POLARIZATION OF LIGHT

4.1. Natural and polarized light

Interference and diffraction phenomena are observed for both transverse and longitudinal waves. At the same time, there are the phenomena for which the light wave transversity has the fundamental importance. Polarization of light is related to such phenomena.

Maxwell's theory states that *the light is the transverse electromagnetic wave, and the electric and magnetic vectors of the light wave oscillate perpendicularly to wave spreading direction*. In fact, the light wave transversity which determines polarization of light has been observed before Maxwell's electromagnetic theory appearing. Particularly, detection of birefringence in Iceland spar crystals (Bartholini, 1670) together with investigation of these rays intensity change (Huygens, 1690) and also study of the light reflection from the glass surface (Malus, 1808) indicated the light polarization, although the meaning of this phenomenon was incomprehensible. Experiments by Fresnel and Arago concerning interference of the polarized rays (1816) stimulated Young to make assumption of transversity of the light waves. Fresnel, independently from Young, has also proposed the conception of the light wave transversity, widely grounded it by means of multiple experiments and explained the phenomena of light polarization and birefringence in crystals.

Although the light waves are transverse ones but they normally have no asymmetry relatively to propagation direction. This is explained by the fact that *in the light radiated by the ordinary sources, the electric vector $\vec{E}$ oscillates in all possible directions that are perpendicular to the wave propagation direction*. Such light is called *natural light*.

According to classical representation, the luminous body radiation consists of the waves radiated by its atoms. Individual atom radiation time equals approximately 10^{-8} s. An atom radiation represents the wavetrain with the length of approximately 3 m where the vector $\vec{E}$ lies in the single plane. After radiation, the atom in some time goes to the excited state and radiates again and again, and every new wavetrain has

its own independent orientation of the vector $\vec{E}$. Many atoms radiate simultaneously and independently and form the resultant light radiated by a body. The light vector $\vec{E}$ oscillations occur in different planes with the same probability. In theory, natural light is represented in Fig. 4.1 where the wave propagation direction is perpendicular to the figure plane.

Light with an ordered plane of the vector $\vec{E}$ oscillations is called *polarized light*. If the oscillations of the vector $\vec{E}$ occur in the single plane only, the light is called *linearly polarized or plane-polarized*. The plane of the vector $\vec{E}$ oscillations is called *the vector $\vec{E}$ oscillation plane or polarization plane*. It passes through the incident ray direction.
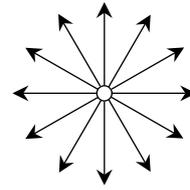
Fig. 4.1

It is known from the theory of oscillations (Module 4 "Oscillations and waves", paragraph 1.9) that addition of two mutually perpendicular harmonic oscillations with the same frequencies generally products motion along the ellipse trajectory. It means that addition of two waves with mutually perpendicular planes of electric vector oscillations

$$E_x = A_1 \cos \omega t , \quad E_y = A_2 \cos(\omega t + \delta) \qquad (4.1)$$

gives the resulting wave in which the end of the vector $\vec{E}$ moves along the ellipse trajectory (Fig. 4.2), the light ray is spreading perpendicularly to the figure plane).

Thus, superposition of two coherent ($\delta = \text{const}$) linearly polarized light waves with the mutually perpendicular oscillation planes forms *the elliptically polarized light wave*. If phase difference $\delta = 0$ or $\delta = \pm\pi$ the ellipse transforms to the straight line and light becomes *the linearly polarized (plane-polarized) light*. The vector $\vec{E}$ oscillations direction and the wave propagation direction form the oscillation (polarization) plane.
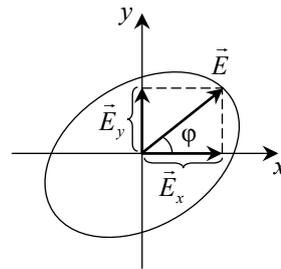
Fig. 4.2

Under condition of the same amplitudes $A_1 = A_2$ and the phase difference $\delta = \pm\pi/2$, the ellipse transforms to the circle, i.e. we observe *rotatory polarization* of the light.

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Left-hand and *right-hand* elliptical and rotatory polarizations are differed depending on the vector $\vec{E}$ rotation direction. If to look in the direction opposite to the direction of the wave propagation and the vector $\vec{E}$ rotates clockwise, the polarization is right-hand, if it rotates counter clockwise, the polarization is left-hand.

Thus, the wave with rotatory or elliptical polarization can be always decomposed into two mutually perpendicular linearly polarized waves with mutually perpendicular planes of the vector $\vec{E}$ oscillations. In this case, phase difference for these two waves remains constant in time ($\delta = \text{const}$). Such waves are called coherent unlike the incoherent waves with the phase difference changing chaotically in time.

The resulting vector $\vec{E}$ (Fig. 4.2) represents the vector sum of electric intensities $\vec{E}_x$ and $\vec{E}_y$. Angle between $\vec{E}$ and $\vec{E}_x$ equals:

$$\text{tg}\varphi = \frac{E_y}{E_x} = \frac{A_2 \cos(\omega t + \delta)}{A_1 \cos \omega t}. \qquad (4.2)$$

If the phase difference is constant ($\delta = \text{const}$) and $\delta = 0$ or $\delta = \pi$ then the angle φ is constant too: $\text{tg}\varphi = \pm A_2/A_1 = \text{const}$. Thus, the resulting oscillation occurs in the fixed direction and the wave is linearly polarized. If $\delta = \text{const} \neq 0$, the vector $\vec{E}$ rotating with the frequency ω around the coordinate origin changes in size according to certain regularity. If the phase difference changes chaotically, both the angle δ and the direction of the vector $\vec{E}$ change chaotically and the mutual position of the vectors $\vec{E}$, $\vec{E}_x$ and $\vec{E}_y$ is arbitrary.

Therefore, natural light can be considered as superposition of two *incoherent* waves polarized in the mutually perpendicular planes and having the same intensity ($I \sim A_1^2 = A_2^2$). In optics, natural light is symbolically shown in Fig. 4.3.
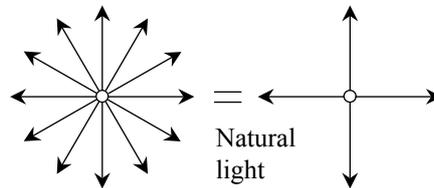

Natural light

Fig. 4.3

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The linearly polarized light can be obtained from the natural light with the help of devices called *polarizers*. These devices transmit only the light vector $\vec{E}$ oscillations which are parallel to the plane called the *polarizer plane of polarization*. Oscillations perpendicular to this plane are retarded completely or partially. A polarizer that completely retards the oscillations perpendicular to the plane of polarization is called the *perfect polarizer*; otherwise, it is called the *imperfect polarizer*.

While passing through the perfect polarizer the natural light becomes the completely linearly polarized. When the light is passing through the imperfect polarizer, the output represents the light in which the oscillations in one direction prevail over the oscillations in other directions. Such light is called *partially polarized*. The partially polarized light as well as the natural light may be considered as a superposition of two *incoherent* linearly polarized waves with the mutually polarized planes of oscillation. The only difference is that the intensity of these waves in the natural light is the same while in the partially polarized light — different. The partially polarized light can be also considered as the sum of the natural and the linearly polarized components as shown in Fig. 4.4.
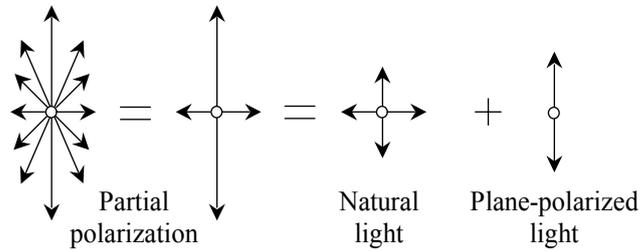

| Partial polarization | Natural light | Plane-polarized light |

Fig. 4.4

It is seen in Fig. 4.4 that the vertical oscillations correspond to the maximum intensity I_{max}, the horizontal oscillations correspond to the minimum intensity I_{min}. Such change of intensity can be observed with the help of a polarizer. The partially polarized light is characterized by the *polarization degree P* which is determined as follows:

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{I_{pol}}{I_0}, \qquad (4.3)$$

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where I_{pol} is the polarized component intensity; I_0 is the total intensity of the partially polarized light: $I_0 = I_{max} + I_{min}$.

