely construction of a multistoried industrial building with flat-slab floors. The girders of the transverse and longitudinal multistoried frames are flat slabs rigidly connected to the columns by the capitals. In both directions, the skeleton-type building behaves as a frame structure. To standardize the dimensions of the slabs and capitals in the outer and middle spans of the flat-slab floor, the exterior self-supporting walls are displaced from the centre line of the outer column row for a distance equal to one half of the drop panel width.

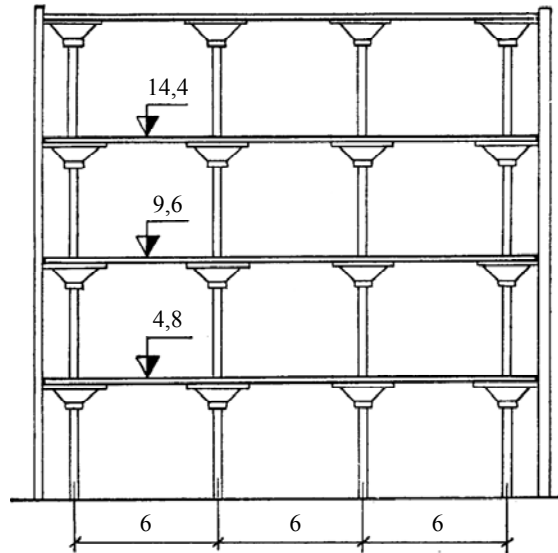


Fig. 19.4 Multistoried industrial building with flat-slab floors

Multistoried industrial buildings with closely spaced supports and column grid sizes of  $6 \times 6$  and  $9 \times 6$  m are not always convenient from the viewpoint of plant layout, and future expansion or re-tooling might require expensive reconstruction.

Therefore, they should be confined to heavy live floor loads running over  $10 \text{ kN/m}^2$ .

A distinction of universal industrial buildings having additional floors in the intertruss space is a larger column grid size, which may be  $18 \times 6$ ;  $18 \times 12$  and  $24 \times 6$  m. The larger spans are bridged by trusses without diagonal web members. The space between the top and bottom chords of such trusses is used to accommodate engineering facilities, utilities or welfare services; it may also be used as storage space or for other auxiliary purposes. The height of intertruss storeys may be 2.4; 3.0 or 3.6 m.

A likely construction of a universal industrial building is shown in Fig. 19.5. The building has three main and three intertruss storeys. The trusses without web diagonals are rigidly connected to the columns. They are members of the multistoried skeleton, and act as frame girders. The outer struts of the trusses have projections on the top and bottom for connection to the columns of the neighboring storeys. The main floors are made of ribbed slabs laid on the top chords of the trusses. The additional floors use hollow-core or ribbed slabs supported on the flanges of the bottom truss chords (Fig. 19.6).

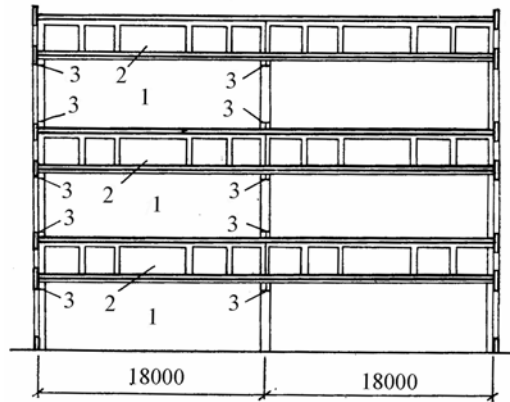


Fig. 19.5. Multistoried industrial building with intertruss storeys: 1 — main storeys; 2 — intertruss storeys; 3 — connections between columns and trusses without web diagonals

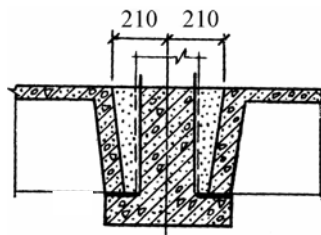


Fig. 19.6. Floor slabs supported on the bottom chord of a truss without web diagonals

## 19.2. Construction Types of Multistoried Frames

Multistoried frames are assembled from off-site prefabricated members adapted to streamlined manufacture and erection. Frame girders are generally designed to be built up from rectilinear members joined together in the plane of the column side by concealed or cantilever joints (Fig. 19.7, *a* and *b*). Columns are likewise designed from several rectilinear members to be joined at every other or every third storey above floor level. Sometimes, in order to enhance the continuity of structure and reduce the number of prefabricated members, multistoried frames are designed as assemblies of single-span one-storey frames (Fig. 19.7, *c*). The joints in multistoried frames using prefabricated members are generally made rigid. Hinged joints would impair the overall rigidity of the building and its resistance to lateral

loading. This is especially pronounced with the increasing number of storeys in a skeleton-type building designed to act as a frame structure.

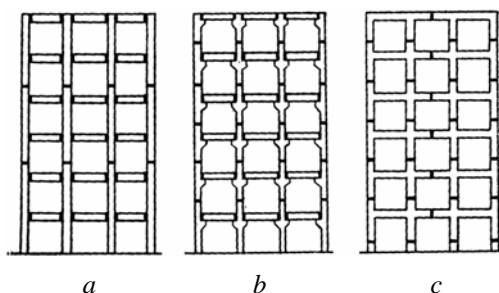


Fig. 19.7. Multistoried frames assembled from several prefab members

Standard 6-m girders are reinforced with non-prestressed steel, and 9-m girders with prestressed steel placed in the span (Fig. 19.8).

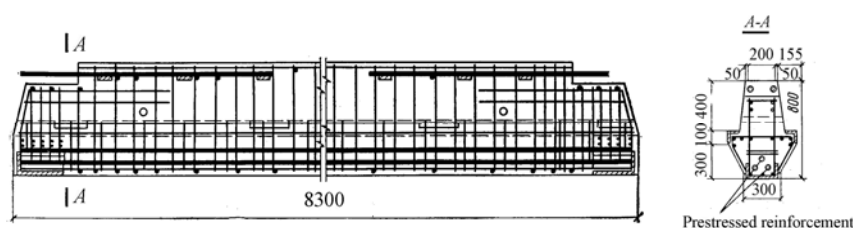


Fig. 19.8. Reinforcement of the 9-m girder of a transverse frame

Two-storeyed columns are congested with longitudinal and transverse reinforcement as eccentrically compressed members (Fig. 19.9).

There are several types of rigid joints in multistoried frames, intended to take up the longitudinal force,  $N$ , and the bending moment,  $M$ , transferred through embedded parts, reinforcing steel and concrete.

Fig. 19.10 shows a concreted joint between the end faces of two columns, produced by welding together embedded angles and plates. For ease of column alignment, the casing formed by the angles and plates has a welded centering pad 30 to 40 mm thick. The longitudinal load-bearing bars of the columns are factory-welded to the casings from the outside. The joint bars placed between the load-bearing bars are welded to the casings during erection. The gap between the column ends is

caulked with stiff mortar, and concrete is placed over welded-wire fabric. This type of joint has the same strength as the columns, but requires a relatively large amount of steel and is too labor consuming. Fig. 19.11 shows a welded joint using end-face steel sheets and a centering pad. To the steel sheets 10 to 20 mm thick are welded four deformed anchor bars and a centering pad 3 to 4 mm thick. The steel sheets of the columns being joined are welded together around their periphery during erection. For this purpose, the edges of the steel sheets are beveled to obtain the desired weld throat. At the ends where the longitudinal reinforcement is cut off, the columns are additionally strengthened with transverse fabric.

In most cases, the bending moment at the joints between the columns in multi-storeyed skeleton-type buildings may be reduced by making the joints closer to the middle of the storey height where the bending moments due to vertical and lateral loads tend to zero, and where it is easier to join the columns.

In-Situ and Precast In-Situ multistoreyed Frames. The girder of an in-situ multistoried frame is reinforced similarly to the girder of an in-situ beam and girder floor, except the marginal support where the girder is rigidly connected to a column (Fig. 19.12). When designing a frame, "bonding" joints must be provided. The purpose of such joints is to bond fresh concrete to that already placed, should an interruption occur in pouring the concrete. In columns, such joints should be located on the topside of the floor. Here, the reinforcement of the lower-storey columns must be extended as appropriate and connected to the reinforcement of the columns of the storey above (Fig. 19.12, b).

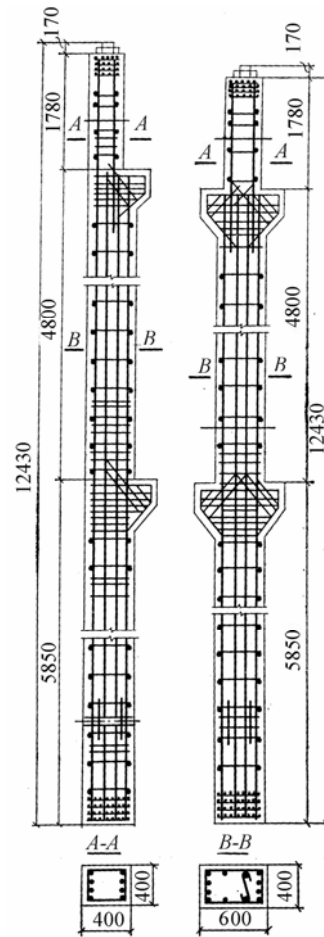


Fig. 19.9. Reinforcement of transverse frame columns



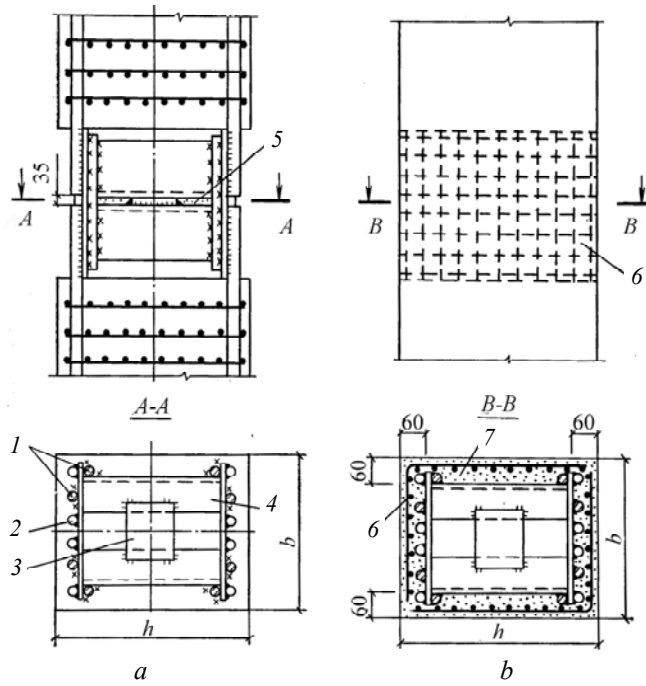


Fig. 19.10. Concreted joint between the end faces of two columns produced by welding together embedded casings:  
*a* — joint before concreting; *b* — concreted joint;  
 1 — fishplates; 2 — longitudinal steel; 3 — steel centering pad;  
 4 — steel casing formed by welded angles and plates; 5 — gap between column ends caulked with stiff mortar; 6 — welded-wire fabric; 7 — in-situ concrete

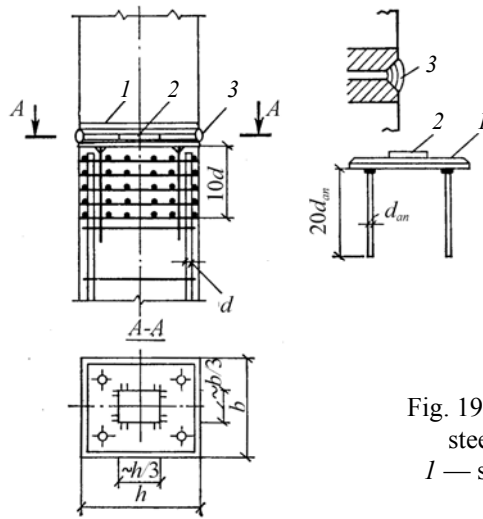


Fig. 19.11. Welded joint using end-face steel sheets and a centering pad:  
 1 — steel sheets, 2 — centering pad,  
 3 — welds

It is good practice to reinforce large-span and high-storeyed in-situ buildings with load-bearing cages. Fig. 19.13 illustrates the load-bearing reinforcing cage of a multistoried frame for a power station building.

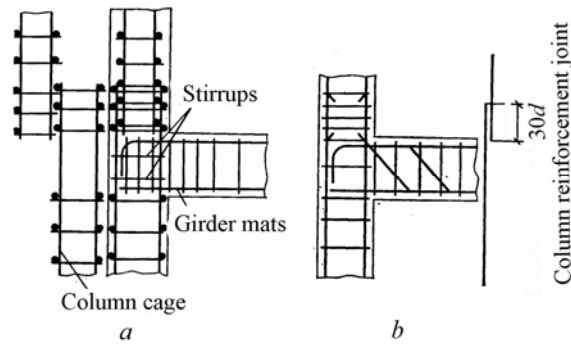


Fig. 19.12. Reinforcement of the joints of an in-situ multistoried frame

For each span of the girder, the welded mats are made in the form of plane trusses with diagonals, and are assembled into a single reinforcing cage with lateral braces at the top and on the underside. For a column the reinforcement is in the form of a cage made of longitudinal bars, stirrups and lateral braces located at the sides. Precast in-situ frames are likewise made with rigid connections.

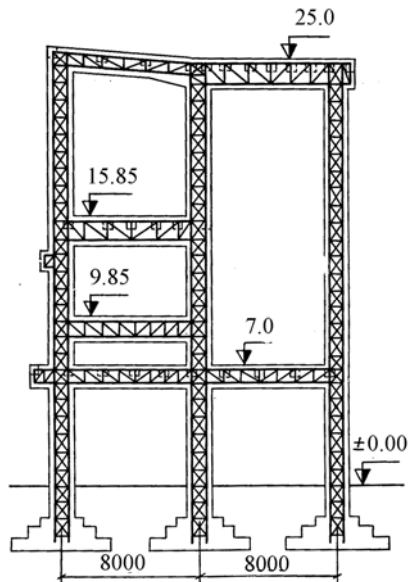


Fig. 19.13. Load-bearing reinforcing cage of an in-situ multistoried frame

The T-section girder has protruding stirrups and bare supporting reinforcement on top of its flange (Fig. 19.14, a). The girder gives support to ribbed slabs placed with a gap of 12 cm between their end faces. The joint between the girder and the columns is made rigid by connecting the top reinforcement of the girder to the column reinforcement. For this purpose, the supporting bars of the joint are passed through a hole in the column. The girders may have ledges to carry the slabs

(Fig. 19.14, *b*). After the prefabricated members have been installed, and the supporting reinforcement of the girder placed and welded, the cavities between the slabs and the gaps between the girder ends and the columns are concreted to provide monolithic connections. Owing to their interaction with the slabs, the girders are regarded as T-sections.

