UDC 004.67 (043.2)

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ESTIMATION OF COMPUTATIONAL WORKLOAD ON THE RESOURCES OF UNMANNED AVIATION COMPLEX IN THE PROCESS OF AERODYNAMIC PROBLEMS SOLVING

A mathematical description of the amount of data and computational operations intended for solving aerodynamic problems in real time of flight on the distributed computing resources of an unmanned aviation complex is considered. The given estimation is made for solution based on the large eddy simulation approach.

The requirements to the quality of movement of modern drones may be formulated for each of the stages and modes of flight. In accordance with the formulation of these requirements, the components of the aircraft control algorithms should also be structured. In particular, in the unsteady regimes of flight, the algorithm of finding the vector of control actions may include procedures for ongoing refinement of the aerodynamic coefficients of the object by means of solving the corresponding computational aerodynamic problems. Given the large amounts of computation when the numerical solution of such problems, it is expedient to distribute computation among onboard and ground-based computing resources. Thus, it is necessary to solve the problem of optimal distribution of computational workload, based on known characteristics of unmanned aerial vehicles piloting, such as the distant location of the operator, presence of time lags in communication channels, the probability of temporary interruptions of communication, and other factors that reduce the quality of controlling the object.

An UAC distributed computing system consists of an onboard computer (with the RAM volume $M_{op\ uav}$ [bytes], CPU speed $P_{proc\ uav}$ [operations per second] and a ground computer with the corresponding factors $M^{op\ gnd}$ [bytes], the speed of the processor $P_{proc\ gnd}$ [operations per second]. In the first approximation, the structure of ground area network and the corresponding impact of this structure on the results of solving the optimization problem can be neglected, considering that the amount of memory of the on-ground computing resources, the performance of these computing resources and the bandwidth of communications between ground nodes are significantly higher than the respective values for the UAV on-board computer and the communication channel "surface-to-air."

For formulation of the problem of information processing for the computational aero-dynamic problem in this system it is necessary to assess the amount of operations required at each step of the calculation. Let us consider solving the aerodynamic problem on the basis of the Large Eddy Simulation (LES) approach, which is a mathematical model for turbulence used in computational fluid dynamics, based on low-pass filtering. This operation is applied to the Navier-Stokes equations to eliminate small scales of the solution. This reduces the computational cost of the simulation and makes affordable the computational cost for solving practical hydro- and aerodynamic problems with complex geometry or flow configurations, such as can be met with turbulent jets, pumps and flows around vehicles. In contrast, direct numerical simulation, which resolves every scale of the solution, is quite expensive for nearly all systems with complex geometry or flow configurations. The governing equations are thus transformed, and the solution is a filtered velocity field. The thresholds of filtering, that is, the length and time scales to be considered as "small" and to be eliminated, are selected as the compromise between the requirements of turbulence theory and available computational resources.

The calculations in the workflow of application of this method include the two basic steps:

1) calculation according to the governing equations, which are, in their essence, the partial differential equations governing the flow field $\rho\mathbf{u}(\mathbf{x},t)$.

2) filtering, which mathematically turns out into calculation of convolution integrals.

An LES filter can be applied in the spatial and/or in the temporal field, and thus perform a spatial filtering operation, a temporal filtering operation, or both these operations, correspondingly. In the general form, for an arbitrary scalar field $\varphi(\mathbf{x},t)$ determined over the space $\{\mathbf{x}\}$, the filtered field is defined as

$$\overline{\varphi}(\mathbf{x},t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\varphi(\mathbf{r},t')G(\mathbf{x}-\mathbf{r},t-t')dt'd\mathbf{r} \quad , \tag{1}$$

and may be written in brief as $\overline{\varphi} = G * \varphi$, where $G(\bullet)$ is the filter convolution kernel. Using the provided filter definition, one may split any field up into a filtered and sub-filtered fractions.

For incompressible flow, the continuity equation and Navier-Stokes equations are filtered. The continuity equation may be used in the differential or in the integral form [1]:

$$\frac{\partial\rho}{\partial t} + \nabla\mathbf{j} = \sigma \quad ; \quad \frac{dq}{dt} + \oint\!\!\oint_S \mathbf{j}\, dS = \Sigma \quad ,$$

where: ρ is the volume density of the fluid, calculated as the amount of the mass of the fluid q per unit volume V;

q is the total amount of the fluid in the volume V; $q = \iiint_V \rho dV$;

$\mathbf{j}$ is the flux of q (a vector function which describes the flow of q per unit area and per unit time);

σ is the generation of the matter per unit volume per unit time. Terms $\sigma > 0$ reflect mass generation (production or income) and referred to as "sources"; terms $\sigma < 0$ reflect mass absorption (disappearing, or removal) and referred to as "sinks";

$\oint\!\!\oint_S \bullet\, dS$ denotes a surface integral over a closed surface S that encloses volume V;

Σ is the total income (if positive, in the case of generation) or loss (if negative, in the case of removal) of matter per unit time due to the sources' and sinks' activity in the volume V; $\Sigma = \iiint_V \sigma dV$;

∇ denotes divergence;

t is time.