For the linearly polarized light $I_{min} = 0$ $\left(I_{pol} = I_0\right)$ and $P = 1$; for the natural light $I_{max} = I_{min}$ $\left(I_{pol} = 0\right)$ and $P = 0$. For the partially polarized light $0 < P < 1$. For the elliptically and rotatory polarized light the polarization degree quantity is not used.

4.1.1. Malus law

Let us consider an experiment. A ray from an ordinary light source is directed perpendicularly to the tourmaline crystal surface T_1 (Fig. 4.5, a) that is parallel to the optical axis OO'. While rotating the crystal around the ray direction no change in the light intensity is observed.
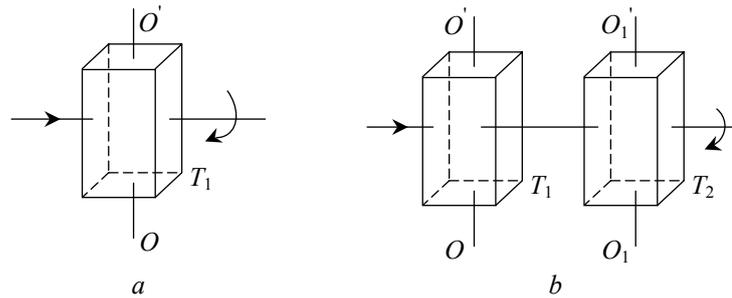

Fig. 4.5

However, if the second similar tourmaline crystal T_2 is placed on the ray path in parallel to the first crystal (Fig. 4.5, b), when rotating one of them (T_1 or T_2) around the ray direction the intensity of light I after passing through them varies depending on the angle φ between the axes OO' and O_1O_1' according to the law discovered *by Malus*:

$$I = I_0 \cos^2 \varphi, \qquad (4.4)$$

where I_0 is the intensity of the light passed through the first crystal T_1.

Such phenomenon can be explained under assumption that: a) light is a transverse wave, b) a tourmaline crystal transmits only the light with

the electric vector $\vec{E}$ oscillations parallel to the optical axis and absorbs the light with the electric vector oscillations perpendicular to this axis.

In fact, as the natural light falling onto the crystal T_1 surface consists of the vector $\vec{E}$ oscillations in all directions perpendicular to the wave propagation direction, during the crystal T_1 rotation around the ray axis (Fig. 4.5, *a*) the oscillations along direction OO' always exist. Therefore, the intensity of the light passed through the crystal does not change.

After passing through the tourmaline crystal the natural light becomes *linearly polarized*, i.e. the electric vector $\vec{E}$ oscillations occur in the single direction OO' only. Let us assume that the crystal T_1 axis OO' forms the angle φ relatively to the crystal T_2 axis O_1O_1'. Then the vector $\vec{E}_0$ of light passed through the crystal T_1 forms the same angle φ relatively to the crystal T_2 axis O_1O_1' as shown in Fig. 4.6.

It is evidently that the electric vector value of light which passed through the crystal T_2 is equal to $E = E_0 \cos\varphi$. Since the light intensity is proportional to the squared amplitude ($I \sim E^2$), the expression $I = I_0 \cos^2\varphi$ is Malus law.

The crystal T_1 which transforms the natural light to the linearly polarized light is a *polarizer*. The second crystal T_2 is used to analyze the light and it is called an *analyzer*.

If the light is transmitted in the reversal direction, the crystals exchange their roles and the crystal T_2 will be the polarizer and T_1 will be the analyzer.

Fig. 4.6

Let us determine the intensity of light passed through two perfect polarizers disregarding reflection and absorption. The first polarizer creates the linearly polarized light with the intensity of $I_0 = I_{nat}/2$. Natural light intensity is I_{nat}. According to Malus law, the second polarizer transmits the light with intensity of $I = I_0 \cos^2\varphi$. Thus, the intensity of light after the polarizers system is:
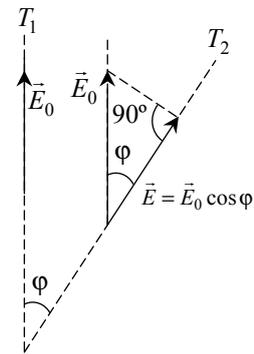
$$I = \frac{1}{2} I_{\text{nat}} \cos^2 \varphi . \qquad (4.5)$$

The maximum intensity $I_{\max} = \frac{1}{2} I_{\text{nat}}$ is observed if $\varphi = 0$ (the polarizers planes are parallel), and the minimum intensity $I_{\min} = 0$ is observed if $\varphi = \pi/2$ (the polarizers planes are crossed).

4.2. Polarization in case of light reflection and refraction on interface of two dielectrics

4.2.1. Brewster's law

If the natural light incidence angle onto interface of two transparent isotropic dielectrics (for example air — glass) is different from zero, the reflected and refracted rays are partially polarized, that is they represent the mixture of the natural light with some portion of the polarized light. The oscillations perpendicular to the incidence plane prevail in the reflected ray (they are shown in Fig. 4.7, *a* by the double-side arrows), and the oscillations parallel to the incidence plane prevail in the reflected ray (they are shown in Fig. 4.7, *a* by points).
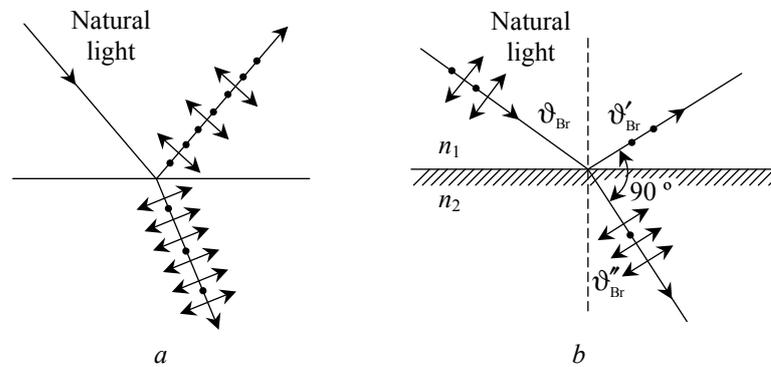

Fig. 4.7

Polarization by reflection and refraction was detected by Malus (1810) who noted by chance that during rotation of a tourmaline crystal around the ray reflected from the glass, the light intensity periodically increases and decreases, i.e. the reflection from the glass acts on the light in the way as it passes through two tourmaline crystals (Fig. 4.5).

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Brewster found experimentally that at a certain incidence angle the ray reflected on the interface becomes completely polarized, and the reflected and the refracted rays form the angle of 90° (Fig. 4.7, *b*). Such incidence angle is called *Brewster angle* ($\vartheta = \vartheta_{Br}$) *or complete polarization angle*. This angle is determined by the law stated by Brewster in 1815:

$$\mathrm{tg}\,\vartheta_{Br} = \frac{n_2}{n_1} = n_{12}\,, \qquad (4.6)$$

where n_{12} is a relative refraction index of the second medium with respect to the first medium.