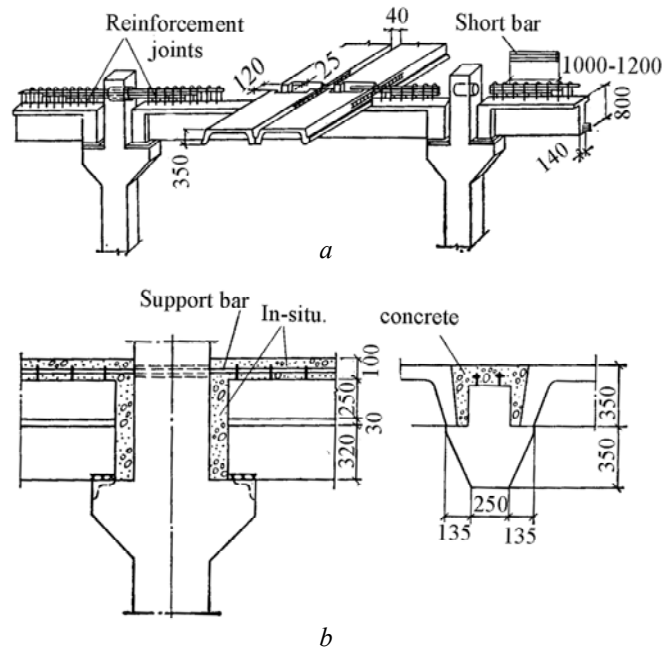


Fig. 19.14. Joints in a precast in-situ multistoried frame:  
*a* — before concreting, *b* — after concreting

### 19.3. Design of the Multi storied Civil Buildings

**Construction Types.** For mass construction, civil buildings of the skeleton and bearing-wall construction are designed to have 12 to 16 storeys. In some cases, they may be high-rise buildings with 20 storeys or even more. The column grid size, the centre-to-centre distance between the bearing walls and the storey height are chosen so that the component members can be adapted to industrialized manufacture and their dimensions standardized. Civil buildings assembled from prefabricated members have a number of advantages over other construction types in that they are uniform in height, use standardized component members, and are clear-cut in layout.

Skeleton-type construction is used for administrative, public and other buildings where large floor area unobstructed by partitions is required. Sometimes, it may be used in apartment houses over 25 storeys high. The main bearing elements in a skeleton-type multistoried civil building are reinforced concrete frames, shear walls and floors.

Under lateral load, the various vertical members of a multistoried building are made to act together owing to the high in-plane flexural rigidity of the floors which behave as lateral diaphragms. The flexural rigidity of the prefabricated floors is ensured by welded embedded parts and concreted joints between the slabs.

In this way, one ensures that multistoried buildings have ample resistance to lateral loading. More specifically, the required space rigidity may be achieved in any one of several ways, according to the manner in which the structure resists lateral loads.

For example, in a skeleton-type building having transverse multistoried frames and transverse shear walls, lateral loads are taken up by the interacting vertical members, and the building behaves in the transverse direction as a shear wall-frame structure. In the longitudinal direction, the building has only shear walls and, therefore, acts as a shear wall structure (Fig. 19.15, *a*).

A building having transverse shear walls and longitudinal multistoried frames behaves as a shear wall structure in the transverse direction, and as a frame structure in the longitudinal direction (Fig. 19.15, *b*). A skeleton-type building in which the girders and the columns are connected by hinges acts as a shear wall structure in both directions. Shear wall-frame multistoried buildings of skeleton construction are the best choice from the technical and economic viewpoint and are widely used in construction practice.

Bearing-wall (large panel) construction is used in apartment buildings, hotels, motels and similar buildings with closely spaced partitions and walls. In such buildings, the main bearing elements are shear walls formed by the panels of the interior bearing walls placed in the transverse direction (or, sometimes, in the longitudinal direction), and floors. The exterior wall panels are suspended on the end faces of the transverse bearing-wall panels. Multistoried bearing-wall (large panel) buildings resist lateral loading as shear wall structures in both directions (Fig. 19.15, *c*).

Fig. 19.16 shows some other construction types of multistoried buildings. These include, for example, skeleton-type single-core buildings, where the internal shear walls are made of combined elevator and ventilation shafts and staircases (Fig. 19.16, *a*); skeleton-type double-

core buildings in which the open-work cores are made of channels and I-sections (Fig. 19.16, *b*); and skeleton-type double-core buildings of complex plan layout tailored to individual architectural designs (Fig. 19.16, *c*). The above construction types resist lateral loading as shear wall or shear wall-frame structures.

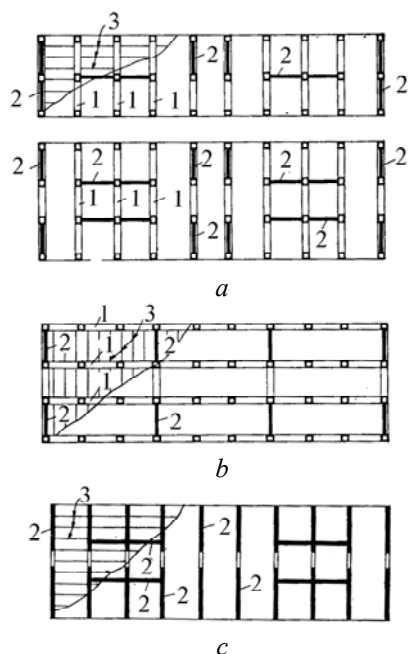


Fig. 19.15. Plan layouts of multistoried other construction types of civil buildings:  
*a* — skeleton-type with transverse frames; *b* — skeleton-type with longitudinal frames; *c* — bearing-wall type;  
 1 — frame girders; 2 — shear walls; 3 — floor slabs

For freedom in layout, a large-size column grid size is used in single-core buildings. Also, in some cases, the internal columns may be omitted so that floor members are supported on the outer columns and the core. Floor girders 12 to 15 m in span are pre-stressed and connected to columns by means of hinges. Floor slabs are made hollow-cored or channeled.

The floors in double-core buildings with complex plan layout consist of in-situ flat slabs installed without capitals. Such buildings are constructed by the lift slab method of construction. With this method, slabs are cast at the ground level, one on top of another. Only side forms

are necessary. Specially designed steel collars for column connections are embedded in the slab. After the slabs attain the desired strength, they are lifted hydraulically upon the columns and fastened to the columns after reaching their final position by welding the steel collars to the embedded steel parts in the columns. This construction type resists lateral loading as a shear wall structure. If, on the other hand, the floor slabs are rigidly connected to the columns so that the flat slabs behave as girders, the building acts as a shear wall-frame structure.

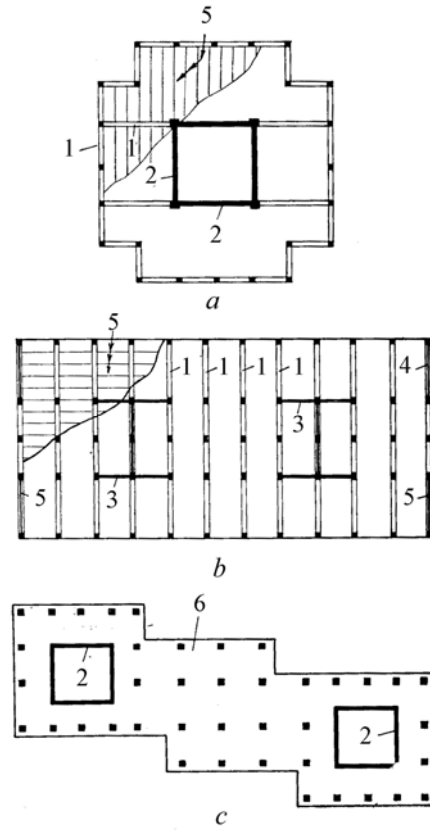


Fig. 19.16. Skeleton-type multistoried buildings:  
*a* — single-core type; *b* — double-core type;  
*c* — double-core type of complex plan layout,  
 constructed by the lift-slab method;  
 1 — frame girders; 2 — closed stiffening core;  
 3 — I-section stiffening core;  
 4 — shear walls; 5 — floor slab; 6 — in-situ flat-slab floor

**Main Vertical Structural Members.** Multistoried frames up to 16 storeys high have uniform-section columns throughout the building height (Fig. 19.17, *a*). The load-bearing capacity of the lower-floor columns is improved by using a high-brand concrete, a larger amount of reinforcing steel, and stiff reinforcement. To reduce labour consumption during erection, the component members of precast columns are made to extend for two to four storeys.

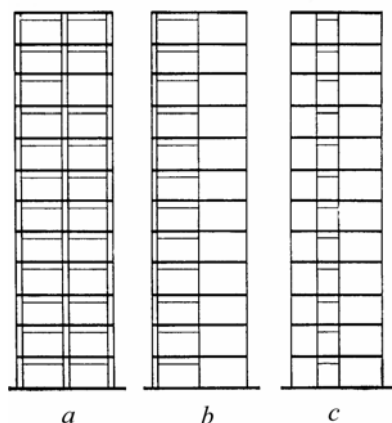


Fig. 19.17. Main vertical structural members of multistoried buildings:  
*a* — regular multistoried frames,  
*b* — combined shear walls;  
*c* — open-work shear walls

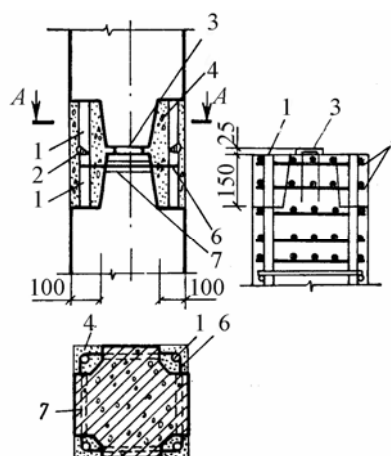


Fig. 19.18. Joint between columns made by welding together with the reinforcement stick-outs in a pool of molten metal contained within a reusable mould: 1 — reinforcement stick-outs, 2 — to be welded during erection; 3 — concrete centering projection, 4 — in-situ concrete, 5 — transverse fabric, 6 — additional stirrup, 7 — slots in column

Combined shear walls, comprising wall and frame parts have regular member and span dimensions throughout the building height (Fig. 19.17, *b*). Shear walls with openings contain reinforced concrete lintels rigidly connected with the wall parts at the supports. They also have regular dimensions throughout the building height (Fig. 19.17, *c*).

The hinged or rigid joints between the girders and the columns are made with or without concealed cantilevers. The rigid joints considerably increase the overall rigidity of the building and cut down the amount of steel in the girders (because such girders have an improved strength and crack resistance and an increased limiting sag).

The joints between the columns are made by welding together the reinforcement stick-outs up to 40 mm in diameter in a pool of molten metal contained within a reusable mould (Fig. 19.18). At the ends, the column members are additionally strengthened with transverse fabric and cast to form centering projections 25 mm high for ease of alignment during erection. The sides of these projections are made 150 mm long so that the reinforcement stick-outs can conveniently be welded together. After the column parts have been installed and aligned, the reinforcement stick-outs are successively welded along the diagonal, and an additional erection stirrup 10 to 12 mm in diameter is placed in the joint area. The joint cavities are concreted in-situ under pressure in a reusable form-work to constitute shear keys. Experience has shown that such joints are strong and reliable. As compared with other types of joints, they take less steel and less labour to make them. At one time, the column members were made with spherical, rather than pointed ends, and the reinforcement stick-outs were welded together in a pool of molten metal contained within a reusable mould to form a spherical joint. This practice required more complex formwork, so it is no longer used in multistoried building construction.

Precast shear walls consist essentially of skeleton columns and panels with flanges intended to carry floor slabs (Fig. 19.19).

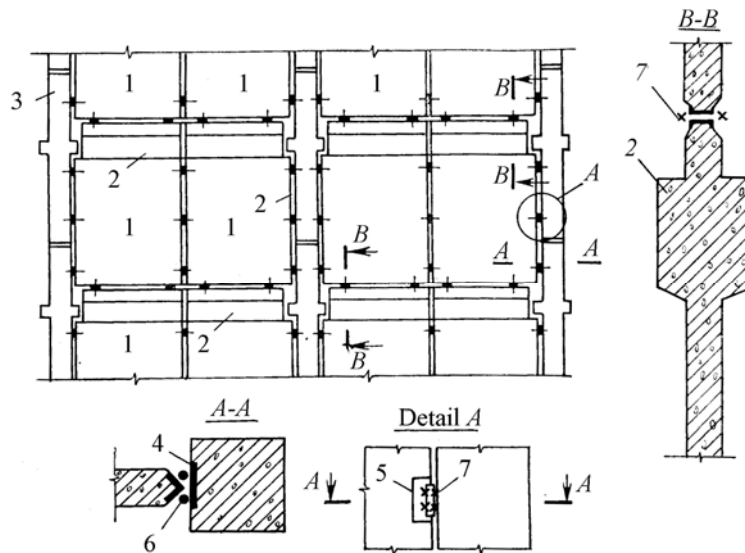


Fig. 19.19. Connection of shear wall members:  
 1 — shear wall panels; 2 — flanges supporting floor slabs; 3 — skeleton columns;  
 4 — column embedded parts; 5 — embedded parts of shear wall panels;  
 6 — joint bars; 7 — to be welded during erection



The component members are joined by welding together embedded steel parts and concreting. Use is also made of panels cast in-situ after reinforcing fabric has been welded to the embedded parts of the columns.

Stiffening-core shear walls are most frequently cast in-situ in sliding formwork. In-situ cast structures are preferable because the component members of the cores cannot be sufficiently standardized to warrant prefabrication. Also, precast products would require a considerable number of welded joints to be made during erection to resist appreciable shearing forces arising at the corners of the shear walls.

