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THE ELECTRODIC PROBLEM IN THE CATHODE ACTIVE ZONE

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The solving of near electrodic problem is required for calculation of the thermal flux density supplied with bombarding ions and the active zone irradiation and removed with the emission cooling and also for calculation of the emission current density on a surface of the cathode active zone.

Thus, the heat flux density generated by ions can be determined as

$$F_1 = \mu_1\left(U_k \pm \frac{3}{2}\cdot\frac{kT_i}{m_i} - \Phi_{\text{Њ}}\right)\beta_k j_i\left(R_A, z, t\right). \quad (1)$$

The thermal flux irradiation of the active zone can be found as the irradiation of the average temperature plasma T_{ucp} and as irradiation of the exited atoms which partially emitted energy under relaxation to their basic state:

$$F_{\text{ЏЌ‘\_}} = \sigma T^4_{\bullet\_p}\,\varepsilon_{\bullet} + \tau_{‹} j_e\left(1 - 2\beta_k\right)\sum_{\alpha=1}^{\alpha=n} U_{bk} W_{bk}. \quad (2)$$

Where ε_n - the plasma blackness degree; T_n – the plasma temperature; P_r – the probability of the exited atoms relaxation to their basic state which depends on the active zone length and on the flux parameter of neutral particles; U_{Bk} – the excitation potential of the k level; β_k – the fraction of ionic current on the cathode; j_e – the emission current density.

$$W_{B_k} = \left[\langle n_c\rangle\langle n_a\rangle S_B\left(\frac{8kT_e}{\pi m_C}\right)^{1/2}\left(\frac{U_{B_k}}{T_{e_2}} + 2\right)\exp\left(-\frac{U_B}{T^*}\right)\right] -$$

Is the excitation function, where $\langle n_e\rangle = j_e / eV_e$ – is average concentration of electrons; V_e – the average velocity of electrons, $\langle n_a\rangle = n_э + n_г$;

$$n_э = \frac{V_{\varphi j}}{m_a V_a}, \quad (3)$$

$$n_r = \frac{G_r}{m_r}\cdot\frac{1}{\pi R_A^2(t)}\sqrt{\frac{2}{3}\cdot\frac{m_\alpha}{kT_r(z)}} \quad \text{or} \quad n_r = \sqrt{\frac{12L\eta G}{kT\left(R_{A,z,t}\right)\pi R_A^4}};$$

V_a – the velocity of evaporated atoms of the cathode material; T_a, m_a and $m_г$ – the temperature and mass of the evaporated atom and the atom mass of working substance; n – the gas viscosity; G – the consumption of working substance; $R_A(t)$ – the radius of the hollow cathode, which depends on the time and axial coordinate; T_e – the electrons temperature, T^*_{ea} – the temperature of excitation reaction.

The emissional cooling is characterized by the density of the removed thermal flux

$$F_e = \frac{\Phi_{\text{Њ}}\left(\text{Њ}^3\text{Ѓ}\right)^{1/2} + 2kT_{(R_A, z, t)}}{\text{Њ}} j_e, \quad (4)$$

where $E = \frac{U_k}{l_D}$ - the electric field strength near the cathode surface (l_D – the Debye length).

To determine ionic flux share on the cathode, the value of cathode drop and other parameters of the near cathode layer let us consider the problem in the collisionless and collision (ionizing) layers [4.11]. As the longitudinal dimensions of the near cathode layer close to cathode are mach greater than the radial ones, then weshall deal the unidimensional problem forit. The boundary 1 conforms with the cathode surface ($r = R_A$), the boundary 2 is between the collision and collisionless layers, the boundary 3 is between the collision and gas-dynamic layers.