As the ray reflected at the angle ϑ_{Br} is completely polarized, the vector $\vec{E}$ oscillations occur only perpendicularly to the incidence plane. The incidence plane is the plane of Fig. 4.7, *b*, thus oscillations of the vector $\vec{E}$ in the reflected ray are shown in this figure by points. At the same time, the refracted ray polarization in case of falling at Brewster angle is maximal but not complete; therefore, the number of arrows just exceeds the number of points in the refracted ray (Fig. 4.7, *b*).

The reflected and the refracted rays form the right angle in case of falling at Brewster angle (Fig. 4.7, *b*). According to Brewster law and refraction law, we get:

$$n_{12} = \frac{\sin \vartheta_{Br}}{\cos \vartheta_{Br}} = \frac{\sin \vartheta_{Br}}{\sin \vartheta''_{Br}}\,.$$

Hence,

$$\cos \vartheta_{Br} = \sin \vartheta''_{Br} = \cos(\pi/2 - \vartheta''_{Br}) \Rightarrow \vartheta_{Br} + \vartheta''_{Br} = \pi/2\,. \qquad (4.7)$$

This result is obtained from Fresnel formulas; let us analyze these formulas in details. It is seen from the formulas (1.18a) and (1.20a) that the signs of the complex amplitudes of *incident* and *refracted* waves at any values of the angles ϑ and ϑ'' are the same (sum of ϑ and ϑ'' cannot exceed π, and cosine of the angles difference is an even function). It means that *phases of incident and refracted waves are the same, i.e. the refraction occurs without the wave phase change under all conditions*.

In case of reflection, the change of the phases depends on conditions. From formula (1.20)

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$$r_\perp = \frac{E'_{0\perp}}{E_{0\perp}} = -\frac{\sin(\vartheta - \vartheta'')}{\sin(\vartheta + \vartheta'')}$$

we see that the ratio of the perpendicular components $E'_{0\perp} / E_{0\perp}$ is always negative (i.e. $E'_{0\perp}$ are $E_{0\perp}$ have the opposite signs) under the condition that $\vartheta > \vartheta''$, i.e. the light is refracted from the optically more dense medium ($n_2 > n_1$). It means that the phase of oscillations perpendicular to incidence plane ($E'_{0\perp}$) changes jump-like by π and becomes opposite to the phase of the incident ray perpendicular component ($E_{0\perp}$).

In case when the light is reflected from the optically less dense medium ($n_2 < n_1$) the incidence angle is less than the refraction angle $\vartheta < \vartheta''$ (or $\vartheta < \vartheta_{cr}$). Thus, $\sin(\vartheta - \vartheta'') < 0$, and $E'_{0\perp} / E_{0\perp} > 0$. The phase change is absent.

If $n_2 > n_1$, the phase of oscillations of a reflected wave parallel to the incidence plane $E'_{0\parallel}$ is opposite to the phase of the parallel component of the incident wave $E_{0\parallel}$ but only for incidence angles *less* than Brewster angle: $\vartheta'' < \vartheta < \vartheta_{Br}$ $(\vartheta + \vartheta'' < \pi/2)$. It is evident that under such condition the ratio $E'_{0\parallel} / E_{0\parallel} < 0$.

While passing through Brewster angle, the phase of a parallel component changes by π jump-like. It means that the phases of the perpendicular $E'_{0\perp}$ and the parallel $E'_{0\parallel}$ components of reflected wave become opposite for the angles *exceeding* ϑ_{Br}, and the phases of the parallel components of the incident $E_{0\parallel}$ and the reflected $E'_{0\parallel}$ waves are the same.

In case of wave reflection on optically less dense medium ($n_2 < n_1$), the situation is opposite. For the incidence angles *less* than Brewster angle $\vartheta < \vartheta'' < \vartheta_{Br}$ $(\vartheta + \vartheta'' < \pi/2)$ the ratio $E'_{0\parallel} / E_{0\parallel} > 0$ is positive, i.e. the phases of the reflected $E'_{0\parallel}$ and the incident $E_{0\parallel}$ waves oscillations are the same. For the incidence angles *more* than Brewster angle (but less than ϑ_{cr}) $\vartheta_{Br} < \vartheta < \vartheta''$ $(\vartheta + \vartheta'' > \pi/2)$, the phases $E'_{0\parallel}$ and $E_{0\parallel}$ are opposite.

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Described above analysis shows that for the incidence angles $\vartheta + \vartheta'' < \pi/2$ the phase of the both components ($\parallel$ and $\perp$) of the reflected wave electric vector is opposite to the incidence wave phase for the case $n_2 > n_1$ and coincides with it at $n_2 < n_1$. The same result was obtained for light normal falling ($\vartheta = \vartheta'' = 0$).

In case of complete polarization of a reflected ray (falling at Brewster angle) the parallel component $E'_{0\parallel}$ in formula

$$r_{\parallel} = \frac{E'_{0\parallel}}{E_{0\parallel}} = -\frac{\operatorname{tg}(\vartheta - \vartheta'')}{\operatorname{tg}(\vartheta + \vartheta'')}$$

is equal to zero.

Hence, $\operatorname{tg}(\vartheta + \vartheta'') = \infty$, i.e. $\vartheta_{Br} + \vartheta''_{Br} = \pi/2$ ($\vartheta = \vartheta_{Br}$, $\vartheta'' = \vartheta''_{Br}$). If an incident wave electric vector lies in the incidence plane (i.e. the component $E_{0\parallel}$ only exists), the reflected wave is absent under the condition of $\vartheta = \vartheta_{Br}$.

4.3. Polarization by birefringence

Sailors transported to Europe from Island the Island spar crystals which had the interesting feature.

While looking at subjects through such crystals, the double image of the subjects was observed. As a result, birefringence (double refraction) phenomenon was discovered.

The Island spar is a kind of chemical chalk ($CaCO_3$); its crystals belong to the hexagonal system. In nature, the Island spar sometimes exists in the form of hexagon columns but often its chips are detected with the rhombohedron shape. It can be explained by the fact that the hexagonal crystals of the Island spar are easily split along the certain planes forming the rhombohedral chips. Rhombs which limit the rhombohedron have angles $101^{\circ}52'$ and $78^{\circ}08'$.

If a narrow beam of light falls onto such crystal, the output represents two spatially separated rays parallel to the incident beam (Fig. 4.8).

Even if a ray incidence angle equals zero, the ray in the crystal is divided into two rays; one of them is a continuation of the incident ray and the other one is deviating (Fig. 4.9).
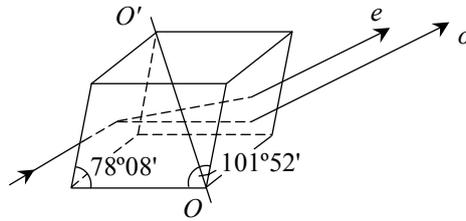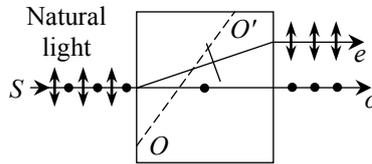
Fig. 4.8

Fig. 4.9

Experiments show that one of these rays propagates along the incidence plane, the refractive index for it does not depend on the incidence angle, and the ordinary refraction law is true for it. The ray velocity does not depend on its propagation direction in the crystal. Such a ray is called *ordinary* and designated with the letter *o* in figures. The second ray does not propagate along the incidence plane and the ratio of sine of the incidence angle to sine of the refraction angle depends on the incidence angle. Hence, the ray velocity depends on its propagation direction in the crystal. This ray is called *extraordinary* and designated with the letter *e*.

Iceland spar, quarts, and tourmaline are the examples of the *single-axis* crystals. These crystals have only *one* direction along which the birefringence is not observed.