To meet sound-insulation requirements, the panels of the interior bearing walls are made of heavy concrete 14 to 16cm thick. This thickness ensures an adequate load-bearing capacity of the panels in buildings up to 16 storeys high. In higher buildings, the load-bearing capacity in the lower floors is enhanced by using higher brands of concrete and reinforced concrete or thicker panels.

The concrete panels of bearing walls are reinforced with vertical steel distributed for structural reasons at each face. The steel area is  $0,3 \text{ cm}^2$  per meter run of the horizontal cross-section of a panel. The amount of horizontal distribution reinforcement at each face should be at least  $0,3 \text{ cm}^2$  per meter run of the vertical cross-section of a panel. The reinforced concrete panels of bearing walls are reinforced with double vertical steel so that the minimum percentage of reinforcement in horizontal cross-sections is 0,1 for B20 concrete and 0,15 for B30 or B40 concrete. To increase the strength of the support sections of reinforced concrete panels (that is, to compensate for the cut-off longitudinal reinforcement), they are indirectly reinforced with fabric.

An advanced type of bearing-wall buildings is one built of box-formed complete room or apartment-size dwelling units. Such units are fully finished at an off-site factory and require minimum labor for erection. According to the manufacturing process involved, three types of box-formed units are available, namely a unit with a separate ceiling panel, a unit with a separate floor panel, and a tube-like unit with separate end-wall panels (Fig. 19.20).

At an off-site factory, these dwelling units can be cast as monolithic structures or assembled from prefab panels. The behavior of this type of building under service loads is governed by the manner in which the dwelling units rest one on top of another. If the units rest on in-situ concrete joints all the way around their perimeter, the building behaves as a bearing-wall type with the walls in compression. If the units are supported at the corners or rest on internal pilasters, the building behaves as a bearing-wall type with the walls subjected to bending in their plane.

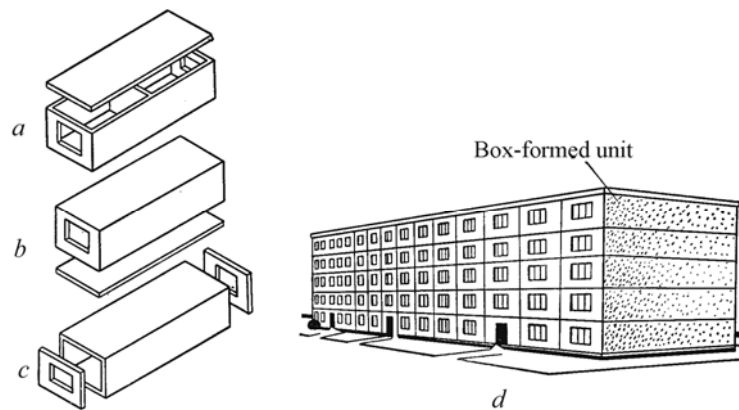


Fig. 19.20. Multistoried apartment building built of box-formed units: *a* — unit with a separate ceiling panel; *b* — unit with a separate floor panel; *c* — tube-like unit with separate end-wall panels; *d* — multistoried building

### ASSIGNMENTS TO CHAPTER 19

1. What construction schemes of multistory industrial buildings are used?
2. What are the constructions of multistory prefabricated frames and column joints?
3. What are the constructions of multistory monolithic and precast-monolithic frames?
4. What construction schemes of multistory civil skeleton-type and panel-type buildings are used?
5. What systems ensuring spatial rigidity of a multistory building are used?
6. What ensures joint work of different type vertical constructions of multistory buildings?
7. What is characteristic of main vertical load-bearing constructions of skeleton-type buildings — diaphragm frames and rigidity kerns?
8. What design schemes of multistory buildings are available?
9. What is characteristic of frame-bond systems with combined diaphragms?
10. What constructive schemes of multi-storey industrial buildings are applied?
11. What are the designs of multi-storey modular frames and joints of columns?
12. What are the designs of multi-storey monolithic and prefabricated-monolithic frames?

13. What constructive schemes of multi-storey civil frame and panel buildings are applied?
14. What systems providing spatial rigidity of a multi-storey building are applied?
15. What provides joint work of various vertical designs of multi-storey buildings?
16. What is characteristic of basic vertical bearing designs of frame buildings- frames, diaphragms, nucleus of rigidity?
17. What calculation schemes of multi-storey buildings are possible?
18. What is moving rigidity of a multi-storey frame skeleton? How is it determined?
19. What is the dependence between movements of a multi-storey frame and transverse force on horizontal loading?
20. How is the equation of balance of transverse forces in horizontal section of a multi-storey frame write down?
21. How can we write down the basic equation of a multi-storey system? What are the linear characteristic and the characteristic of rigidity?
22. How does the pliability of joints influences work of prefabricated RCCs and how is it taken into account in calculations?
23. How should we determine movement and effort in frame connecting systems?
24. What is characteristic of frame-connecting systems with combined diaphragms?
25. How are moves and efforts in diaphragms with openings of connecting systems determined?
26. How can deflections and effort in design sections of multi-storey systems be determine making use of tables?
27. What is the essence of calculation of a system with two different type vertical designs?
28. How does pliability of bases influence the work of a multi-storey design?
29. How does plane the bend of overlapping in its influence the work of a multi-storey design?
30. How are dynamic characteristics of frame systems determine?
31. How are dynamic characteristics of frame connecting systems determined?
32. How are dynamic characteristics of diaphragms with openings of connecting systems determined?
33. How can the factor of the form of fluctuations determined?
34. How can we determine the pulse component of wind loading on a multi-storey building?
35. Name basic constructive schemes of multi-storey constructions
36. What bearing systems of skeletons of multi-storey constructions by providing spatial rigidity are applied?
37. How are efforts in spatial systems of multi-storey buildings under vertical and horizontal loadings determined?

## 20. CALCULATION OF STONE CONSTRUCTIONS

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### 20.1. Stone materials. Building mortars

**General information.** Manufactured stones with mass less than 40kg and prefabricated stone wares with the mass restricted by the load carrying capacity of transport and assembling equipment are used as stone materials for masonry. Stone materials include ceramic bricks, ceramic stones, natural stones of regular shape, rubbles of irregular shape and concrete stones. Stone wares are produced as concrete blocks for different purposes, brick blocks and ceramic stones, blocks from natural stone, vibratory panels made of brick and ceramic stone, and concrete panels. Stone materials, applied for masonry, must meet durability and frost-resistance requirements to provide durability and reliability of stone constructions.

Mixtures from inorganic binding materials (i.e. cement, lime and clay), fine aggregate (sand), water and special admixtures are used as building mortars for mason work. According to type of used binding materials the building mortars are subdivided into cement, lime and mixed (cement-lime, cement-clay) ones. Freshly made building mortars must possess mobility and water keeping ability, and in the hardening state they should provide necessary durability of masonry. To enhance bearing capacity of masonry the different methods of its reinforcing by means of steel armature are used. Such masonry is named a reinforced masonry. The usage of the reinforced masonry allows considerably extend the masonwork application field in constructions.

Stone materials are classified as follows:

*By the origin:*

- natural stones, mined in stone careers (stone blocks, rubble, crushed stone);
- manufactured stones, made by burning (brick, ceramic stone, facing tiles) and the unborn stones (silicate brick, slag brick, concrete stones made of heavy and light concrete);

*By the structure:*

- dense brick and solid stones;
- hollow brick and stones with hollows of different structure.

For manual mason work the next types of bricks are applied:  
 – usual ceramic brick manufactured by means of plastic and semidry pressing;

- ceramic hollow brick manufactured by means of plastic pressing;
- silicate brick;
- slag brick.

A brick is produced as a single block with dimensions  $250 \times 120 \times 65$  mm and as a modular (thickened) block with dimensions  $250 \times 120 \times 88$  mm. Single brick is usually performed as a dense one or with technological hollows. Modular brick is prefabricated with technological hollows or as a hollow one with circular or chinked hollows in order to decrease its mass (Fig. 20.1).

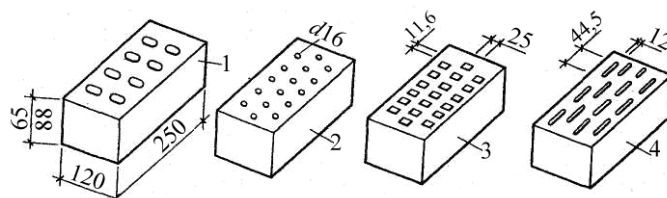


Fig. 20.1. Types of brick:

- 1 — usual ceramic brick of semidry pressing with blind hollows;  
 2...4 — ceramic brick of plastic pressing  
 (2 — with circular hollows; 3, 4 — with chinked hollows)

For manual stone masonry the manufactured stones are used, i.e. ceramic, concrete, natural stones of regular shape and rubble stone. Ceramic stones are made as hollow ones, blocks manufactured by means of plastic pressing with dimensions  $250 \times 120 \times 138$  mm (7 or 18 pieces) and with longitudinal hollows (Fig. 20.2). Solid and hollow concrete stones are made of heavy and light concretes (e.g. concrete with porous aggregates, cellular concrete, popcorn concrete, porous concrete and silicate one). Natural stones of regular shape are sawed out from array of rocks or sawed up from stocking blocks. Hard rocks and sedimentary rocks (i.e. marble, cuts, limestone, and volcanic tuff) are used for these wares. Natural rubble stone (irregular shape) is mined from sedimentary porous rocks (dense limestone, dolomite, sandstone). Stones from natural materials shouldn't have weathering marks, clay layers, stratifications and cracks.

**Durability and frost-resistance.** Durability of stone materials is determined on base of results of standard samples compression test. Brick is additionally tested on bending. Stone and concrete are brittle materials and their ultimate compression strength is higher than the ultimate tensile

strength by 10...15 times. Ultimate axial compression strength, which is used for setting stone's strength grade, is the main strength characteristic for such materials.

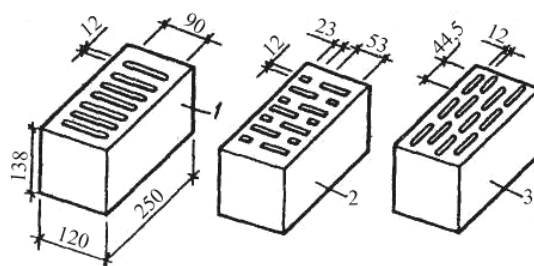


Fig. 20.2. Ceramic stones:  
with transversal hollows (1, 2) and longitudinal (3) hollows

Brick and stones for masonry and reinforced masonry constructions can be produced of the following grades according to ultimate axial compression strength: stones with low durability (light concrete and natural stones): 4, 7, 10, 15, 25, 35, 50; stones with middle durability (brick, ceramic, concrete and natural stones): 75, 100, 125, 150, 200; stone with high durability (brick, natural and concrete stones): 250, 300, 400, 500, 600, 800, 1000.

For building mortars the next grades are established according to the ultimate axial compression strength — 4, 10, 25, 50, 75, 100, 150, and 200. Mortars with density of  $1500 \text{ kg/m}^3$  and more in dry state are called heavy and the mortars with density up to  $1500 \text{ kg/m}^3$  — light ones.

Stone grade by the frost-resistance is established by means of standard samples tested on alternate freezing and thawing. Maximum amount of cycles, at which standards reduce their axial compression strength not more than by 20...25 % and do not have destruction features, is accepted as a stone grade. For stone materials frost-resistance there are set the grades from F10 to F300 according to the norms.

For masonry constructions reinforcing the next classes of armature are used: for the reticulated reinforcing — A-I and Bp-1; for longitudinal and transversal armature, anchors, connections — A-I, A-II and Bp-1.

**Types of the plain masonries.** Masonry is a qualitatively new building material, because of its difference from initial materials by its physical and mechanical properties and at the same time masonry property depends on the properties and sizes of its components. The basic requirement, which masonry must satisfy, is monolithic nature, provided due to the stones cohesion with mortars and stones bond in

horizontal rows. Different types of walls masonry are used: manual masonry from manufactured masonry materials, which include continuous masonry and different types of light-weight masonry; masonry from precast stone wares (ceramic blocks and large-sized blocks from natural stone). Continuous masonry is performed from all types of ceramic stones. In order to provide monolithic nature and durability of the continuous masonry the bond of vertical and horizontal joints is followed. Single row (chain) and multi-row systems of masonry bonding are widely used (Fig. 20.3, *a, b*).

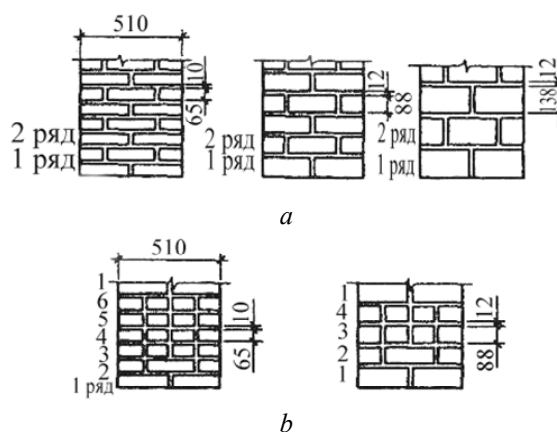


Fig. 20.3. Scheme of masonry bond for walls with thickness in two bricks:  
*a* — single row (chain); *b* — multi-row

Average thickness of the masonry vertical joints is 10 mm, horizontal joints with thickness 10... 12 mm, but no more than 15 mm.

Continuous masonry from bricks is used in the bearing structures of the ground floors in multistory buildings. Continuous masonry of hollow brick, ceramic and light concrete stones is used for filler constructions of dry apartments as well as apartments with normal humidity. It can be also used in the external walls of buildings with the wet mode but with protection of internal wall surface by the layer of steam insulation. Continuous stones of heavy concrete are used for the continuous masonry in socles, basement walls and in unheated building walls. Stones from cellular concrete are used in the filler constructions of buildings.

In the light-weight masonry the part of basic masonry material is replaced by heat insulation materials with less durability. Constructions,

which are performed from the light-weight masonry, are called multi-layered ones.