Under condition that the Larmor radius of ions is much greater than Larmor radius of the electron, and the last one, in its turn is much greater than the Debye screening radius then the elecrodynamic layer can be divided in two: collisionless or the zone of the cathode potential jump and the collision or ionizing zone. Then the equations system, describing the processes in the collisionless zone, can be written down in the form of the Poisson's equation

$$\frac{\partial^2\varphi}{\partial r^2} = \frac{j_e}{\varepsilon_0}\cdot\left(\frac{1}{\upsilon_{ey}} - \frac{\beta_k}{\upsilon_{iy}}\right) \tag{5}$$

and the equations of motion for electrons and ions

$$\upsilon_{Ib}\frac{\partial\upsilon_{Ib}}{\partial r} = -\frac{eE}{m_e}, \qquad \upsilon_i\frac{\partial\upsilon_i}{\partial r} = \frac{eE}{m_i} \tag{6}$$

with boundary conditions:
in the case of

$$y = 0, \varphi = 0, U_{co} = \sqrt{\frac{3}{2}\cdot\frac{kT_e}{m_e}}, \tag{7}$$

and in the case of

$$y = d, \varphi = U_k, E = E_k, U_{iy} = U_{ir} = \sqrt{\frac{3}{2}\cdot\frac{kT_e}{m_i}}, \tag{8}$$

where - $\beta_k = \frac{j_{ik}}{j_{ek}}$ - is a fraction of ionic current on the cathode; U_{ey}, U_{iy} – the transversal velocities of electrons and ions; T – the electron temperature in the cathode material; T_i – the ions temperature on the boundary I; U_k – the cathode potential jump; $\delta_k = l_D$ – the thickness of electrodynamic layer, which is equal to Debye radius.

In the collisional (ionizing) layer the electrons are collided with the neutrals and plasma components of the ionizing layer of the cathodic zone. The electrons are maxwellizing quickly ionizing and excitating neutral atoms and in so doing they lose that energy which they took in the zone of cathode jump.

Then, the equations system for ionizing layer can be reduced to the Poisson's equation, to the balance of particles fluxes and to the energy balance:

$$\frac{\partial}{\partial\rangle}(E) = \frac{\rho_0}{\varepsilon_0};$$

$$\frac{\partial}{\partial x}(n_a;\upsilon) = i_a - r_a = W_a; \tag{9}$$

$$\frac{\partial}{\partial x}\left\{\left(\frac{1}{2}n_a m_a V_{ak}^2 + \frac{3}{2}n_a T_a\right)kV_{ak} + (\pi a_{ik}V_{ai}) + q_{ak}\right\} = e_a n_a E_i V_{ak} + k_a,$$

where α - the plasma components (electrons, ions and neutrals correspondingly): V_{ak} – the K components of the particles velocity of the α - variety as a whole; m_α, n_α, T_α - the mass, concentration and temperature in the energy units of the gas of α - variety; l_a – the gas charge of this variety; $\pi_{a\,ik}$ – the viscous stresses tensor; q_{ak} – the thermal flux which is removed by heat conductivity in particles of the given variety; E – the electric field strength near the cathode; W – the function of ionization – recombination (the velocity of number increase of the ionized particles); K_a – the component taking into account the losses and the energy sources in the gas particles of the α - variety as the result of collisions with particles of other varieties; ρ_c – the volumetric charge density; $\varepsilon_0 = 8.85\cdot10^{-12}\,F/m$.

The boundary conditions to the (9) equations system:
on the boundary 2

$$(n_e V_e) = \frac{I_э}{2}, \qquad \frac{m_e V_e^2}{2} = eU_k, T_e = T_{e_1}; \tag{10}$$

on the boundary 3

$$(n_e V_e) = \frac{I_{e_2}}{2}, T_e = T_{e_2}. \tag{11}$$

The combined solution of equations (5) – (11) makes it possible to obtain the value of cathode potential jump of the electrodynamic layer for compensation of all energy losses and also a value the ions and electrons fluxes on the cathode.

The electrodic problem [equations (5) – (11)] embraces practically all processes passing in the collisionless and collision layer, but represents considerable difficulty even in the case of its computerized solving. That is why we solved the system differential equations (22.31) – (22.35) on the base of the front processes from the combustion theory, i.e. we carry out the linearization of the differential equations.