Double-axis crystals (mica, gypsum) have *two* directions along which the birefringence is not observed. Both rays are extraordinary in the double-axis crystals; the refractive indices for them depend on the incidence angle.

Iceland spar crystals have only one direction along which the birefringence is absent; the refractive indices for both rays are the same in this direction. Such direction is called the *crystal optical axis*. An infinite number of the parallel crystal optical axes can be directed through the crystal.

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4.3.1. Ordinary and extraordinary rays

Any plane parallel to the uniaxial crystal optical axes is called its *main section* or *main plane*. An infinite number of the main planes can be traced through the crystal. A line of any two main planes crossing is the crystal optical axis. As a rule, the main section is selected in such a way that the optical axis and the incident ray are located on the section.

The experiments show that the ordinary and the extraordinary rays are completely polarized in the mutually perpendicular directions. The ordinary ray electric vector is perpendicular to the main plane; the extraordinary ray electric vector is parallel to the plane. Fig. 4.9 represents oscillations of the extraordinary ray electric vector by two-sided arrows; the ordinary ray electric vector is normal to the main plane and its oscillations direction is represented by points. If, after passing through the first crystal, the rays are directed onto the next crystals, they again are doubled (Fig. 4.10).
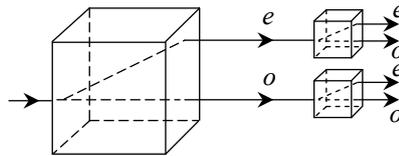

Fig. 4.10

Therefore, a double refraction exists for both *natural* and *linearly polarized* light. A difference is that intensities of both rays are identical for double refraction of natural light; intensities of ordinary and extraordinary rays depend on the angle of incidence in case of double refraction of polarized light.

Let us explain the phenomenon. If the polarized light with the amplitude E falls onto a crystal at the angle φ between the light oscillation plane and the crystal main plane, the electric vectors of the extraordinary e and ordinary o rays create accordingly angles φ and $\pi/2 - \varphi$ with the plane of oscillations of the falling polarized light. Then, the amplitudes of the electric vectors oscillations for the ordinary and extraordinary rays are:

$$E_o = E \sin \varphi; \quad E_e = E \cos \varphi,$$

and their intensities are:

$$I_o = I\sin^2\varphi; \quad I_e = I\cos^2\varphi. \tag{4.8}$$

The equations (4.8) represent Malus law for the ordinary and extraordinary rays. Then, we can get the relation of intensities:

$$I_o / I_e = E_o^2 / E_e^2 = \sin^2\varphi / \cos^2\varphi = \operatorname{tg}^2\varphi. \tag{4.9}$$

As it follows from (4.9), the intensities of o and e rays are identical only for $\varphi = 45^\circ$, it is confirmed by the experiments.

Double refraction is explained by the crystals anisotropy. Physical nature of anisotropy is conditioned by the structure of molecules and their placing and interaction in crystals. The phenomenon of double refraction is observed in all optically transparent crystals except the cubic system crystals. An index of refraction of the cubic system crystals does not depend on the direction of light propagation. Therefore, cubic system crystals are optically isotropic. Crystals of tetragonal, trigonal and hexagonal systems are uniaxial crystals; crystals of rhombic, monoclinic and triclinic systems are biaxial.
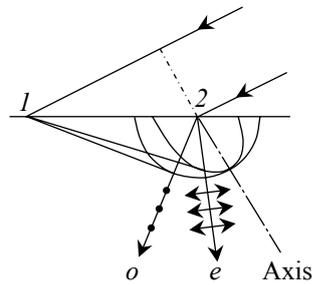

Fig. 4.11

Singularity of propagation of the ordinary and extraordinary rays in a crystal is possible to explain by Huygens principle. Fig. 4.11 shows wave surfaces of o and e rays with a center in a point *2* on the crystal surface. The flat wave front reaches the point *1*. According to Huygens principle, the straight lines from the point *1* that touch ellipsoid and spherical surfaces show the wave fronts of the flat extraordinary e and ordinary o waves. The ordinary ray o coincides with a normal to the spherical surface. At the same time, the extraordinary e ray deviates from this normal.

Let us consider three cases of a light normal falling on the crystal surface. In the first case (Fig. 4.12, *a*), the rays o and e propagate along the optical axis and coincide. In the second case (Fig. 4.12, *b*), the extraordinary ray deviates from the normal to this surface. In the third case (Fig. 4.12, *c*), the optical axis is parallel the surface of the crystal; the ordinary and extraordinary rays propagate one-way but their speeds are different, that is why increasing with time difference of phases appears between them. Oscillations of the vector E of the ordinary o ray

are perpendicular to the main plane; oscillations of the vector E of the extraordinary e ray are parallel to the main plane.
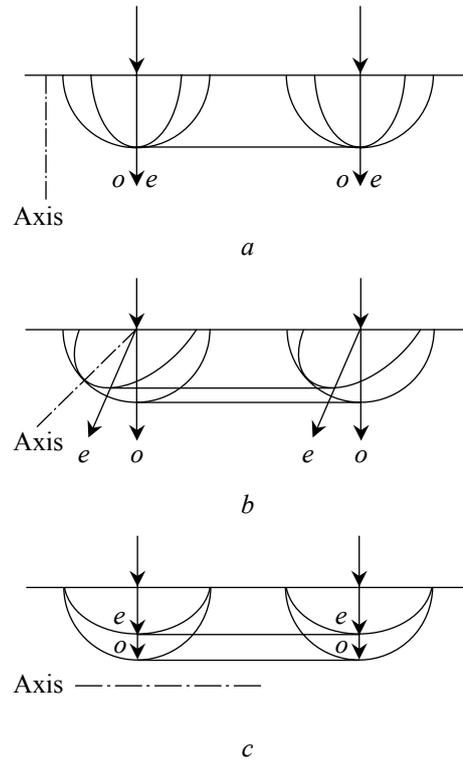

a

b

c

Fig. 4.12

4.3.2. Polarization devices (prisms)

The index of refraction of the Iceland spar changes from 1.486 to 1.658; it can be applied for separation of two polarized in mutually perpendicular planes rays.

The first polarization prism was made by Scottish physicist Nicol (1828) and it has the name «Nicol». It consists of two prisms of the Iceland spar that are glued together by the Canadian balsam, which index of refraction is between n_o and n_e.

Fig. 4.13 shows the main cut of the Nicol prism and the rays path in the prism. An optical axis is represented by a two-sided arrow; it forms

an angle 64° with the long rib of the prism. At the proper angle of incidence on the prism face, the ordinary ray *o* undergoes complete internal reflection on the layer of the Canadian balsam and then it is absorbed by the prism blackened lower face.

To prevent heating of the large prisms, an ordinary ray is taken away from the prism by the little prism glued to the Nicol prism (it is shown by the dotted line, Fig. 4.13).
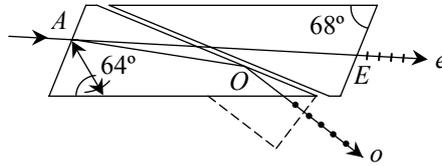

Fig. 4.13

The Foucault prism (Fig. 4.14) is built as the Nicol prism but the function of the Canadian balsam is executed by the layer of air. Due to that, the Foucault prism can be used for ultraviolet, while the Nicol prism is not suitable for this purpose as the Canadian balsam absorbs ultraviolet light. The Foucault prism is considerably shorter and, consequently, more cheap.
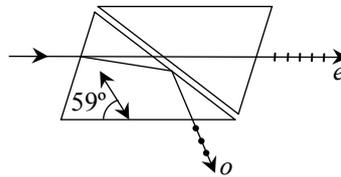

Fig. 4.14

Absorption of the ordinary and extraordinary rays is identical in the common transparent uniaxial crystals. However, there are crystals in which one of the rays is absorbed stronger than the other. Such phenomenon is named *dichroism*. A crystal of tourmaline has strong dichroism of the visible rays.