In building practice the next types of light-weight masonry are used: masonry with heat insulation slabs, located at walls surface (Fig. 20.4, *a*);

– masonry with heat insulation slabs inside the wall and flexible connections between the external brick walls (Fig. 20.4, *b*);

– brick pit masonry with vertical transversal walls and slab by heat insulation material inside the wall (Fig. 20.4, *c*);

– brick pit masonry with vertical transversal walls and the aggregate of light concrete or mineral backfilling (Fig. 20.4, *d*);

– brick and concrete masonry with horizontal flat arches in course of headers (Fig. 20.4, *e*);

– masonry with a broadened joint, filled with heat-insulation material (Fig. 20.4, *f*).

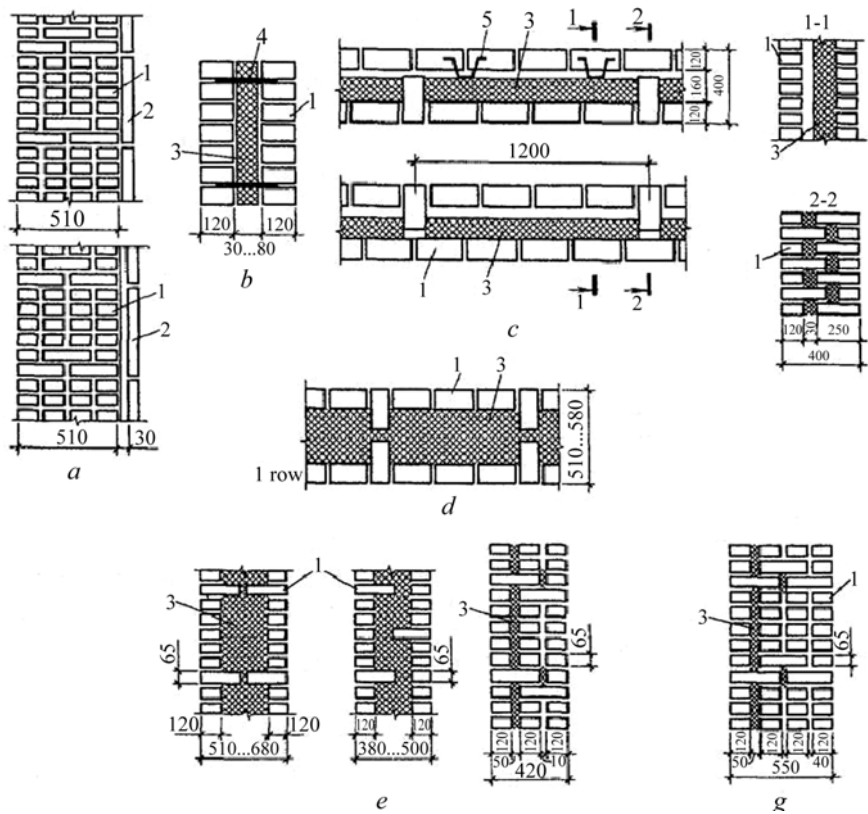


Fig. 20.4. Types of masonry:  
*1* — brick; *2* — facing tiles; *3* — heat-insulation material; *4, 5* — anchors



Joints bond in masonry with external heat-insulation is performed by the ordinary methods. Heat-insulation slabs are installed at mortar close to masonry or with an air layer. Monolithic nature of masonry with internal heat insulation is provided with the help of steel brackets, the section area of which is not less than  $0.4 \text{ cm}^2$  per  $1 \text{ m}^2$  of wall. Brackets are located in the places of heat-insulation slabs joints with minimum space of 600 mm. In the brick pit masonry monolithic nature is provided by means of vertical flat arches. Vertical flat arches are performed with thickness equal to 0.5 brick thick, at distance along the wall length not more than 1,2 m and not more  $10/h$ , where  $h$  is a thickness of the thinner layer. Horizontal flat arches are formed by paired course of headers, laid on the masonry height at distance not more than 62 cm and not more than  $5h$ . Such connections are called the rigid ones; they provide combined work of separate masonry layers.

In case of wells filling with mineral backfilling, the reinforced diaphragms are arranged in each 0,5 m along the wall height. In the brick and concrete masonry the monolithic nature of wall is reached due to laying the header courses, penetrating into a concrete to 0,5 brick thick (12 cm) after each 5 rows of masonry along the height. The monolithic nature of masonry with a broadened joint is provided in the same way.

The light-weight masonry is used for filler constructions of apartments with dry and normal humidity modes. The light-weight masonry isn't used for apartments with wet mode. The laying with backfilling is used for the external walls of buildings with two floors maximum. All other types of the light-weight masonry are recommended for bearing walls (up to 5 floors), for self-bearing walls (up to 9 floors), for non-bearing (curtain) walls (number of storeys isn't limited).

Manual masonry of bricks and stones provides durability of masonry constructions, but it requires the high labor intensiveness, rather high cost and relatively large terms of constructions erection. That's why, presently, the preference is given to the wall constructions, erected from precast stones, i.e. concrete blocks and panels, blocks of brick and ceramic stone, large-sized blocks of natural stone, vibrated brickwork and ceramic panels, in the mass construction.

Masonry of walls from large-sized concrete blocks, from blocks of natural stone, from ceramic bricks blocks or stones is performed with observance of bonding rules of vertical joints, which provide walls interconnection and combined work of blocks. Large-sized concrete blocks are used in the constructions of strip foundations, basement walls, socles. They are performed from the cement concrete B10 and higher, silicate concrete B15 and higher; the frost-resistance grade of large-sized concrete blocks is accepted not less than F25.

For the external walls of residential buildings with height up to 5 floors the concrete blocks of class B5 and higher are used. Thus, the grade of mortar for horizontal joints should be not less than 25, for vertical joints — 100. External walls from large-sized blocks are performed with joints bonding according to the followings cutting systems:

- double-course cutting of wall field at usage of three blocks types, i.e. pier, windowsill and flat arch (Fig. 20.5, *a*). It is used for external bearing walls;

- three- and four course cutting of pier block along height of two or three blocks (Fig. 20.5, *b*). It is used for bearing and self-bearing sectional walls made of silicate concrete;

- two-sectional (stripped) cutting at division on the piers and half-length blocks (Fig. 20.5, *c*). It is used for self-bearing and curtain external walls.

Half-length and flat arch blocks are fastened together by means of steel connections, welded to the blocks imbedded items. They form continuous floor belts, providing monolithic nature of walls masonry and combined work of blocks. Vertical and horizontal joints are filled with mortar.

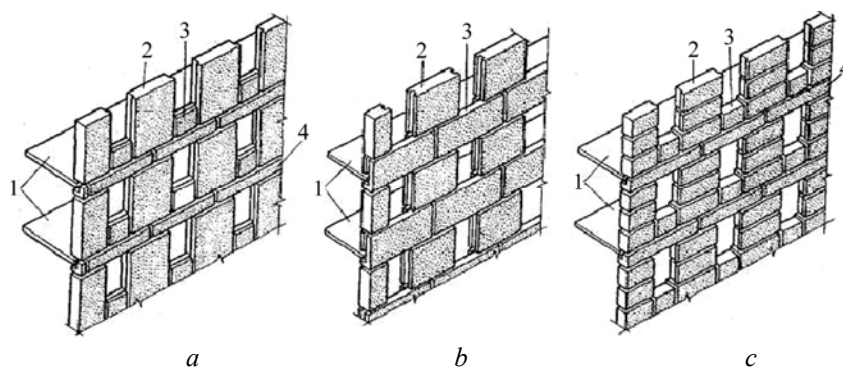


Fig. 20.5. Walls of large-sized blocks:

*a, b* — double course uncoupling; *c* — four course uncoupling;

1 — overlapping panels; 2 — pier block;

3 — windowsill block; 4 — flat arch block

Large-sized brick blocks are used for external and internal walls, basement walls, socles cornices. They are performed from continuous or light-weight masonry. The cutting system of the external walls made of the brick blocks is similar to the cutting for concrete blocks. It is established depending on the floor's height, blocks' mass and load-carrying capacity of assembling and transport means.

The brick blocks are prefabricated in the factories workshops or on grounds with usage of all types of ceramic brick or stones. Concretes with density up to  $1500 \text{ kg/m}^3$  and ultimate axial compression strength not lower than 1,5 MPa are used for hollows filling in blocks of the light-weight masonry. If mortar in the brick blocks is compacted by means of vibration the axial compression durability of masonry increases in 2...2,5 times in comparison with manual masonry. This fact allows increase brick blocks sizes to the size of panel and at the same time to decrease their thickness to 1,5...1,0 brick instead of 1,5...2,5 in case of manual masonry.

Monolithic nature of masonry made of panels is achieved by means of mutual interconnection of external and internal walls, connection of external walls with overlapping elements with the help of steel connections, welded to the embedded items in panels as well as filling of vertical and horizontal joints by mortar.

Panels made of ceramic brick and stones are subdivided according to:

- construction — single layer and multi-layer;
- purpose — panels of external and internal walls, partitions;
- type of loadings — bearing, self-bearing, nonbearing (curtain).

Ceramic and silicate brick, ceramic stones of grades 75, 100, 125, 150, 200 and 300, building mortar of grades 75, 100, 150, 200 are used for producing the panels. Rigid slabs from glass and mineral wool with synthetic and bituminous copula, blocks and slabs from porous plastics, slabs from fibrolite and cellular concretes are used as heat-insulation material. Single-layer panels from dense brick are used in the masonry of external walls of unheated apartments, internal bearing walls of multistory buildings and partitions of industrial, residential and public buildings (Fig. 20.6, *a*). Single-layer panels from hollow brick, perforated and multi-hollow ceramic stones are used for masonry of external walls.

Multi-layer panels (two- and three-layer ones) are used for curtain and self-bearing external walls of buildings mainly (Fig. 20.6, *b*). In the two-layer wall panels the bearing layer in 0,5 brick can be external or internal. Accordingly, rigid slabs of heat insulation can be arranged at the external or internal side of a panel.

Monolithic nature of panel is achieved due to the mortar layers, i.e. internal and finishing ones. Finishing layer must have a thickness not less than 40 mm and should be reinforced using the wire with  $d = 3 \text{ mm}$  and cells dimensions not more than  $150 \times 150 \text{ mm}$ . In the three-layer panels the heat-insulation material with thickness  $\frac{1}{4}$  or  $\frac{1}{2}$  brick each is located between two rows of masonry. Monolithic nature of the panel is provided with the help of mortar layers and reinforcement by welded

frameworks, which are placed in the mortar layers along the external panel's perimeter and perimeter of windows or doors openings.

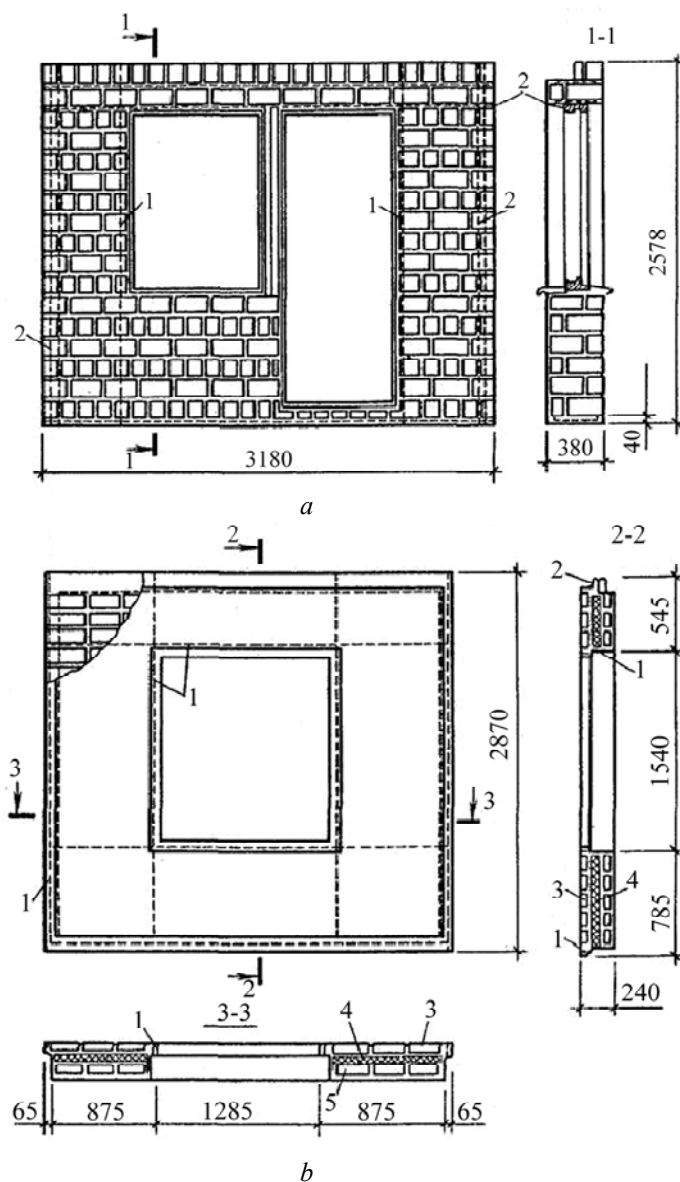


Fig. 20.6. Construction of vibrated masonry panels of external walls: *a* — single-layer panel in one and a half bricks thick; *b* — three-layer panel; 1, 2 — armature; 3, 5 — bricks; 4 — heat-insulation material

Concrete panels of external walls are also produced as single-, two- and three-layer ones. Single-layer panels are performed of light concrete. Their internal surface is covered by lime-cement mortar and external surface — by finishing's materials. The internal layer in two- and three-layer panels is executed of dense concrete, and external one — of light concrete (light-weight aggregate, shungezite, perlite) or cellular concrete.

At small step of transversal walls (up to 3m) panels measuring on a room are used (Fig. 20.7, *a*), and at big step of transversal walls panels on two rooms are used (Fig. 20.7, *b*).

Curtain walls are performed from strip hinged panels, located horizontally and alternated with the pier panels (Fig. 20.7, *c*). Curtain walls with vertical division are performed from vertical pier panels with height till two floors and window insertions (Fig. 20.7, *d*).