After reconstruction of energy equation (11) we can obtain the expression for the portion of ionic current on the cathode:

$$\beta_k = \frac{A_0\beta_p + \left(\frac{m_e V_e^2}{2} + \frac{5}{2}kT_{e_2}\right)\frac{\rangle W_e \langle}{1+\beta_p} + \left(eU_k + \frac{5}{2}kT_{e_1}\right)\rangle W_e \langle (1+\beta_p)}{A_0 - \left(\frac{m_e V_e^2}{2} + \frac{5}{2}kT_e\right)\frac{\rangle W_e \langle}{1+\beta_p}} + \tag{12}$$

$$+ \frac{\frac{e}{j_e}\left[(\pi j_k V_{ek}) + qe_R\right]\rangle W_e \langle (1-\beta_p)}{A_0 - \left(\frac{m_e V_e^2}{2} + \frac{5}{2}kT_e\right)\frac{\rangle W_e \langle}{1+\beta_p}},$$

where A_0 = $<k_e>$ - $<E_k j_{ek}>$ - the difference between consumption of energy for collision of electrons with another variety particles and Joule's heating in the ionizing layer (angular brackets <> mean the averaging out of the value over ionized layer);); $q_{eR} = q_{e\,2}$ - $q_{e\,i}$ – the power consumption on the heat conductivity in electronic gas; $(\pi_e j_k V_{ij}) = (\pi_{eik} V_{ej})$ - $(\pi_{eik\_}$ - $V_{ej})$ – the power consumption on the viscosity; β_p – the ionic current share on the boundary with gasodynamic layer.

The expression (12) is general but it contains a lot of values which can be eliminated without the accuracy loss of the ionic current calculation. The following parameters can be related to them: the power consumed on viscosity, heat conductivity, changing of the potential and kinetic energies and also the Joule's heating energy.

Then the expression (12) after simple transformmations will have a form of:

$$\beta_k = \beta_0 + \frac{eU_k \rangle W_e \langle (1+\beta_0)}{e\rangle K_e \langle}, \tag{13}$$

where $$\rangle W \langle = n_e n_a S_i \left(\frac{8kT_e}{\pi m_e}\right)^{1/2}\left(\frac{I}{T_e} + 2\right)\exp\left(-\frac{T_e}{T_{ei}}\right); \tag{14}$$

$S_i = C_i T_{e_r}$ - the ionizing section under the linear approximation of the section dependence from the particle energy;

$T_{ei}^* = \frac{I}{0.865 + 0.602 \cdot 1/U_k}$ - the temperature of the ionization reaction; 1 – the ionization potential;

$$\rangle K_e \langle = \rangle W_e \langle I + \sum_{a=0}^{k} \rangle W_{b_j} \langle U_{b_j};$$

$$\rangle W_{b_j} \langle = kn_e \langle \rangle n_a \langle C_b T_{e_2} \left(\frac{8kT_{e_2}}{\pi m_e}\right)^{1/2}\left(\frac{U_{b_j}}{T_{e_2}} + 2\right)\exp\left(-\frac{U_{b_a}}{T_{b_2}^*}\right) -$$

the velocity of the concentration increase of excited atoms;

$T_{B_2}^* = \frac{U_{B_2}}{0.875 + 0.602 U_{b_k} / U_k}$ - the reaction temperature of the level excitation.

The ionic current fraction on a boundary of the ionizing and gas dynamic layers can be written down with diffusion neglecting as:

$$\beta_p = \frac{\tau_{P_2}}{\tau_{i_2}} \cdot \frac{m_e}{m_i} \cdot \frac{1+(\omega_e \tau_{e_2})^2}{1+(\omega_i \tau_{i_2})^2}, \qquad (15)$$

where $\tau_{e\,2}$ and $\tau_{i\,2}$ – the time of the electron-electron and ion-ion collisions; m_e and m_i – the mass of the electron and ion; $\omega_e\tau_{e\,2}$ and $\omega\tau_{i\,2}$ – the Holl's parameter for electrons and ions.