This crystal of 1 mm thickness almost entirely absorbs the ordinary ray. As a result of the removal of one ray, the plate of tourmaline becomes a polarizer. The phenomenon of dichroism is used for production of polarizers as nonselective light filters; they are named *Polaroid*.

4.3.3. Elliptic, circular and linear polarizations of light.
Transmission of the polarized light
through a crystalline plate

Let us consider a case that has a practical application, when the linearly polarized light propagates in a uniaxial crystal crosswise to its optical axis. If the linearly polarized light falls crosswise onto a crystal plate that is cut parallel to the optical axis $O'O$, division of the ordinary and extraordinary rays is absent (see Fig. 4.12, *c*) but they have different speeds and different indexes of refraction.

Let the plane of oscillations of the vector E of falling linearly polarized light form an angle φ with an optical axis. The light propagates perpendicular to the plane of Fig. 4.15. E_o and E_e are vectors of the ordinary and extraordinary rays, their amplitudes are:

$$E_o = E \sin\varphi, \; E_e = E \cos\varphi. \quad (4.10)$$

Fig. 4.15

Depending on the thickness of the crystal plate *d*, both rays go out from it with the difference of phases δ that depends on the optical path difference:

$$\Delta = d(n_o - n_e). \quad (4.11)$$

Taking into account that $\delta = 2\pi\Delta/\lambda_0$, where λ_0 is a wavelength in vacuum, we get:

$$\delta = 2\pi \frac{d(n_o - n_e)}{\lambda_0}. \quad (4.12)$$

Consequently, if the linearly polarized light falls normally onto a uniaxial crystal, the ordinary *o* and extraordinary *e* rays are coherent and the difference of phases between them is determined by the formula (4.12). Taking into account the difference of phases, the electric fields intensity oscillations of *o* and *e* rays at exit from the crystal are:

$$E_{ox} = E_o \cos\omega t, \quad E_{ey} = E_e \cos(\omega t + \delta), \quad (4.13)$$

where indexes of *x* and *y* state that the oscillations of the electric vectors of the ordinary *o* and extraordinary *e* rays take place in the mutually perpendicular planes (Fig. 4.15).
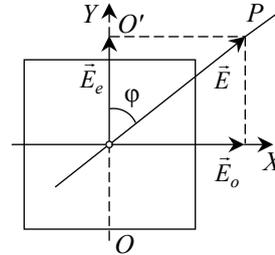
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It is known from the theory of oscillations (module 4 "Oscillations and waves", subunit 1.9) that (4.13) is a formula of an ellipse in a parametric form; its general form is:

$$\frac{E_{ox}^2}{E_o^2} + \frac{E_{ey}^2}{E_e^2} - 2\frac{E_{ox}E_{ey}}{E_o E_e}\cos\delta = \sin^2\delta. \qquad (4.14)$$

Expression (4.14) represents an arbitrarily oriented in relation to coordinate axes ellipse (Fig. 4.16). The orientation of the ellipse depends on the difference of phases δ.
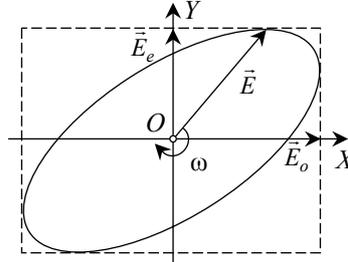

Fig. 4.16

Therefore, if linearly polarized light passes through a uniaxial crystal plate, the light becomes the elliptic polarized light with a resulting vector *E* rotating with a cyclic frequency ω.

Addition of two mutually perpendicular coherent oscillations leads to the formation of the elliptic polarization. The elliptic polarized light can be presented as a sum of two mutually perpendicular coherent oscillations. Depending on the values of amplitudes and the difference of phases, the elliptic polarized light can turn into the circle polarized or linearly polarized light.

Transformations of light depend on the crystal plate thickness *d* parallel to the optical axis. Let us consider special important cases when the polarized light falls normally on the uniaxial crystal plate.

1. *A plate thickness is equal to the fourth part of wavelength.* Therefore, the path difference of *o* and *e* rays is the fourth of the wavelength:

$$d(n_o - n_e) = \lambda_0/4, \text{ or}$$

$$d(n_o - n_e) = (m + 1/4)\lambda_0, \quad m = 0, 1, 2,... \qquad (4.15)$$

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According to the equation (4.12), $\delta = \pi/2$ and the equation of an ellipse (4.14) becomes

$$\frac{E_{ox}^2}{E_o^2} + \frac{E_{ey}^2}{E_e^2} = 1 .$$

The ellipse is oriented relatively to the main axes (Fig. 4.17). If $E_o = E_e$, the ellipse turns into a circle, its equation is:

Fig. 4. 17

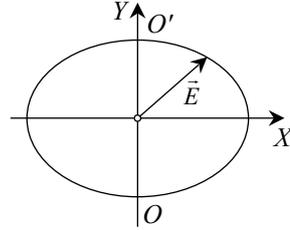$$\frac{E_{ox}^2}{E_o^2} + \frac{E_{ey}^2}{E_o^2} = 1 . \tag{4.16}$$

Consequently, to get a circle polarized light, we have to add two coherent waves with the identical amplitudes that have the difference of phases $\pi/2$ and are polarized in two mutually perpendicular planes. It can be achieved if the linearly polarized light passes through a fourth of wavelength plate so that the plane of polarization of the falling light forms an angle 45° with the crystal plate optical axis.

Depending on the plate orientation, the difference of phases can be positive or negative. So, the resulting vector E can rotate clockwise or counter-clockwise.

2. *A plate thickness is equal to the half wavelength.* Therefore, the path difference of *o* and *e* rays is:

$$d(n_o - n_e) = \lambda_0/2 , \quad \text{or} \quad d(n_o - n_e) = (m + 1/2)\lambda_0 , \tag{4.17}$$

hence, $\delta = \pi$. In this case an ellipse turns into a line:

$$\frac{E_{ox}}{E_o} + \frac{E_{ey}}{E_e} = 0 \Rightarrow E_{ey} = -\frac{E_e}{E_o} E_{ox} . \tag{4.18}$$

The equation (4.18) states that the linearly polarized light after passing of the half wavelength plate remains linearly polarized, but the «minus» sign shows that the oscillations direction is changed by the angle 2φ relatively to the plate main optical plane (Fig. 4.18).
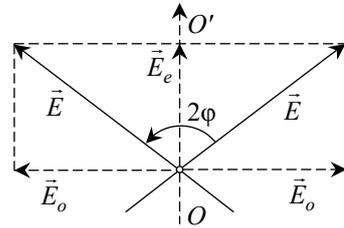
3. *A plate thickness is equal to the wavelength*. Therefore, the path

Fig. 4.18

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difference of o and e rays is:

$$d(n_o - n_e) = \lambda_0, \quad \text{or} \quad d(n_o - n_e) = m\lambda_0, \qquad (4.19)$$

i.e. $\delta = 2\pi$. An ellipse transforms into a straight line:

$$\frac{E_{ox}}{E_o} - \frac{E_{ey}}{E_e} = 0 \Rightarrow E_{ey} = \frac{E_e}{E_o} E_{ox}, \qquad (4.20)$$
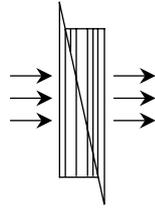

Fig. 4. 19

and a ray passed through the plate remains linearly polarized in the same plane as the falling light. Thus, a wavelength plate does not change orientation of the plane of oscillations of the falling light.