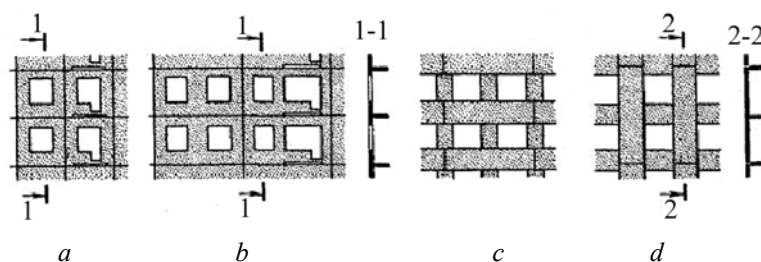


Fig. 20.7. Schemes of building facade cutting on panels:  
*a* — on a room with window; *b* — on two rooms with windows;  
*c* — strip hinge panel; *d* — pier panels on two floors with window insertions

#### **Strength and deformation characteristics of the plain masonry.**

Durability and deformability of masonry depend on many factors, namely on durability and deformability of stone and mortar; size and shape of stone; mobility of mortar and degree of vertical joints filling by mortar; masonry qualities (providing of uniform thickness and density of horizontal joints); cohesion of mortar with stone, etc (Fig. 20.8).

Masonry materials are the brittle ones and nonlinear deformations on the diagram « $\sigma - \varepsilon$ » (Fig. 20.9) appear only at very high levels of tensions (in the area of ultimate strength values). Building mortars are elastic-plastic materials in the hardening state that gives the nonlinear dependence of tension deformations during testing on compression. Masonry, the carrying capacity of which is provided due to combined work of these materials, is nonlinear deformed material.

Transversal deformations of building mortars in horizontal joints considerably exceed transversal deformations of masonry materials during

perception of compressive effects by masonry. Therefore masonry is destructed from tensile efforts in a stone, arising up under the influence of transversal mortar deformations.

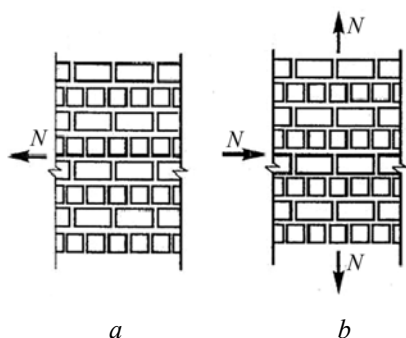


Fig. 20.8. Destruction of masonry in case of tension along the bonded (a) and unbonded (b) sections

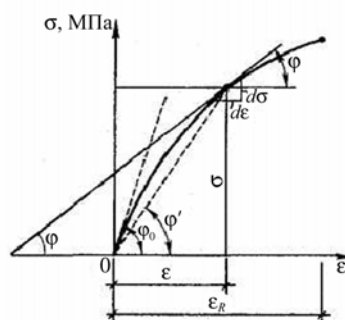


Fig. 20.9. General graph of masonry deformations in case of compression

Increasing of joint thickness leads to decrease of masonry strength. Destruction of masonry at compression begins from the opening of vertical joints and appearance of local vertical cracks in separate stones. At further increase of loading the shallow vertical cracks are combined along the height and masonry is dismembered to separate posts. Then, the non-considerable increase of loading leads to loss of these posts stability and the masonry is destructed.

Strength and deformation characteristics of masonry are obtained by means of statistical processing test results for multiple prismatic standards, the sizes of base of which are  $38 \times 3$  cm or 51 cm, height 110...120 cm.

*Basic strength characteristics of masonry include:*

- temporal compression resistance  $R_u$  (average ultimate strength);
- designed axial compression resistance  $R$ ;
- designed axial tension resistance  $R_{bt}$ ;
- designed tension resistance at bending  $R_{tb}$  ( $R_{tv}$ );
- designed shear resistance  $R_{sq}$ .

*Basic deformation characteristics of masonry include:*

- elasticity module of masonry (initial module of deformations)  $E_0$ ;
- elastic response of masonry,  $a$ ;
- deformation module of masonry  $E$ ;
- creep coefficient of masonry  $y_{cr}$ ;
- linear expansion coefficient  $a_i$ ;
- friction coefficient,  $\mu$ .

The value of temporal compression resistance  $R_u$  is obtained directly from the results of masonry standards testing. Designed masonry resistance to axial compression is found as follows:

$$R = R_u / k, \quad (20.1)$$

where,  $k$  — is a coefficient, which equals 2,0 for masonry of brick and all kinds of stones, large-sized blocks, lacerated rubble and rubble concrete and 2,25 for masonry of large-sized and small-sized blocks of cellular concretes.

Coefficient of work conditions  $\gamma_c$  is taken into account during setting of designed masonry resistance, i.e.  $\gamma_c$  for the summer masonry and  $\gamma_{cl}$  for the winter masonry, performed by means of freezing method.

*Values of the coefficient  $\gamma_c$  in accordance with the type of construction and masonry material, condition of masonry hardening*

Posts and piers $A < 0,3 \text{ m}^2$ .....	0,8
Elements with circular section, prefabricated from the non-radial brick .....	0,6
The same, in the case of loading application after the period of masonry hardening more than 1 year .....	1,15
Masonry from a silicate brick on the mortars with admixture of potash .....	0,85
Masonry of large blocks and stones, prefabricated of heavy concretes or natural stones ( $p > 1800 \text{ kg/m}^3$ ) .....	1,1
Masonry of blocks and stones of silicate concretes of class B25 and higher .....	0,9
Masonry of blocks and stones of macro porous or cellular concretes of A type .....	0,8
Masonry of blocks and stones of cellular concretes of B type .....	0,7.

For light-weight types of masonry the design compression resistances are accepted for separate masonry layers laying in accordance with materials, used in these layers.

Designed axial tension resistances  $R_t$ , tension at bending  $R_{tb}$  ( $R_{tw}$ ), shear  $R_{sq}$  depend on the type of section, through which the masonry destruction is observed. Two cases of masonry destruction are possible, namely:

– along the unbonded sections being the horizontal masonry joints (Fig. 20.8, *b*);

– along the unbonded sections being the vertical masonry joints; in these cases the section has a stepped form (Fig. 20.8, *a*).

Values of the design resistances  $R_t$ ,  $R_{tb}$  ( $R_{tw}$ ),  $R_{sq}$  for unbonded sections, passing along the horizontal layers, and for the unbonded

sections, passing along the vertical masonry layers, are presented in СНиП-П-22-81, table 10.

For the continuous masonry of brick or stones of regular shape the work on tension and shear is allowable only along the unbonded section. In that case, when destruction of masonry, working on tension, bending or shear, takes place along the bonded sections, which pass along brick or stone, the values of the design resistances  $R_t$ ,  $R_{tb}$  ( $R_{tv}$ ),  $R_{sq}$  are accepted depending on the brick or stone grade.

Basic deformation characteristics of masonry are determined according to the diagram « $\sigma - \varepsilon$ », which are obtained during masonry standards testing on short-term axial compression. The « $\sigma - \varepsilon$ » dependence for masonry is nonlinear one; its generalized graph is shown in the Fig. 20.9.

In case of short-term loading the elasticity module of masonry  $E_0$  is accepted to be equal to the slope of the tangent line to the curve « $\sigma - \varepsilon$ » passing through the beginning of coordinates, i.e. it is also proportional to the temporal axial compression resistance:

$$E_0 = \alpha R_u, \quad (20.2)$$

where  $\alpha$  is the elastic masonry characteristic for the main types of masonry;  $R_u$  is the temporal axial compression resistance (average ultimate strength) of the masonry.

Fig. 20.8. Destruction of masonry in Fig. 20.9. General graph case of tension along the bonded (a) of masonry deformations and unbonded (b) sections in case of compression.

During calculation of masonry constructions as the subject to action of permanent and long-term loading and taking into account the creep the elasticity module of masonry is decreased by means of its division by the creep coefficient  $\nu$ , which is accepted: 1,2 — for masonry of the plastic and haft dry pressing ceramic brick; 1,8 — for masonry of ceramic stones with vertical cinked hollows (height of stone is 138 mm); 2,8 — for masonry and large-sized blocks or stones, prefabricated of heavy concrete; 3,0 — for masonry of silicate brick, continuous and hollow stone, and also of stones, prefabricated of concrete with porous aggregate (porous or silicate).

Deformation module of masonry  $E$  is accepted to be equal to the slope of the tangent line to the curve « $\sigma - \varepsilon$ » in the point with given tensions level  $E = \text{tg } \varphi$  (Fig. 20.9).

Deformation module is used during calculations according to the I and II groups of masonry constructions limited states, which worked in



constructions together with constructions elements from other materials (concrete, metal), here

$$E = 0,5E_0. \quad (20.3)$$

During determination of masonry deformations in the statically indeterminate frame systems rigidity of masonry constructions or period of their vibrations is  $E = 0,8 E_0$ .

Elasticity and deformation modules of masonry from natural stones are accepted according to the results of experimental research. Relative deformation of masonry taking into account the creep can be found as follows:

$$\varepsilon = \nu\sigma / E_0 \quad (20.4)$$

where,  $\nu$  is coefficient, taking into account the influence of masonry creep;  $\sigma$  is the stress in masonry at long-term loading.

**Strength and deformation characteristics of the reinforced masonry.** For the reinforced masonry the followings materials are used:

- a) brick of all types — not lower than the 75 grade; ceramic, natural and manufactured stones — not lower than the 35 grade;
- b) building mortars — not lower than the 50 grade;
- c) for the mesh reinforcing — armature of the classes A-I, A-II and Bp-I;
- d) for longitudinal and transversal armature, anchors and tiers — armature of the classes A-I, A-II and Bp-I.

The height of masonry row must not exceed 150 mm for the mesh reinforcing.

Normative and design resistances of armature  $R_{sn}$  and  $R_s$  are multiplied by the coefficients of work conditions  $\gamma_{sc}$  depending on the type of reinforcing.

Main strength characteristics of the reinforced masonry include: temporal compression resistance (average ultimate strength) of the reinforced masonry  $R_{sku}$ ; design axial compression resistance of the reinforced masonry  $R_{sk}$ ; design eccentrically compression resistance of the reinforced masonry  $R_{skb}$ .

*Temporal compression resistance of the masonry with mesh reinforcing*

$$R_{sku} = kR + 2R_{sn}\mu / 100, \quad (20.5)$$

with longitudinal reinforcing:

$$R_{sku} = kR + R_{sn}\mu / 100 \quad (20.6)$$

where,  $\mu$  is the percent of masonry reinforcing.

The percent of mesh masonry reinforcing is determined from the ratio of volume of mesh armature  $V_s$  to the correspondent volume of masonry

$$\mu = (V_s / V_k)100 = (2A_{st} / cs)100,$$

$V_k$ , where  $V_s = 2A_{st}$  is a doubled sectional area of mesh bars with square cells and the same armature diameter in both directions;  $V_k = cs$  is a product of cells dimension  $c$  by the distance between meshes along the masonry height  $s$ .

The percent of masonry with longitudinal armature reinforcing is determined from the ratio of armature sectional areas  $A_s$  and masonry one  $A_k$  in a section:

$$\mu = (A_s / A_k)100.$$

The percent of masonry reinforcing with mesh reinforcement in case of eccentrically compression is accepted within the limits.

The percent of transversal masonry reinforcing in the case of central compression is accepted within the limits:

$$0,1\% < \mu \leq \frac{50R}{(1 - 2e_0 / y)R_s},$$

$$0,1\% < \mu \leq 50R / R_s.$$

Design resistance of the masonry with mesh reinforcing in case of central compression:

$$R_{sc} = R + \frac{2\mu R_s}{100} \leq 2R. \quad (20.7)$$

Design resistance of the reinforced masonry in the case of eccentrically compression and mortar of grade 50 and higher

$$R_{skb} = R + \frac{2\mu\mu_s}{100} \left( 1 - \frac{2e_0}{y} \right) \leq 2R. \quad (20.8)$$

Design characteristics of the masonry with mesh reinforcing  $R_{sk}$  and  $R_{skb}$  must not exceed doubled design resistance of the correspondent plain masonry ( $2R$ ) according to condition of transversal reinforcing efficiency.

Basic deformation characteristics of the reinforced masonry are elasticity module (initial module of deformation  $E_0$ ) and elastic characteristic  $\alpha_{sk}$ . The elasticity module of the masonry with mesh reinforcing is accepted like for the plain masonry.

$$E_0 \alpha_{sk} R_u; \quad (20.9)$$

Elasticity module for the masonry with longitudinal reinforcing:

$$E_0 = \alpha R_{sku}, \quad (20.10)$$

where,  $a_{sk}$  is elasticity characteristic of the masonry with mesh reinforcing

$$\alpha_{sk} = \alpha R_u / R_{sku}, \quad (20.11)$$

where,  $\alpha$  is elasticity characteristic of the plain masonry;  $R_u$ ,  $R_{sku}$  is the temporal compression resistances of the plain and reinforced masonry correspondently.

**Application.** It is necessary to take into account the special application requirements of certain types of masonry materials and wares for concrete conditions during designing masonry and reinforced masonry constructions of new, extended and reconstructed buildings and constructions. For masonry of the external walls of apartments with dry and normal humidity modes it is recommended to use the following manufactured masonry materials: brick, hollow ceramic and concrete stones, continuous stone and blocks of concretes with porous aggregates, porous and cellular concretes. Brick and stone of the 150 grades and more are used in buildings with more than five floors height.

It is recommended to use panels and large blocks as a masonry wares for external walls. Local natural masonry materials are used for external walls. For the apartments with moist mode it is admitted to use silicate brick, stones and blocks of cellular concretes, hollow brick and ceramic stones, ceramic brick of the haft-dry pressing under the conditions that vapor insulation would be spread on the internal walls surfaces. It is not admitted to use mentioned materials for the walls of apartments with wet mode, and also for the external walls of basements and socles.

It is necessary to take into account the humidity mode of apartments, type of constructions, and also their life time during appointment of the frost-resistance grade of masonry materials. For buildings, erected in the middle climatic conditions, the frost-resistance grades of masonry materials are established depending on the reliability degree of constructions, which is determined according to their design life time. Three degrees of constructions reliability are established: I — with life duration not less than 100 years; II — not less than 50 years; III — not less than 20 years. Design frost-resistance grades of masonry materials for external walls part (at thickness 12 cm) and foundations (at entire thickness) are presented in table 20.1.

The frost-resistance requirements are not established for regions with design winter temperature higher than 10°C. The design frost-resistance grades are raised on one stage for regions with unfavorable climatic conditions, and it is permitted to reduce them on one stage but not lower then F10, for regions with favorable climatic conditions.