For low currents and voltages of magnetic field the fraction of ionic current on the boundary with gas dynamic layer will be:

$$\beta_P \le \frac{\tau_{i_2}}{\tau_{e_2}} \cdot \frac{m_e}{m_i},$$

где / where $\frac{1}{\tau_i} = \frac{1}{\tau_{e_i}} + \frac{1}{\tau_{i_a}}, \frac{1}{\tau_e} = \frac{1}{\tau_{e_i}} + \frac{1}{\tau_{e_a}}$. (16)

The time of the electron-atomic and ion-atomic collisions is:

$$\frac{1}{\tau_{e_a}} = \rangle n_a \langle \sigma_{ea} \upsilon_e; \frac{1}{\tau_{ia}} = \rangle n_a \langle \sigma_{i_a} \upsilon_i.$$

The time of the ion-electronic collisions is:

$$\tau_{ie} = \frac{\lambda e_i n_e}{3.5 \cdot 10^4 (T_{e_2})^{3/2}}.$$

An average concentration of the neutral component can be determined as a mean value between concentrations on boundaries and in the case of fulfillment of the neutrals maxwellization condition in the layer we have:

$$\rangle n_a \langle = \frac{V_\Phi j}{m_a \sqrt{\frac{8kT}{\pi m_a}}} + \sqrt{\frac{G_\text{‹} 2L\eta}{kT\pi R_A^4}}, \qquad (17)$$

where $G_г$ – the neutral component consumption through the cathode; πR^2_A – the section over which this consumption is distributed; T – the working medium temperature during its coming out from the cathode which can be taken equal to the cathode temperature.

The (15) – (17) equations system for the collisionless layer after integration and with taking into account the boundary conditions will take the from:

$$E_k = \frac{2U_k}{\Lambda}\left[\sqrt{\frac{m_e U_e^2}{2eU_k}} + \beta_k \sqrt{\frac{m_i}{m_e}\left(1+\frac{m_i V_i^2}{2eV_i}\right)}\right]^{1/2}, \qquad (18)$$

where

$$\Lambda^2 = \frac{(1+\beta_k)\varepsilon U_k}{j_k}\sqrt{\frac{2eU_k}{m_e}};$$

$$U_{e_0} = \sqrt{\frac{3}{2}\frac{kT_e}{m_e}}; \; U_i = \sqrt{\frac{3}{2}k\frac{T_i}{m_i}}.$$

The temperature of ions on a boundary of the ionizing and collisionless layer can be taken equal to the temperature of atoms $T_{e\,i} = T_a$.

The balance of particles in the ionized layer (18) makes it possible to obtain the expression for the ionizing layer thickness δ_φ required for calculation of neutral particles concentration on a boundary with the gas dynamic layer $n_{a\,2}$:

$$n_{a_2} = n_a \exp\{-\alpha\delta_{\varphi\varphi}\};$$

$$\delta_{\varphi} = \frac{(\beta_k - \beta_P)j}{e(e + \beta_P)(1 + \beta_k)\rangle W_e \langle}, \qquad (19)$$

Is a layer thickness taken from the equation (18).

The ionization coefficient can be found with a such way as:

$$\alpha = \sqrt{\frac{\left[n_e S_i \left(\frac{8kT_e}{\pi m_e} \right)^{1/2} \left(I/T_{e_2} + 2 \right) \right] \exp\left\{ - \frac{I}{T^*_{e_i}} \right\}}{D_a}},$$

where $D_a = \frac{1}{3} \lambda_{\Sigma} \overline{V_a}$ - the diffusion coefficient; $V_a = \sqrt{\frac{8kT_{a_2}}{\pi m_a}}$ - the velocity of the working medium maxwellized atoms; $\lambda_{\Sigma} = \frac{1}{\rangle n_a \langle \sigma_{\Sigma}}$ - the smallest length of the free path related to the electron-atom and ion-atom collisions; $\langle n_a \rangle$ - the mean concentration of the particles proper variety; σ_{Σ} - the section related to the collisions of appropriate variety .

To close the equations system (1) – (19) we use the equation of the cathode power balance:

$$- F_{\square\text{Њ“.}} + F_j \pm F_{\text{ђ„,}} - F_{\text{Џ–.}} - F_e + F_{\text{ЏЌ‘.}} - F_{\text{ЏЌ‘}} \pm F_k = 0, \qquad (20)$$

where F_i, F_e, $F_{\text{изл п}}$ depend from U_k.

By the solution of this equation we can obtain the value u_k on the each next step. At the calculation beginning a value of the cathode potential drop is equal to o,8 u_0 (u_0 – the voltage on electrodes after a break-down of the cathode-anode path).

The system of equations makes it possible to obtain the ionic current fraction, cathode drop and other parameters of the cathode layer.

Reffereses

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