4. *Compensator*. The simplest compensator consists of two quartz wedges (Fig. 4. 19), optical axes of which are parallel. It is possible to move one wedge relatively to the other by a micrometrical screw and change the overall thickness of the plate. Consequently, the difference of phases δ between ordinary o and extraordinary e rays changes from 0 to 2π. As a result, it is possible to see how a character of polarization changes with the growth of the plate thickness and the difference of phases. Direction of polarization changes periodically (Fig. 4. 20).
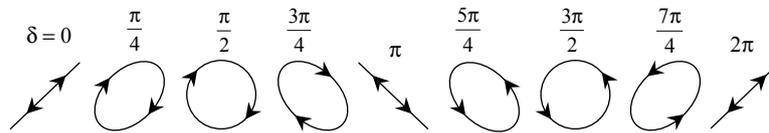

Fig. 4.20

4.3.4. Analysis of polarized light

In practice, there is a necessity to recognize a character of a light polarization. Let us consider the most typical cases.

1. *Linearly polarized light*. During the polarizer rotation, a falling light intensity changes after passing through the polarizer. When the polarizer optical plane becomes perpendicular to the plane of oscillation of the falling light, the light completely disappears after passing through the polarizer. Consequently, the falling light is linearly polarized.

2. *Natural and circle polarized light*. How to distinguish them? One polarizer is not enough for this purpose; a falling light intensity does not change during the rotation of the polarizer in both cases.

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If primarily insert a plate of the fourth part of wavelength thickness, the circle polarized light turns into the linearly polarized light as the plate creates the additional difference of phases $\pi/2$ that compensates the present difference of phases converting it into zero or π. Such method allows converting the circle polarized light into the linearly polarized light that can be fully liquidated by a polarizer. A natural light remains the same after passing the plate and cannot be liquidated by the polarizer.

3. *Elliptic and partly polarized light*. The elliptic polarized light can be detected similarly to the circular polarized light. But unlike to the circular light, the plate optical axis has to coincide with the ellipse main axes of the elliptic polarized ray.

Thus, to distinguish the elliptic and partly polarized light, they have to be taken through the fourth part of wavelength plate and then analyzed by a polarizer. If we can completely liquidate a falling ray by rotation of the plate, it is the elliptic polarized light. If the ray intensity changes after the polarizer but we cannot completely liquidate the ray by the polarizer rotation, it is the partly polarized light.

4.3.5. Interference of the polarized waves

The ordinary *o* and extraordinary *e* rays in the uniaxial crystals *are non-coherent* because they are created by the different wave trains. However, both rays can be coherent if on the way of the natural light we put the polarizer *P* before the crystal plate *K* (Fig. 4.21), so that the falling light becomes *linearly polarized*. But interference cannot be observed during addition of the coherent waves polarized in the mutually perpendicular planes. Therefore, for interference of the ordinary *o* and extraordinary *e* rays they must have one direction; it can be got by passing these rays through the second polarizer (analyzer) *P'*.
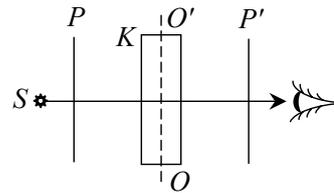

Fig. 4.21

Let us consider interference of the polarized light when an angle between the polarizer *P* and the crystal plate *K* is 45°. In this case, the amplitudes of the ordinary *o* and extraordinary *e* rays are identical; consequently, interference is perfect. Two cases are considered here:

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1. Optical planes of both polarizers are parallel. After the polarizer P the linearly polarized wave is divided by the plate K into the mutually perpendicular waves with the identical amplitudes.

To provide interference, the waves are taken to one direction of the oscillations by the second polarizer P'. Their amplitudes are identical.

$$E'_o = E'_e = E / 2 , \qquad (4.21)$$

where E is the linearly polarized light wave electric field intensity after passing the polarizer P.

Interference of the ordinary o and extraordinary e rays depends on the difference of phases, which they get after passing through the crystal plate. This difference of phases is represented by a vectogram (Fig. 4.22). The amplitude of the resulting wave is:

$$E''^2_{\|} = 2\left(\frac{E}{2}\right)^2 + 2\left(\frac{E}{2}\right)^2 \cos\delta = E^2 \frac{1+\cos\delta}{2} = E^2 \cos^2\frac{\delta}{2} . \quad (4.22)$$
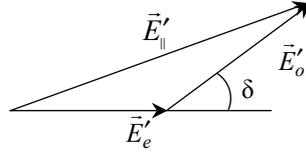

Fig. 4.22

Intensity is proportional to the amplitude squared:

$$I'_{\|} = I \cos^2\frac{\delta}{2} . \qquad (4.23)$$

2. Optical planes of both polarizers are perpendicular. The amplitudes are identical $E'_o = E'_e$ and directed oppositely. It means that after passing the polarizer, the additional difference of phases π appears. Therefore, to find intensity by the formula (4.23), we have to write down $\delta + \pi$ instead of δ:

$$I'_{\perp} = I \sin^2\frac{\delta}{2} . \qquad (4.24)$$

We see from the formulas (4.23) and (4.24) that the intensities $I'_{\perp}$ and $I'_{\|}$ are complementary; it means that their sum equals the light intensity I that passed through the polarizer P. If

$$\delta = 2\pi m \quad (m = 1, 2, 3 \ldots) \tag{4.25}$$

(the plate K thickness is equal to the wavelength), the intensity of light $I'_{\parallel}$ which passed through an analyzer is maximal, and the intensity $I'_{\perp}$ equals zero.

It means that an interference maximum is observed for the parallel polarizers, and an interference minimum is observed for the crossed polarizers. If

$$\delta = \pi(2m + 1) \quad (m = 0, 1, 2 \ldots), \tag{4.26}$$

(the plate K thickness is equal to the half wavelength), the result of the interference is reverse.

The intensity $I'_{\parallel}$ is minimal, and the intensity $I'_{\perp}$ is maximal. Actually, we don't see a classic interference sequence of light and dark sections characteristic for it.

Interference of the ordinary o and extraordinary e rays shows up as the redistribution of the light energy between mutually perpendicular planes.

If the *white light* falls on the optical system (Fig. 4.21), the light color after the analyzer changes during rotation of the polarizer or analyzer.

This phenomenon is explained by the fact that the difference of phases (4.12), which determines the result of interference, depends on the wavelength.

If the thickness d of the crystal plate is different in different places, the values δ are also different. Therefore, during observation of the monochromatic light after the analyzer and such a plate, characteristic system of the dark and light interference sections is observed instead of the homogeneous light.

During observation of the white light (sunlight) after the analyzer and such a plate, the varicolored light is observed. These colors change during the polarizer or analyzer rotation.

4.3.6. Artificial double refraction

Anisotropy under deformation. Zeebek (1813) and Brewster (1815) opened the phenomenon of double refraction in transparent bodies under mechanical deformations. For example, in the case of one-sided compression or extension of a glass plate, it acquires properties of

a uniaxial crystal which optical axis coincides with the direction of compression or extension. Experience shows that difference of indexes of refraction of the ordinary o and extraordinary e rays in perpendicular to the optical axis direction is proportional to normal tension:

$$n_o - n_e = k\sigma, \qquad (4.27)$$

where $\sigma = F/S$ is a mechanical tension; k is a coefficient of proportion that depends on properties of the substance.

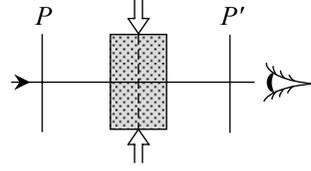The difference can be both positive and negative.

Fig. 4.23

For the double refraction observation, a glass plate is placed between the crossed polarizers $P \perp P'$ (Fig. 4.23). While the glass plate is not deformed, such a system does not transmit light. If the plate is deformed, the system transmits light, at that white light develops into varicolored bars. Every colored bar responses the identically deformed places of the plate.