Design grades of masonry materials

Type of constructions	Value F for the supposed life duration of constructions, years		
	100	50	30
External walls or facing in buildings with apartments mode like:			
– dry and normal ones	25	15	15
– moist	35	25	15
– wet	50	35	25
Foundations and underground parts of walls:			
– from brick of the plastic pressing	35	25	–
– from natural stone	25	15	15

## 20.2. Stone elements of constructions

**Axial compression.** Internal bearings posts of multistory stone buildings can be considered as the example of the centrally-compressed elements of masonry constructions. Posts perceive loadings by roof, attic and interstorey overlapping. If adjoining to the posts girders are equal span and value of temporal loading on overlapping is small in comparison with coverage self-weight, then one-sided load of post by temporal loading is not dangerous. In this case posts are calculated for the central compression.

During estimation of sections strength the stress diagram in the centrally-compressed element of masonry is accepted as rectangular one with an ordinate equal to the design axial compression resistance  $R$  of masonry.

The possibility of compressed elements destruction till the durability exhausting due to the influence of longitudinal bending and increasing of deformations because of material creep in case of long-term loading is taken into account by means of the coefficients of  $\varphi$  and  $m_g$ .

Bearing capacity of masonry constructions elements in case of central compression is considered to be provided if the following condition is fulfilled:

$$N \leq m_g \varphi R A, \quad (20.13)$$

$$m_g = 1 - \mu N_g / N, \quad (20.14)$$

where,  $N$  is a design longitudinal force;  $m_g$  is a coefficient, which takes into account influence of long-term loading;  $\varphi$  is a buckling

coefficient (tabl. 20.2); R is a design compression resistance of masonry; A is a cross-section area of element;  $\eta$  is a coefficient taken in accordance with table 20.2;  $N_g$  is a design longitudinal force at long-term loadings.

*Note.* The coefficient  $\varphi$  is determined by means of interpolation at the intermediate values of flexibility.

The coefficient  $m_g$  is accepted to be equal to 1 for rectangular sections with the less dimension  $h > 30$  cm and for the sections of any form with the less inertia radius  $i > 8,7$  cm.

The values of coefficients  $\eta$  and  $\varphi$  depend on the masonry material and flexibility of the compressed elements:

– for the arbitrary shape sections

$$\lambda_i = l_0 / i; \quad (20.15)$$

– for the rectangular continuous section

$$\lambda_h = l_0 / h, \quad (20.16)$$

where,  $l_0$  is a design height (length) of element; h is the less dimension of rectangular section; i is the minimum inertia radius of section.

Table 20.2

Values of  $\eta$  and  $\varphi$  coefficients

Flexibility		Coefficient $\eta$ for plain masonry made of ceramic brick and stones	Coefficient $\varphi$ in case of elastic masonry characteristics			
$\lambda_h$	$\lambda_i$		$\alpha = 1500$	a = 1000	a = 750	a = 500
6	21	0,00	0,98	0,96	0,95	0,91
8	28	0,00	0,95	0,92	0,90	0,85
10	35	0,00	0,92	0,88	0,84	0,79
12	42	0,04	0,88	0,84	0,79	0,72
14	49	0,08	0,85	0,79	0,73	0,66
16	56	0,12	0,81	0,74	0,68	0,59
18	63	0,15	0,77	0,70	0,63	0,53
20	70	0,20	–	–	–	–
22	76	0,24	0,69	0,61	0,53	0,43
24	83	0,27	–	–	–	–
26	90	0,31	0,61	0,52	0,45	0,36
30	104	–	0,53	0,45	0,39	0,32

Design height of compressed walls and posts  $l_0$  depends on condition of their resting on the horizontal supports (overlapping). In case of immobile hinge supports it is accepted that  $l_0 = H$  (Fig. 20.10, *a*) ( $H$  is a distance between overlapping and other horizontal supports).

In case of elastic upper support and rigid fixing in lower support the  $l_0$  equals  $1,5H$  for single-span buildings,  $l_0$  equals  $12,5H$  for multi-span buildings (Fig. 20.10, *b*);  $l_0$  is accepted in accordance with the actual degree of fixing but no less than  $0,8H$  for constructions with partly fixed supporting sections. For the self-supporting constructions the  $l_0$  equals  $2H$  (Fig. 20.10, *c*). Values of the coefficients  $m_g$  and  $\varphi$  along the height of the compressed walls and posts are accepted according to the Fig. 20.10, *a, b, c*.

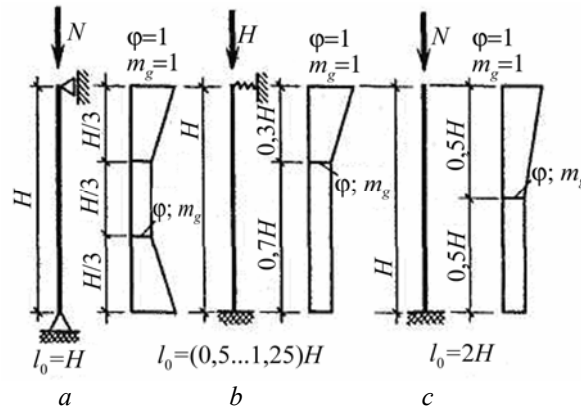


Fig. 20.10. Values  $m_g$  and  $\varphi$  along length of the compressed walls and posts:  
*a* — hinged supports; *b* — fixing and hinged support; *c* — cantilever

**Eccentric compression.** Eccentric compression is the most widespread case of the stress state in the masonry construction elements. The eccentric compression is observed due to action of the bearing walls and piers of masonry buildings, internal bearing posts in case of unequal spans of overlapping girders or in case of one-sided loading of floors by temporal loading, basements walls.

Under the eccentric compression the stress state of masonry is characterized by the presence of tension and compression stresses in a section. Elements strength is determined by bearing capacity of the section compressed area, within the limits of which stress diagram is accepted with rectangular shape with ordinate equal to the design compression resistance  $R$  of masonry (Fig. 20.11).

Tension stresses, arising up in eccentrically compressed masonry elements, can lead to the formation and opening of cracks in horizontal joints, which increase masonry deformation and decrease its bearing capacity. The value of tension stresses and cracks opening in tension area of masonry depends on the size of longitudinal force eccentricity with respect to the section center of gravity  $e^0$ . Therefore the restriction of eccentricity size is introduced into norms.

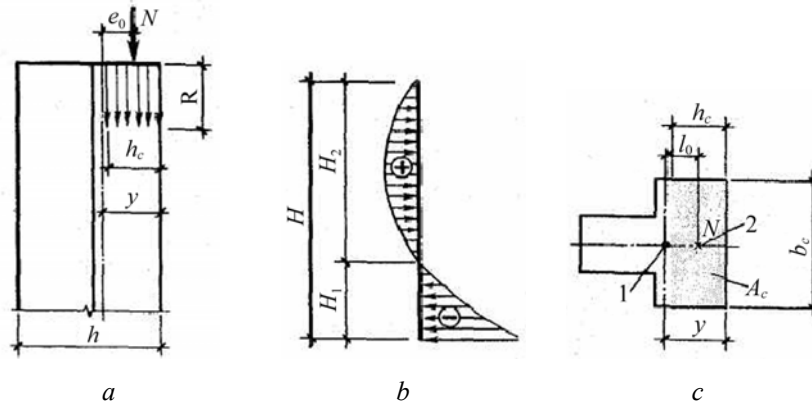


Fig. 20.11. To the calculation of pier:

*a* — design scheme; *b* — diagram of moments; *c* — design section;  
*1* — section center of gravity; *2* — center of the section compressed area

Bearing capacity of masonry constructions elements in case of eccentric compression is considered to be provided if the following condition is fulfilled:

$$N \leq m_g \phi_1 R A_c \omega, \quad (20.17)$$

where,  $N$  is design longitudinal force;  $R$  is design compression resistance of masonry;  $A_c$  is the cross section area of the compressed part of element at the rectangular stress diagram; it is determined from the condition, that its centre of gravity coincides with the application point of  $N$ ;  $\omega$  is a coefficient, which takes into account possibility of the compressed masonry area design resistance increasing due to the influence of the less stressed section part.

The coefficient  $\omega$  is accepted to be equal to 1 for the sections of rectangular and arbitrary shapes and masonry from stones and large-sized blocks, prefabricated of cellular and macro porous concretes, from natural stones (including rubble).

For the other types of masonry and sections of rectangular shape:

$$\omega = 1 + e_0 / h \leq 1,45, \quad (20.18)$$

where,  $e_0 = M/N$  is eccentricity of design force  $N$  with respect to the section gravity center;  $h$  is the section height in the plane of bending moment action.

Coefficient  $m_g$  has the same physical sense that is used during the calculation of centrally-compressed elements

$$m_g = 1 - \eta \frac{N_g}{N} \left( 1 + \frac{1,2e_{0g}}{h} \right) \quad (20.19)$$

where,  $\eta$  is the coefficient accepted according to the table 20.2;  $N_g$  is the design longitudinal force due to long-term loadings;  $e_{0g}$  is the eccentricity of the longitudinal force  $N_g$  with respect to the section center of gravity.

It is accepted that  $m_T = 1$  if  $h > 30$  cm or  $I > 8,7$  cm. Buckling coefficient of eccentrically compressed masonry element equals

$$\varphi_1 = (\varphi + \varphi_c) / 2, \quad (20.20)$$

where,  $\varphi$  is the buckling coefficient for entire plane section of bending moment action;  $\varphi_c$  is the buckling coefficient for the compressed section part under the values of element's flexibility  $\lambda_{hc}$  or  $\lambda_{ic}$ ;

$\lambda_{hc} = H/hc$  — for the rectangular section ,

$\lambda_{ic} = H/hc$  — for the section of arbitrary shape.

His the design height of element;  $h^c$ ,  $i^c$  am the height and inertia radius, respectively, of the compressed part of cross-section  $A_c$  in the plane of bending moment action.

For the rectangular section

$$h_c = h - 2e_0. \quad (20.21)$$

For the T-shaped section (at  $e_0 > 45y$ ) it is admitted to accept

$$h_c = 2(y - e_0) \quad (20.22)$$

where,  $y$  is the distance from the center of gravity of element's section to its edge to side of eccentricity.

In case of alternating bending moment diagram along the element's height (see Fig. 20.11) the buckling coefficient  $\varphi_c$  is determined along the height of element's part within the limits of single-valued bending moment diagram at the following flexibilities:

$$\lambda_{h1c} = H_1 / h_{c1} \quad \text{and} \quad \lambda_{i1c} = H_1 / i_{c1},$$



$$\lambda_{h2c} = H_2 / h_{c2} \text{ and } \lambda_{i1c} = H_2 / i_{c2},$$

where,  $H_1, H_2$  is the part of the element's height with single-valued bending moment diagram;  $h_{c1}, i_{c1}, h_{c2}, i_{c2}$  are the heights and radiuses of inertia of the compressed elements' part in sections with maximal bending moments.

Area of the compressed part  $A_c$  for the rectangular section is determined by the formula:

$$A_c = A(1 - 2e_0 / h), \quad (20.23)$$

where,  $A$  is the section area of element.

For T-shaped section (when  $e_0 > 0, 45y$ ) it is allowable to accept approximately

$$A_c = 2(y - e_0)b, \quad (20.24)$$

where,  $b$  is the width of the compressed flange or wall thickness of the T-shaped section in accordance with the eccentricity position with respect to the section gravity center.

Casual eccentricities  $e_a$  are taken into account during calculation of the elements with thickness of 25 cm and less, namely for bearings walls  $e_a = 2$  cm; for self bearings walls and separate layers of the three-layered bearings walls  $e_a = 1$  cm.

Casual eccentricity is summarized with the eccentricity of longitudinal force:

$$e_0 = e_a + M / N.$$

In these elements the total eccentricity must not exceed:

- for basic loadings combinations –  $0,8y$ ;
- for special loadings combinations –  $0,85y$ .

During calculation of the eccentrically compressed masonry elements with thickness more than 25 cm, the maximum value of eccentricity must not exceed:

- for basic loadings combinations –  $0,9y$ ;
- for special loadings combinations –  $0,95y$ .

If eccentricity  $e_0 > 0,7y$ , the eccentrically compressed masonry elements are checked on the cracks opening in masonry joints together with checking the bearing capacity according to the formula (20.17). If the cross-section height of eccentrically compressed masonry elements (piers, posts) is more than its width ( $b < h$ ), the bearing strength of such sections is checked by means of calculating the central compression in the plane which is perpendicular to the plane of bending moment action according to the formula (20.13).

**Oblique eccentric compression.** The oblique eccentric compression occurs if compressed force  $N$  is applied with eccentricity with respect to the element's section gravity center in both planes. Initial eccentricity in the plane of section height is designated as  $e_h$  and in the plane of section width —  $e_b$ .

Bearing capacity of section in case of oblique eccentric compression is checked according to the formula (20.17) twice: 1) the coefficients  $m_g$ ,  $\varphi$ ,  $\omega$  are determined at sections height  $h$  or radius of inertia  $i_i$ , eccentricity  $e_h$  in the direction  $h$ ; 2) these coefficients are calculated at sections width  $b$  or radius of inertia  $i_b$  and eccentricity  $e_b$  in the direction  $b$ .

The least from these two values is accepted as design bearing capacity. Area of the compressed section part  $A_c$  is conditionally accepted in the form of rectangle, in which the gravity center coincides with the point of the compressed longitudinal force  $N$  application (Fig. 20.12). In case of complicated section form it is assumed to accept area of its compressed part as the rectangular shape, without taking into account areas, which complicate its form.

It is permitted to determine  $A_c$  approximately according to the formula (20.25):

$$A_c = 4c_h c_b, \quad (20.25)$$

where,  $c_h = h_c/2$  is a distance of a force application point to the nearest section border in the plane  $h$ ;  $c_b = b_c/2$  the same one, but in the plane  $b$ .

If one of initial eccentricity exceeds the values  $e_b > 0,7c_b$  or  $e_h > 0,7c_h$ , then section is additionally checked on cracks opening in the correspondent direction.

**Crushing.** Parts of walls or piers, on which the girder ends, beams, trusses rest on; foundation blocks or beams, the loading from posts is transmitted to the last ones; pilasters of walls, which loading from crane girders is transmitted on are worked on the crushing (local compression). In case of the masonry work on crushing its design resistance  $R$  should be increased, as the pressure transmitted to masonry is perceived not only by the crushing area directly, but also by the nearby masonry areas, located within the limits of design section area.