The filtered continuity equation and the filtered Navier-Stokes equations for the incompressible case have the forms:

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \; ; \qquad (2)$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_i}\left(\overline{u_i u_j}\right) = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + 2\nu \frac{\partial}{\partial x_j} S_{ij} \, , \qquad (3)$$

where $\overline{p}$ is the filtered pressure field, ν is viscosity, and S_{ij} is the rate-of-strain tensor.

The nonlinear filtered advection term $\overline{u_i u_j}$ is the main cause of difficulty in LES modeling. Its evaluation requires knowledge of the unfiltered velocity field, which is unknown, so it must be modeled [2]. In addition, the nonlinearity of the problem leads to interaction between large and small scales, thus preventing separation of scales. The filtered advection term can be split up as:

$$\overline{u_i u_j} = \tau_{ij}^{\gamma} + \overline{u}_i \overline{u}_i \;\; , \qquad (4)$$

so that the filtered Navier-Stokes equations (3) get the form

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_i}\left(\overline{u}_i \overline{u}_j\right) = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + 2\nu \frac{\partial}{\partial x_j} \overline{S}_{ij} - \frac{\partial \tau_{ij}^{\gamma}}{\partial x_j} \quad , \qquad (5)$$

where τ_{ij}^{γ} is the residual stress tensor, which includes all unclosed terms. This stress tensor may be decomposed as

$$\tau_{ij}^{\gamma} = L_{ij} + C_{ij} + R_{ij} \quad , \qquad (6)$$

where L_{ij} (referred to as the Leonard tensor) represents interactions among large scales; R_{ij} (referred to as the Reynolds stress-like term) represents interactions among the sub-filter scales (SFS), and C_{ij} (referred to as the Clark tensor) represents cross-scale interactions between large and small scales [3].

For the governing equations of compressible flow, one should take into account and also filter the equation that describes the conservation of mass. In order to avoid necessity to model the sub-filter scales associated to the mass conservation equation, the Favre density-weighted filtering operation, or Favre filtering, was proposed [4]. This type of filtering is defined for an arbitrary quantity φ as $\overline{\varphi} = \overline{\rho\varphi}/\overline{\rho}$, which, in the limit of incompressibility, becomes the ordinary filtering operation.

The Favre-filtered momentum equation for compressible flow gets the following form [5]:

$$\frac{\partial \overline{\rho}\widetilde{u}_i}{\partial t} + \frac{\partial \overline{\rho}\widetilde{u}_i\widetilde{u}_j}{\partial x_j} + \frac{\partial \overline{p}}{\partial x_i} - \frac{\partial \overline{\sigma}_{ij}}{\partial x_j} = \frac{\partial \overline{\rho}\tau_{ij}^{\gamma}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\overline{\sigma}_{ij} - \widetilde{\sigma}_{ij}\right) \, , \qquad (7)$$

where σ_{ij} is the shear stress tensor; for Newtonian fluids

$$\sigma_{ij} = 2\mu(T)S_{ij} - \frac{2}{3}\mu(T)\delta_{ij}S_{kk} \, .$$

For each step (e.g. for each iteration) of solving the problem in accordance with the enumerated equations, the amounts of data and calculations may be estimated as provided in the Table 1. (Supposed the 3-dimentional problem, the fineness of integration is L, the tensor representing the $\mathbf{u}(\bullet)$ field consisting of N numerical components.)

Table 1

Estimations for the amounts of data and calculations

Equation No.	Data volume	Amount of calculations
(1)	Not less than N	Order of the $(N{\cdot}L)^3$
(2)	N	Not less than $3N$
(4)	$2N$	Not less than $2^3{\cdot}N$
(5)	$4N$	$9{\cdot}12N$
(6)	$4{\cdot}3N$	Not less than $9{\cdot}3N$
(7)	$4{\cdot}9N$	$9{\cdot}17N$

Conclusions

1. An estimation of the amount of computational operations intended for solving aerodynamic problems in real time of flight of an unmanned aviation complex is proposed. The used model supposes using the Large Eddy Simulation approach. Some estimations are obtained in the form «not less than», because the corresponding equations include the terms that need additional evaluation, which may be done in various ways. These ways differ in the mathematical approaches and, respectively, in required computational resources.

2. The possible directions of the further research may include numerical modeling, accounting for the features of an on-ground computing structure, application of the methods of the theory of mass service (queueing theory).

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