Such method of research of deformation is used for aircraft constructions; it simplifies control of tensions in new constructions of airplanes.

A Plexiglas model of an airplane is subjected to the tensions similar to the real. Observation of light transmitted through the model gives an opportunity to draw a conclusion about distributing of tensions and their magnitudes in the construction.

Anisotropy under electric field. Kerr effect. Kerr discovered in 1875 that solid and liquid transparent dielectrics become optically anisotropic under strong electric field action. The phenomenon was named *Kerr effect.*

The similar phenomenon was discovered for gases in 1930.

Experiments show that under the action of electric field a liquid in a flat capacitor is polarized and its optical properties become similar to the uniaxial crystals properties if its optical axis is parallel to the electric field intensity vector. The difference of indexes of refraction of the polarized liquid for the ordinary o and extraordinary e rays is proportional to the intensity $\vec{E}$ squared:

$$n_e - n_o = K\lambda_0 E^2, \qquad (4.28)$$

where λ_0 is a wavelength in vacuum; K is a Kerr constant.

It has especially large value for nitro-benzol ($C_6H_5NO_2$, $K = 2.2 \cdot 10^{-12}$ m^{-1}V^{-2}). The Kerr constant $K > 0$ for most materials that is $n_e > n_o$; that responses a positive uniaxial crystal. But there are materials with $K < 0$ (alcohol, ethyl ether). The Kerr constant depends on nature of material, wavelength, and usually quickly diminishes with the temperature increase.

A path difference appears in the Kerr cell on the way l between the rays o and e:

$$\Delta = (n_e - n_o)l = K\lambda_0 lE^2$$

or phase difference

$$\delta = \frac{2\pi}{\lambda_0}\Delta = \frac{2\pi}{\lambda_0}(n_e - n_o)l = 2\pi KlE^2 . \qquad (4.29)$$

Kerr effect was explained by Lanzheven (1910) and Born (1918). Molecules obtain a dipole moment in the direction of the external electric field and reorient so that the dipole moments coincide with the direction of the field. Thus, the material becomes anisotropic.

Kerr effect is almost noninertial. Time of the molecules orientation in the direction of the electric field after its appearance or disorientation after its shutdown is approximately 10^{-10} s and even 10^{-12} s . Therefore, the Kerr cell placed between the crossed polarizers is used as a fast-acting noninertial keyer or a shutter of light. In particular, Kerr effect is used for the management of work of lasers for creation of high-powered ultrashort pulses.

Anisotropy under electric field. Pockels effect (linear electro-optic effect). Some *polar crystals* in the external electric field obtain double refraction properties. At that difference of indexes of refraction of e and o rays is proportional to the first degree of the electric field intensity.

This phenomenon was named *Pockels effect* (1894).

Induced by the field an artificial optical axis is perpendicular to the light direction and the light undergoes double refraction relatively to this axis

$$n_e - n_o = \alpha E , \qquad (4.30)$$

where α is a constant that equals $10^{-11} - 10^{-10}$ m/V .

The potential difference for Pockels effect is approximately ten times less than the potential difference in the case of Kerr effect. It is an important advantage of Pockels effect for practical application. Pockels effect as well as Kerr effect is noninertial.

Linear dependence of the difference of the indexes of refraction on the electric-field intensity (4.30) and relatively small regulatory potential difference give Pockels effect advantage in technical application compared to Kerr effect.

Anisotropy under magnetic field. Cotton-Mouton effect. The analogue of Kerr effect is *Cotton-Mouton effect* discovered in 1910. Without the external magnetic field molecules of a substance are oriented chaotically.

If the substance is placed in the strong magnetic field, the orientation of the molecules magnetic moments appears. As a result, an optically isotropic substance in the magnetic field acquires properties of a uniaxial crystal which optical axis is parallel to the direction of the magnetic field induction $\vec{B}$.

Double refraction under magnetic field has application similar to Kerr effect. The laws of both phenomena are absolutely identical. Dependence of difference $n_e - n_o$ on $\vec{B}$ if light propagates crosswise to direction of the magnetic field induction is:

$$n_e - n_o = C\lambda_0 B^2, \tag{4.31}$$

where C is constant depending on the substance properties.

It is determined experimentally and, for example, for nitrobenzene $C_6H_5NO_2$ $C = 2.25 \cdot 10^{-2}$ $m^{-1}T^{-2}$. This substance in the magnetic field with the induction 1 T creates the difference of phases between e and o rays after passing 1 m distance

$$\delta = \frac{2\pi}{\lambda_0}\Delta = \frac{2\pi}{\lambda_0}(n_e - n_o)l = 2\pi ClB^2 = 0{,}14 \text{ rad}.$$

4.3.7. Rotation of polarization plane

Natural rotation. A lot of substances named *optically active* have a property to revolve direction of polarization of a linearly polarized light. In particular, such properties are characteristic for crystalline substances (quartz, cinnabar, crystalline sugar), clean liquids (turpentine, nicotine),

and also for solutions of optically active substances in inactive solvents (water solution of sugar, glucose).

A phenomenon of rotation of polarization plane was discovered by Arago (1811) during observation of propagation of the linearly polarized light *along the optical axis of quartz plate*. If the linearly polarized light falls on the optical system that consists only of two crossed polarizers $P \perp P'$, the optical system does not transmit the light.

However, if the quartz plate K, cut out crosswise to the crystal optical axis is placed between two crossed polarizers (Fig. 4.24), the system begins to transmit light. Revolving the analyzer P' (Fig. 4.24) can shut down the light and find the angle of rotation of polarization plane by the quartz plate K.
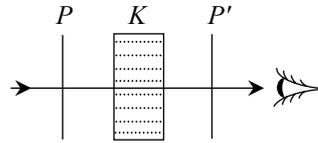
Fig. 4.24

Experiments show that the angle of rotation depends on the path l of light in a crystalline plate

$$\varphi = \alpha l, \qquad (4.32)$$

where α is a *rotation constant*, which is measured in radian per meter or in degree per millimeter.

It significantly depends on a falling light wavelength that means rotatory dispersion of light. Indeed, for a quartz plate in the case of red light $\alpha \approx 15^o / mm$, green $\alpha \approx 27^o / mm$, violet $\alpha \approx 51^o / mm$.

Rotatory property of quartz is caused exceptionally by its crystalline structure, as melted quartz is not optically active. There are distinguished *levorotary and dextrorotatory modifications* of quartz depending on the direction of rotation of the polarization plane. The magnitude of rotation in both cases is identical. All other optically active crystals, liquids, and solutions also exist in two modifications.

The direction of rotation changes if we change the direction of propagation of light (Fig. 4.25). In other words, the direction of rotation (levorotary or dextrorotatory) is «tied» to the ray direction.

Therefore, when the linearly polarized light passes through a quartz plate, reflects on a mirror, and passes through the plate again, the direction of the linear polarization restores. In solutions, the angle of rotation of the polarization plane is determined by *Biot law* (1832)
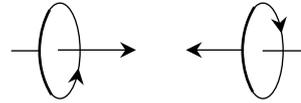
Fig. 4.25

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$$\varphi = [\alpha]cl,\qquad\qquad (4.33)$$

where c is a solution concentration; l is a ray path in the solution; $[\alpha]$ is a *specific rotation constant*, which depends on the wavelength and temperature.

Experiments show that the dependence on the temperature is weak, and the dependence on the wavelength is approximately inversely proportional to the square of the wavelength. For chemically clean liquids $[\alpha] = \alpha/\rho$, where ρ is a liquid density. For solutions it means *the density of active substance* that is its mass in the solution unit volume.