If pressure to the masonry is transmitted through the mortar layer then the pressure diagrams in masonry become of triangular shape in the places of local loading transmission. If installation of distribution slabs is foreseen under the supporting elements areas, which transmit local loadings on masonry, then the pressure diagrams become of rectangular shape.

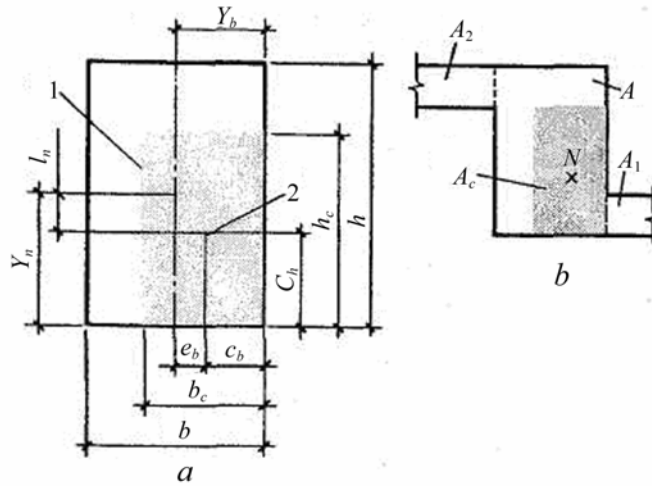


Fig. 20.12. Oblique eccentric compression:  
*a* — simple section form; *b* — complicated section form;  
 1 — section's center of gravity; 2 — center of the compressed section area

Calculation of masonry bearing capacity for crushing during loading distribution along the part of section area is performed according to the formula

$$N_{loc} \leq \psi d R_{loc} A_{loc}, \quad (20.26)$$

where,  $N_{loc}$  is the longitudinal compressive force of the local loading;  $\psi$  is the fullness coefficient of pressure diagram due to local loading;  $d$  is the coefficient, which depends on masonry material;  $R_{loc}$  is the design resistance of masonry to crushing, determined by the formula (20.27);  $A_{loc}$  is the crushing area being the subject to loading.

It is accepted that the coefficient  $\psi = 1$  in case of rectangular pressures diagram within the limits of crushing area, and it is accepted that the coefficient  $\psi = 0,5$  in case of triangular pressures diagram. If local loadings are transmitted to the brick or vibrated brick masonry, as well as to the masonry made of continuous stones or blocks, prefabricated of heavy and light concrete, then coefficient  $d = 1,5 - 1,5 \psi$ .

For masonry of hollow concrete stones or continuous stones and blocks of macro porous and cellular concretes the coefficient  $d = 1$ . If installation of the distributive slabs under the bending elements supports is not required, than it is permitted to accept product of coefficients  $\psi$  and  $d$  in accordance with table 20.3.

Design masonry resistance to crushing

$$R_{loc} = \xi R, \quad (20.27)$$

where,  $\xi = \sqrt[3]{A/AC} < \xi_1$  is the coefficient, which takes into account the work of the nearby masonry areas;  $\xi_1$  is the coefficient, which depends on masonry material and place of loading application (see table 20.3); A is the design section area, determined according to the different cases of local compression.

Table 20.3

Values of coefficients  $\xi_1$  and  $\psi d$

Material of masonry	$\xi_1$ for loadings according to the schemes				$\Psi d$ when distributing plates are absent
	local loading	sum of local and basic loading	local load	sum of local and basic loading	
– Solid brick, stones, large-sized blocks of heavy concrete or concrete on porous aggregates;	2,0	2,0	1,0	1,2	0,75
– Building stones with gap hollows, hollow brick, rubble concrete	1,5	2,0	1,0	1,2	0,75
– Hollow concrete stones and blocks, stones and blocks of cellular concrete and natural stone	1,2	1,5	1,0	1,0	0,5

In case of combined action of local (supporting reactions of beams, trusses, girders, etc.) and basic loadings (self-weight of upper situated masonry with loading, transmitted on it) the calculation for crushing is performed separately: at first only for the local loading, then — for sum of local and basic loadings.

**Bending and central tension.** External walls of multistory buildings from the action of wind loading, external walls of counter fort retaining wall, etc. work on bending. Work of masonry elements on bending along the unbonded section is not permitted according to norms. The calculation of masonry on bending is performed in supposition of its elastic work, i.e. the formulas of materials strength are used. Bearing capacity of masonry at

perception of normal stresses along the bonded joint and tangent stresses along the sloping section is checked during calculation of the plain masonry.

Design bending moment of the plain elements from external loadings

$$M \leq R_{tb}W, \quad (20.28)$$

where,  $R_{tb}$  is the design tension resistance of masonry at bending along bonded section;  $W$  is moment of masonry section resistance at elastic work.

Design transversal force

$$Q \leq R_{tw}bz, \quad (20.29)$$

where,  $R_{tw}$  is the design masonry resistance to the main tensile stresses in case of bending;  $b$  is the width of section;  $z$  is the arm of the internal pair of forces (for the rectangular section  $z = 2/3h$ ).

Masonry constructions, which work on central tension (walls of circular reservoirs, silos and other capacities), are calculated according to the masonry strength along the bonded section or stone:

$$N \leq R_t A_n, \quad (20.30)$$

where,  $N$  is the design axial force at tension from external loadings;  $R_t$  is the design axial tension resistance of masonry along the bonded section;  $A_n$  is the design net area of masonry section.

**Shear.** For example, the masonry elements, which perceive thrust from arches strings, work on shear along horizontal joint. Shear resistance of masonry along the horizontal joint consists of self-bearing capacity of masonry on shear and friction resistance of masonry along the horizontal joint.

Design shear effort in the plain masonry can be found as follows:

$$Q \leq (R + 0,8n\mu\sigma_0)A, \quad (20.31)$$

where,  $R_{sq}$  is the design shear resistance;  $n$  is the coefficient, which is equal to 1,0 – for masonry of continuous brick and stones and 0,5 – for masonry of hollow brick and stone;  $\mu$  is the friction coefficient along masonry joint, which is equal to 0,7 for the masonry of brick and stones of regular shape;  $\sigma_0$  is the average compression stress in the masonry at the minimum design longitudinal loading, determined with the overload coefficient 0,9;  $A$  is the design area of section.

If we have case of eccentric compression with eccentricity beyond the core of section (for the rectangular sections  $e_0 < 0,17h$ ) in a design section checked on a shear, then the design section area includes only the area of the compressed section part  $A_c$ .

**Calculation of the multi-layered walls.** Walls, which are prefabricated of the light-weight masonry or masonry with finishing, are called multi-layered ones. The calculation of bearing strength of the multi-layered wall elements has its own features in comparison with calculation of elements of continuous masonry. Methods of multi-layered wall elements calculation depend on the type of connections, which provide combined work of separate layers.

There are distinguished two types of connections between structural layers: rigid and elastic connections.

Rigid connections are performed in accordance with the followings requirements:

- distance between the axes of vertical diaphragms from the headers courses of bricks or stones is not more than  $10h$  and not more than 120 cm ( $h$  is a thickness of the least structural layer); heat-insulation layer can be any one;

- distance between the axes of headers horizontal bearing courses (horizontal diaphragms) along the masonry height is not more than  $5h$  and not more than 62 cm if heat-insulation layer is performed of monolithic concrete with ultimate compression strength not less than 0,7 MPa and masonry of stone not lower than 10 grade;

- facing layers have the rigid contact with the main walls masonry by means of headers courses, in this case it is necessary to foreseen bonding of facing layer. It is recommended to perform facing layer of bricks or stones, which have height equal to height of main row of masonry.

If rigid connection has been provided then all the layers participate in perception of loading. But bearing capacity of each layer is conditionally reduced to the bearing capacity of the main bearing layer by means calculation of reduced section area using the ratio of strength characteristics of separate layers. In this case, the layers' thickness is accepted in accordance with the actual one, and the layers' width is changed proportionally to the ratio of design resistances and usage coefficients of layers:

$$b_{red} = bm_i R_i / (mR), \quad (20.32)$$

where,  $b_{red}$  is reduced width of layer;  $b$  is an actual width of layer ;  $m$ ,  $m_i$ , are the coefficients of the strength usage of main bearing layer and other layers;  $R_s$ ,  $R_i$  are the design resistance of main bearing layer and any other wall layer.

The main bearing layer of multi-layered wall is a layer with the highest strength characteristics; more often it is a layer of brick masonry.

Heat-insulation layers are taken into account during calculation if they are performed of light and cellular concretes with ultimate compression strength not lower than 1,5 MPa. Heat-insulation layers of materials with strength lower than 1,5 MPa as well as of mineral or organic slabs and porous plastics are taken into account as loading only.

Calculation of bearing capacity of multi-layered walls with rigid connections is performed according to the formula (20.13) at the central compression and according to the formula (20.17) at the eccentric compression. In these formulas the section area  $A$  is replaced by the reduced area of the section of  $A_{red}$ ; the area of the compressed section part  $A_c$  is replaced by the area of compressed part of the reduced section  $A_{cred}$ ; design layer resistance  $R$ , which section is reduced to, is replaced by the  $mR$ . Buckling coefficients  $\varphi$  and  $\varphi_1$  as well as coefficient  $mg$  are determined for the material's layer, which section is reduced to.

During calculation of walls with a facing layer, tightly connected with the main masonry, coefficients of layers strength usage  $m$  and  $m_i$  are accepted according to the norms. Walls section is reduced to the material of wall main bearing layer.

During calculation of such walls on eccentric compression the eccentricity of loading  $e_0$  toward facing must not exceed the value  $e_0 < 0,25 y$  and coefficient  $\omega$  is accepted to be equal to 1,0 in this case. If eccentricity is directed toward the internal border of wall and its value  $e_0 > y(1 - m)/(1 + m)$ , but no less than  $0,1y$ , then calculation is carried out according to the formulas (20.13) and (20.17) without taking into consideration  $m$ ,  $m_i$ , as for single-layer section according to the material of main bearing layer of wall.

Connections between separate structural layers, performed from corrosive proof steels, or steels protected from corrosion, as well as from polymeric materials are considered as the flexible ones. Total section area of flexible steel connections must be not less than  $0,4 \text{ cm}^2$  per  $1 \text{ m}^2$  of wall surface.

Calculation of bearing capacity of multi-layered walls with flexible connections is performed for each layer separately on the loading, which it perceives. Internal layer of bearing wall is calculated on the self-weight, loading of coverage and overlapping and part of loading from self-weight of heat-insulation material. External layer is calculated on the self-weight and part of heat-insulation material weight. Loading from the weight of heat-insulation material is distributed on masonry bearings layers proportionally to their section.

Bearing capacity of separate layers is determined according to the formulas (20.13) and (20.17) depending on the design characteristics of

each layer; the coefficients  $\varphi$ ,  $\varphi_i$ ,  $m_g$  are accepted in accordance with the tables and formulas (20.14), (20.19) and (20.20) for the conditional wall thickness

$$h = 0,7 \sum h_i. \quad (20.33)$$

If masonry layers are performed from different materials, than elastic characteristic  $\alpha$  is replaced by the reduced one:

$$\alpha_{red} = \frac{\alpha_1 h_1 + \alpha_2 h_2}{h_1 + h_2}, \quad (20.34)$$

where,  $\alpha_1$ ,  $\alpha_2$  are the elastic characteristics of separate layers;  $h_1$ ,  $h_2$  are thickness of layers.

**Elements with mesh and longitudinal reinforcing.** In order to increase bearing capacity the next methods of masonry reinforcing are used: a) mesh (transversal) reinforcing when the wire mesh is located in the horizontal masonry layers (Fig. 20.13, *a*, *b*); longitudinal reinforcing when the armature is located in furrows or channels, missed in masonry with their subsequent embedding by mortar (Fig. 20.13, *b*). Increasing of masonry bearing capacity on compression, strengthened by wire mesh reinforcing, is conditioned due to the perception transversal tension efforts by wire meshes, because wire meshes deformability is considerably lower than the deformability of masonry.

In calculations the increase of masonry bearing capacity is taken into account by means of introduction of conditionally increased strength and deformation characteristics of reinforcing. Longitudinal masonry reinforcing increases its bearing capacity due to the combined work of masonry and armature. Monolithic nature of masonry, its seismic stability, combined work of separate building parts are provided as well.

It is recommended to use the wire mesh reinforcing of horizontal masonry joints, when increasing of brick, stone and mortars grades does not provide the required strength of masonry, and the transversal section area of element can not be increased.

It is not necessary to use wire mesh reinforcing, when eccentricity of compression forces action transcend the section core (for rectangular sections if  $e > 0,17/h$ , as well as for flexible elements if  $e > 15$  or  $\lambda_i > 53$ ). In these cases it is more rational to use the longitudinal masonry reinforcing.