It appears that all optically active substances in the liquid state (including solutions) have the same properties in the crystalline state. But some optically active crystals (quartz) are not active in the liquid state. Consequently, optical activity can be determined both by the structure of molecules and their ordering in the crystal lattice.

If we add the same amount of dextrorotatory molecules to the optically active solution with levorotary molecules, the solution loses optically active properties and becomes the *racemic mixture*.

The dependence of the polarization plane rotation on concentration (4.33) is a basis of methods of determination of concentration of the optically active substances, in particular, sugar (sugar refining) and biological objects (blood).

Fresnel made the first theoretical explanation of rotation of the polarization plane. He assumed that a linearly polarized ray passing through a crystal along the optical axis is separated into two circular polarized rays that move with the different phase speeds and have different indexes of refraction.

Quick rays rotate clockwise in the dextrorotatory crystals, if we look towards the rays; quick rays rotate counter-clockwise in the levorotary crystals.

Fresnel confirmed the hypothesis experimentally. The linearly polarized light is transmitted through a complex prism that consist of the levorotary and dextrorotatory quartz (Fig. 4.26, the optical axes of the three prisms are parallel to the basis *AD*). As a result, he got two spatially divided circular polarized rays. Analysis by means of the $\lambda/4$ plate states that a declined downward ray is dextrorotatory, and a declined upwards ray is levorotary. Hence, Fresnel fixed the

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phenomenon of double circular refraction. These rays are added after the prism and create again the linearly polarized ray but its plane of polarization is turned to the angle φ.
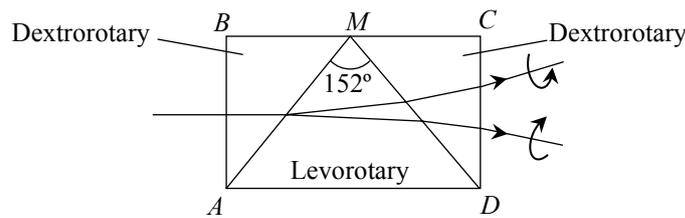

Fig. 4.26

Magnetic rotation of polarization plane. In 1846 Faraday discovered that the optically inactive substances in the strong magnetic field rotate a light polarization plane when the light propagates along the magnetic field direction. Such phenomenon is named *Faraday Effect*.

Experiments of Faraday and then more detailed experiments of Verdet had showed that for a certain wavelength the angle of rotation of polarization plane is proportional to the light path and intensity of the external magnetic field:

$$\varphi = VlH \,, \tag{4.34}$$

where V is a *Verdet constant or specific magnetic rotation*; l *is* a light path in the substance; H is a magnetic-field intensity.

Verdet constant depends on the nature of the substance, the temperature, and the wavelength. The equation (4.34) describes the angle of rotation of the polarization plane for paramagnetic and diamagnetic materials. For these materials values of V are small, that is why the angles of rotation are small too. For ferromagnetic materials the angle of rotation of polarization plane is expressed through the magnetization J.

Thus, in the formula (4.34) H is replaced by J, and a Verdet constant is replaced by a Kundt constant. Very huge rotation of the polarization plane is observed in ferromagnetic materials (Fe, Ni, Co), which thin to such a degree that they are transparent. In particular, for Fe $l = 0.1$ mkm thickness in the magnetic field with intensity $\sim 8 \cdot 10^4$ A/m, the angle of rotation is $\varphi = 2^{\mathrm{o}}$.
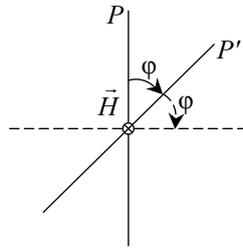
Fig. 4.27

The direction of rotation of the polarization plane is determined by the magnetic field direction and does not depend on the direction of the light propagation. It allows creating the optical valves that let pass light only one-way. It is explained on Fig. 4.27; the polarizers P and P' form the angle $45°$. Natural light passes through the polarizer P and becomes linearly polarized, then the magnetic field rotates the light polarization plane by $\varphi = 45°$ and the light passes through the analyzer P'. After that the light reflects on the mirror surface and passes through the magnetic field again; the field rotates the light polarization plane by $\varphi = 45°$. As a result, the light polarization plane becomes perpendicular to the polarizer P plane and it cannot pass backward.

? **Test Questions**

1. What is the difference between polarized and natural light?
2. What types of polarization of light do you know?
3. Give a definition of linearly polarized light.
4. Give a definition of elliptically polarized light.
5. Give a definition of partially polarized light.
6. What methods of polarization of light do you know?
7. Give a definition of Brewster's angle.
8. Give a definition of Malus law.
9. What is the nature of birefringence?
10. Give a definition of optical anisotropy.
11. Give a definition of the crystal optical axis.
12. Give a definition of the uniaxial crystals.
13. What is the feature of the polarized light interference?
14. How can the artificial anisotropy be obtained?

✓ **Sample Problems**

Problem 1. The intensity of light transmitted through the polarizer and the analyzer is four times less than the intensity of light transmitted through the polarizer. Find the angle between the basic planes of the polarizer and the analyzer. Absorption of light is neglected.

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Solution: Let us denote the intensity of light transmitted through the polarizer by I_0 and the intensity of light transmitted through the polarizer and the analyzer by I_1. According to the Malus law $I_1 = I_0 \cos^2\alpha$. Therefore

$$\cos^2\alpha = \frac{I_1}{I_0} = 0.5 \text{ , } \cos\alpha = 0.25 \text{ and } \alpha = 45°.$$

Problem 2. The beam of the natural light falls on the polished surface of a glass plate submerged in a liquid. The light beam reflected on the plate forms the angle of 97° with a falling ray (Fig. 4.28). Find the liquid index of refraction if the reflected beam is completely polarized.
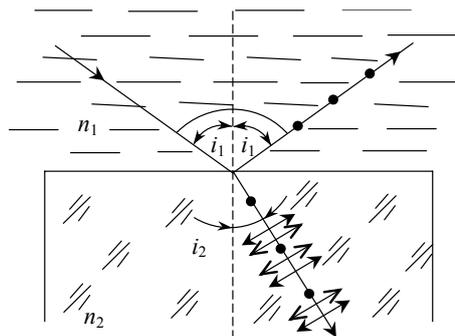

Fig. 4.28

Solution. By Brewster law, reflected on a dielectric light is completely polarized if the incidence angle tangent is:

$$\text{tg}i_{Br} = n_{21},$$

where n_{21} is a relative index of refraction of the second substance (glass) relatively to the first one (liquids).

The relative index of refraction equals the relation of the absolute indexes of refraction of these substances. Thus,

$$\text{tg}i_{Br} = \frac{n_2}{n_1}.$$

According to the problem data and the reflection law, we get:

$$i_{Br} = 97°/2 \,; \quad \text{tg}(97°/2) = n_2/n_1 \,; \text{ hence,} \quad n_1 = \frac{n_2}{\text{tg}(\phi/2)}.$$

Calculations give: $n_1 = 1.33$.

Problems

1. Find the angle between the Sun ray and the horizon when the ray reflected on the surface of a lake is maximally polarized. (37°)

2. A beam of natural light falls on the surface of glycerin ($n = 1.47$). Find the angles of incidence and refraction of the beam if the reflected beam is completely polarized. (55°; 34°)

3. The limit angle of complete reflection of the beam on the liquid-air border is 45°. Find the Brewster angle for the beam falling from the air on the liquid surface. (55°)

4. Find the angle between the main planes of the polarizer and the analyzer if a natural light intensity is diminished by five times after passing the polarizer and the analyzer. (51°)

5. A natural light passes through two polarizers the main planes of which are placed under the angle of 30°. Then the light falls on a mirror and reflects on it. The reflected light passes through the two polarizers again. What is the light intensity after two passing through the two polarizers?