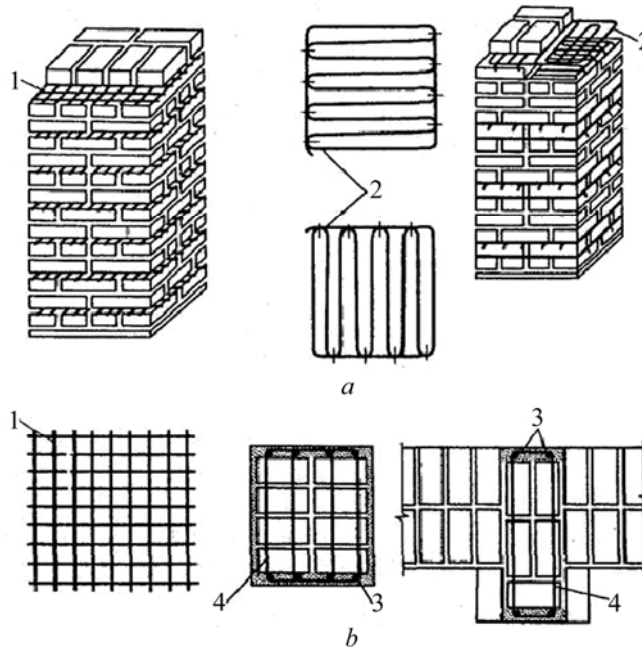


Fig. 20.13. Methods of masonry strengthening:  
*a* — transversal reinforcing; *b* — longitudinal reinforcing;  
 1 — rectangular wire mesh; 2 — «zigzag» wire mesh;  
 3 — longitudinal armature; 4 — clips

The wire meshes reinforcing leads to increasing the masonry bearing capacity if the following conditions would be kept:

- eccentricity must not transcend the section core (for the rectangular elements  $e_0 < 0,17/h$ );
- flexibility of the compressed elements must not exceed the following values:  $\lambda_h < 15$  or  $\lambda_i < 53$ ;
- minimum percent of reinforcing is 0,1 %;
- the height of masonry course must be not more than 150 mm;
- the percent of masonry reinforcing must not exceed the value, determined according to the formula:

$$\mu \leq \frac{50R}{(1 - 2e_0 / y)R_s} \quad (20.35)$$

Centrally-compressed elements of masonry constructions, reinforced by means of wire meshes (e.g. posts, foundations) are calculated like the plain elements according to the formula:

$$N \leq m_g \varphi R_{sk} A. \quad (20.36)$$

Eccentrically compressed elements of masonry constructions, reinforced by means of wire meshes (e.g., piers, parapets, foundations), in case of any form of transversal section, are calculated according to the formula:

$$N \leq m_g \varphi_1 R_{skb} A_c \omega. \quad (20.37)$$

For the transversal sections of the rectangular shape the formula (20.37) looks like the following:

$$N \leq m_g \varphi_1 R_{skb} A (1 - 2e_0 / y) \omega. \quad (20.38)$$

The wire mesh reinforcing of masonry is permitted in those cases, when increasing of bricks, stones or mortars grades doesn't provide the required durability of masonry, and the area of transversal element section can not be increased. It is recommended to lay the wire meshes with step  $s$ , but not rarer than in each 5 courses of masonry from usual brick, in each 4 courses of masonry from the thickened brick or in each 3 courses of masonry from ceramic stones.

The distance between the wire mesh bars is accepted within the limits  $3 \text{ cm} < c < 12 \text{ cm}$ , and bars diameter is accepted not less than 3 mm. The thickness of masonry joints must exceed the armature diameter not less than by 4 mm.

The maximum armature diameters in the horizontal masonry joints must not exceed 6 mm in case of armature crossing in courses and 8 mm — in case of «zigzag» wire meshes application.

### 20.3. Calculation stone elements of constructions

For masonry and reinforced masonry constructions calculations by cracks formation and by deformations are performed on the action of normative loadings at their main combinations. Calculation of eccentrically compressed and plain masonry constructions at  $e_0 > 0,7 y$  by cracks opening (joints of masonry) are performed on the action of design loadings. If appearance and opening of cracks in the masonry joints are not dangerous for constructions durability and stability (internal partitions, nonbearing walls), the calculation is performed for action of normative loadings.

It is necessary to perform calculation by formation and opening of cracks (masonry joints) in the followings cases:

– for eccentrically compressed elements of constructions at  $e_0 > 0,7y$  on the action of design loadings;

- for adjacent structural elements, which work together and prefabricated from materials with different deformation characteristics or at considerable difference in stresses, arising up in these elements;
- for buildings, in which the formation of cracks is not permitted or their opening is limited due to the operation conditions (reservoirs, silos);
- for self-bearings walls, connected with frameworks and working on transversal bending.

The followings positions are laid in the basis of calculation;

- distribution of stresses in the eccentric compressed section is linear one, as for elastic body;
- consideration of the section tension area, at the same time the stresses in it are accepted to be equal to conditional edge tension stress.

Checking of the eccentrically compressed plain sections on the formation and opening of cracks is performed according to the formula:

$$N \leq \frac{\gamma_r R_{tb} A}{[A(h-y)e_0 / I] - 1}, \quad (20.39)$$

where,  $\gamma_r$  is coefficient of masonry work conditions of laying during calculation on cracks opening (masonry joints);  $R_{tb}$  is design tension resistance of masonry during bending along the unbonded section;  $y$  is a distance from the section center of gravity to the extreme compressed fibred;  $I$  is the section moment of inertia.

Calculation on deformations is performed for the constructions' elements, on the surfaces of which the cracks formation is forbidden according to the operation conditions. The calculation is performed on the normative loadings, applied to construction's element after facing coatings have been deposited on its surface.

Relative deformations of the tension plastered surfaces of plain masonry must not exceed the values of relative deformations  $\epsilon_u$ . The calculation on deformations is performed for the elastic stage masonry material work at different cases of stress state.

During axial tension

$$N \leq EA\epsilon_u; \quad (20.40)$$

During bending

$$M \leq EI\epsilon_u / (h-y); \quad (20.41)$$

During eccentric compression

$$N \leq \frac{EA\epsilon_u}{A(h-y)e_0 / I - 1}; \quad (20.42)$$

During eccentric tension

$$N \leq \frac{EA\varepsilon_u}{A(h-y)e_0 / I + 1}; \quad (20.43)$$

where,  $E$  is the masonry module of deformations;  $A$  is section area of element;  $\varepsilon_u$  are the ultimate relative deformations;  $I$  is the moment of inertia of section;  $y$  is the distance of the center of gravity of masonry section to the most remote tension edge of coverage;  $N$ ,  $M$  are the longitudinal force and moment from normative loadings respectively.

#### ASSIGNMENTS TO CHAPTER 20

1. What classes of stone by strength are established by norms?
2. What grades of stone are established by norms?
3. Explain the meaning of «class of stone strength for compression».
4. What is the value of guaranteed stone strength for the given stone class? What supplies is it given?
5. What technical and economical recommendations are given for choosing the stone class?
6. How do time and hardening conditions influence stone strength?
7. What basic advantages have constructions made of simple and reinforced stone?
8. What basic faults have constructions from simple and reinforced stone?
9. What are the basic requirements to building constructions according to complex criterion of efficiency?
10. What are the rational regions of stone constructions application?
11. What are the most perspective directions of various types of stone constructions development in Ukraine?
12. What is the basic idea of stone constructions creation?
13. What is the basis of compatible work of stone constructions?

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**Словник  
англійсько-українських термінів**

Discipline .....	дисципліна
subject.....	предмет
term.....	семестр
course .....	курс (навч.)
building, engineering, construction.....	будівництво
structure, design building .....	споруда
construction .....	конструкція
projection, projecting, designing .....	проектування
purpose .....	ціль
support.....	підтримка, опора
condition.....	умова
task .....	задача, завдання
calculation, computation, design .....	розрахунок
construction, designing .....	конструювання
perform .....	виконувати, здійснювати
supporting construction .....	несуча конструкція
assembling, mounting.....	монтаж
repair, up keep, maintenance .....	ремонт
education .....	освіта, виховання
training .....	навчання, тренування
regulation.....	правило
include .....	включати, охоплювати
act .....	акт, закон, документ
employee .....	службовець, працівник
deal .....	розподіляти, роздавати
maintenance.....	поточний ремонт
aerodrome, air field .....	аеродром
concrete .....	бетон
reinforced concrete, armored concrete .....	залізобетон
specialty.....	спеціальність, фах
specialization .....	спеціалізація
industry.....	промисловість
industrial.....	промисловий
highway .....	автострада
lecture .....	лекція



independ .....	незалежність
requires .....	вимагас
analysis .....	аналіз
capacity .....	здатність
bearing .....	несучий
interaction .....	взаємодія
provide .....	забезпечувати, постачати
depend .....	залежати, покладатись
manufactory .....	виготовлення, виробництво
labor .....	праця
result .....	результат
slab, plate, flagstone, flag .....	плита
coefficient of safety, safety factor .....	коефіцієнт надійності
load, weight .....	навантаження
super imposer load, surcharge, duration .....	тимчасова, тривалий
safety, security, reliability, dependability .....	надійність
armature .....	арматура
hot-rolled reinforcement .....	гарячекатана арматура
reinforcing .....	армування
reinforced .....	армований
reinforce .....	армувати
varying, alternative, changeable, variable .....	змінний
prestressed reinforcement .....	попередньо напружена арматура
stab, flagstone, flag, plate .....	плита
diagram, scheme, schematic drawing .....	схема
pivot, spindle, stem, rod, bar .....	стрижень
reinforcing .....	арматурний
strength, endurance, strong, durability .....	міцність
element, member, unit .....	елемент
built up, precast, prefabricated, sectional .....	збірний
axial .....	осьовий
resistance, strength .....	опір
arch, dome, cove, covering, crown .....	склепіння

property .....	властивість
deformation, strain .....	деформація
shrinkage, contraction .....	усадка
filler, fill, aggregate .....	заповнювач
coarse .....	крупний
fine .....	дрібний
crush .....	давити
ultimate .....	тимчасове
resistance, strength .....	опір
rupture .....	перелом, злом, розрив
destruction, demolition, collapse, breakdown .....	руйнування
shear, shearing, section, cut .....	зріз
force, power, strength .....	сила (N, Q)
cast-in-situ .....	монолітний бетон
bending moment .....	згинаючий момент
bending, curvature, curve, flexure, bendзгин longitudinal .....	подовжній
transverse, cross, lateral, diametrical .....	поперечний
transversally loaded .....	з поперечним навантаженням
width .....	ширина
rectangular .....	прямокутний
flange .....	полка, кромка, фланець
depth .....	глибина, висота
compression .....	стиснення
tension .....	розтягнення, натягування
transformed .....	приведений, перетворений (переріз)
equal .....	що дорівнює, рівний
hinge .....	з'єднання
produced .....	виконується, виробляється
weld .....	зварювати
sheet .....	лист
beam, girder, joist .....	балка (ферма)
joist, secondary .....	другорядна
main girder, main beam .....	головна
I-girder (beam), double T-rail, flagged beam .....	двотаврова

Jib .....	кранова
Multispan.....	багато прогонна
Embed .....	вставляти, впроваджувати, закладна деталь
washer, collar, disk.....	шайба
joint, link, articulation, hinge .....	шарнір
girder, framework, truss .....	ферма
space truss .....	просторова ферма
lattice truss .....	ґратчаста ферма
truss, truss frame, roof truss .....	кроквяна ферма
piece .....	шматочки, комбінація
column, shaft, pillar.....	колонна
solid.....	суцільна
dimension .....	вимір, величина
cantilever, console.....	консоль
intermediate.....	проміжна (колона)
marginal.....	крайня (колона)
ensure .....	забезпечувати, гарантувати
rigidity.....	жорсткість, неподатливість
adequate.....	адекватний, відповідний вимогам
experience .....	випробування, досвід
consider .....	вважати
implicate.....	включати
level.....	рівень, висота
less.....	менший
convenient .....	зручний, відповідний
passage .....	прохід, проїзд
installed .....	встановлювати, влаштовувати
assume .....	приймати
strut.....	стояк, підпорка, підкіс
achieve.....	досягати, виконувати
brace, bracing .....	зв'язок
beam anchor .....	анкерний
vertical bracing.....	вертикальний
top.....	верхній
top lateral bracing.....	горизонт в площині верхніх поясів

bottom lateral bracing.....	горизонт в площині нижніх поясів
chord.....	струна, пояс
thrust.....	штовхати, гальмівні (зусилля)
advantage.....	перевага
longevity.....	довговічність
fail .....	недоліки
appearance.....	поява
crack .....	тріщина
previous .....	попередній
tense.....	напружений, натягнутий
cohesion.....	зчеплення
mixture .....	суміш
astringent.....	в'язучий
creep of concrete .....	повзучість бетону
specify .....	визначати
protective layer.....	захисний шар
longitudinal reinforcement .....	поздовжня арматура
condition.....	стан
simultaneous.....	одночасний
achieve.....	досягати
loss.....	втрата, збиток
temporal.....	тимчасовий
action .....	дія
liquid .....	рідина
cause .....	причина
violation.....	порушення
uneven .....	нерівномірний, нерівний
previous .....	попередній
simultaneous.....	одночасний
equilibrium .....	рівновага
peculiar, feature .....	спец., особливий
circumflex.....	огинаючи
boundary.....	межа
indirect.....	посередній, побічний
comparison .....	порівняння
curvature.....	кривизна, згин
firm .....	твердий, стійкий
collapsible.....	збірний, складаний
sedimentary .....	осадовий

stitch .....	шов
assemblage .....	збирання навантажень
effort .....	зусилля, напруження
hangar .....	ангар
flight .....	проліт, прогін
brace .....	зв'язок, підпора, скоба
spatial .....	просторовий
stonework .....	кам'яна кладка
artificial .....	штучний
intestine .....	внутрішня (пустота)
appear .....	з'являться
squeeze .....	стиснення
iron .....	обойма, залізний виріб
stirrup .....	хомут, поперечна арматура
purlin .....	прогін
multistory, multistage, many-storied, multispan .....	багатоповерховий
skeleton, carcass .....	каркас
frame .....	рама
bearing wall .....	несуча стіна
fill .....	наповнювати, заповнювати
stress-strain behavior .....	деформативність (поведінка в стресовому стані)
shear .....	зріз, зсув
fatigue .....	втома, стомленість
developed .....	розвинена (деформація)
duration .....	тривалий, тимчасовий
drawing .....	креслення, малюнок
eccentric .....	позацентровий
crane (under crane) girders .....	балка підкранова
two branch .....	дво гілковий
strand .....	зв'язка
decreasing, reduction .....	зменшення
single force .....	зосереджена сила
column in compression .....	колона стиснута
rafter .....	кроква
slopes .....	нахили, скати
prestressed .....	попередньо-напружена

eccentrically compressed.....	позацентрове стиснутий
thrust.....	розпір
trussed (sub diagonal) girder .....	шпренгельна балка
stone .....	камень
artificial .....	штучний
hollow.....	пустотелий
laying.....	кладка
masonry .....	каменна
heat-insulation .....	теплоізоляційний
brick pit .....	колодіз цегляний
barrier .....	огороження
non-bearing wall.....	стіна ненесуча
enclosing wall.....	стіна огорожувальна
hang up wall .....	стіна навісна
layer.....	шар
brick.....	цегла
eccentric.....	позацентровий
garret .....	чердак
span .....	пролет
eccentricity .....	ексцентрицитет
flexibility .....	гнучкість
counterfort .....	контрфорс
patching up .....	заделка
heat-insulation .....	утеплювач

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