AVIATION IN THE XXI-ST CENTURY

NATIONAL ACADEMY OF SCIENCES OF UKRAINE
MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL AVIATION UNIVERSITY

PROCEEDINGS

THE SEVENTH WORLD CONGRESS "AVIATION IN THE XXI-st CENTURY"

"Safety in Aviation and Space Technologies"

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1.12. Computer systems

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Methods of spherical data modeling overview

The paper presents the basic features of spherical data: application area, visualization techniques, coordinate system types, the main distributions, methods of simulation. The results of the random vectors modeling are given in polar coordinates.

Introduction

The measuring data can be represented in several forms: linear, circular, and spherical. Thus, the directional data analysis is the important scientific and technical part of the measurement theory. The spherical observation results, or data samples in the form of directions in space, is applicable for such fields as: navigation, astronomy, geodesy, cartography, geology, geophysics, meteorology, physical oceanography, mathematics, crystallography, animal physiology, bioinformatics, text analytics, etc. The examples of the vectorial data processing potential tasks are: risk management, the determination of the density of wind direction, hotline phone calls, earth temperatures, or earthquakes, etc.

Random vectorial data modeling is the important stage of data origination for random vectors statistical experiment. The simulation of the samples on the sphere of unit radius can be useful for evaluating a new statistical procedure, assessing the variability of an estimate from data [1-6].

Spherical data representation

Let ψ to be the arbitrary vector with a direction to (α, β). The spherical observation results used to be described as a collection of points $P_1, \ldots, P_n$, $n = 1, 2, \ldots, N$ on the surface of the unit-radius sphere centered at the origin O of the three-dimensional coordinates x, y and z, where P_i can be identified with a unit vector $\overrightarrow{OP}$. P_i is corresponding to $\overrightarrow{OP}$ by polar coordinates (θ_i, φ_i) and the direction cosines [1]:

$$x_i = \sin\theta_i \cdot \cos\varphi_i,$$
$$y_i = \sin\theta_i \cdot \sin\varphi_i, \qquad (1)$$
$$z_i = \cos\varphi_i,$$

where $i = 1, \ldots, n$; $0 \le \theta \le \pi$, $0 \le \varphi < 2\pi$.

The spherical coordinates definition depends on coordinate system and summarized in the Table 1.

Depending on the task purpose and the scope of application spherical data visualization techniques can be divided into: polar or equatorial projections (Wulff, or “stereographic”, projection; Lambert, or Schmidt, projection; orthographic projection; gnomonic or central projection), sunflower plots, density contour and shade plots, directional/spatial plots.

Table 1

Spherical coordinate systems

Spherical coordinate system	Angles definition
1. Polar coordinates	Colatitude, longitude
2. Geographical coordinates	Latitude, longitude
3. Geological coordinates	Strike azimuth, dip azimuth
4. Astronomical coordinates:	
4.1 The horizon system	Azimuth, altitude (elevation angle)
4.2 The equatorial system	Declination, right ascension
4.3 The ecliptic system	Celestial longitude, celestial latitude
4.4 The galactic system	Galactic longitude, galactic latitude
4.5 The super-galactic system	

The major spherical distributions include: the point distribution, the uniform distribution on sphere, the Fisher distribution, the Watson distribution, the Kent distribution, the Wood distribution, the Bingham distribution, the Arnold distribution, the Selby distribution, the Kelker-Langenberg distribution, the General Fisher-Bingham distribution, the Fisher-Watson distribution, the Bingham-Mardia distribution, the Saw distribution, the Mardia-Gadsden distribution.

Methods of spherical data modeling

It is possible to formulate the following ways for unite vectors modeling:

1. Using the routine technique for samples of θ and φ pseudo-random variates modeling described in [1]. This way is the most suitable, but only in case if the technique for specified distribution is available.

2. Using the marginal probability density functions $f(\theta)$ and $f(\varphi)$ of each polar coordinate, θ and φ, with the next inverse function method application.

The method of spherical data modeling by usage of the marginal probability density functions of polar coordinates includes the following steps:

1) obtaining the cumulative function of the $f(\theta)$;

2) application of the inverse function method [2] to the cumulative function for modeling the sample of θ-coordinates $\theta_1, \ldots, \theta_n$, $n = 1, 2, \ldots, N$, $0 \le \theta \le \pi$;

3) execution of steps 1-2 for modeling the sample of φ-coordinates $\varphi_1, \ldots, \varphi_n$, $n = 1, 2, \ldots, N$, $0 \le \varphi < 2\pi$ [1-4];

4) rotation of the set of modeled coordinates (θ_i, φ_i) directed in (0, 0) to the specified direction (θ, φ) with a rotation matrix given in [1].

The imperfection of the method is in the particularity of the marginal functions definitions.

3. Using the probability density function $f(\theta, \varphi)$ of the arbitrary vector $\psi(\alpha, \beta)$. The approach provides modeling unite vectors samples by given probability and its parameters, such as location and shape parameters. The disadvantage of the approach is in lack of methodical and engineering-software assurance. The issue of spherical data modeling directly by its probability density function is perspective for future scientific-engineering researches.

Modeling results

Based on the method of spherical data modeling by usage of the marginal probability density functions of polar coordinates the samples of points on the unit sphere are modeled for the Fisher (Figure 1) and the Watson (Figure 2) distributions in the MatLab software.

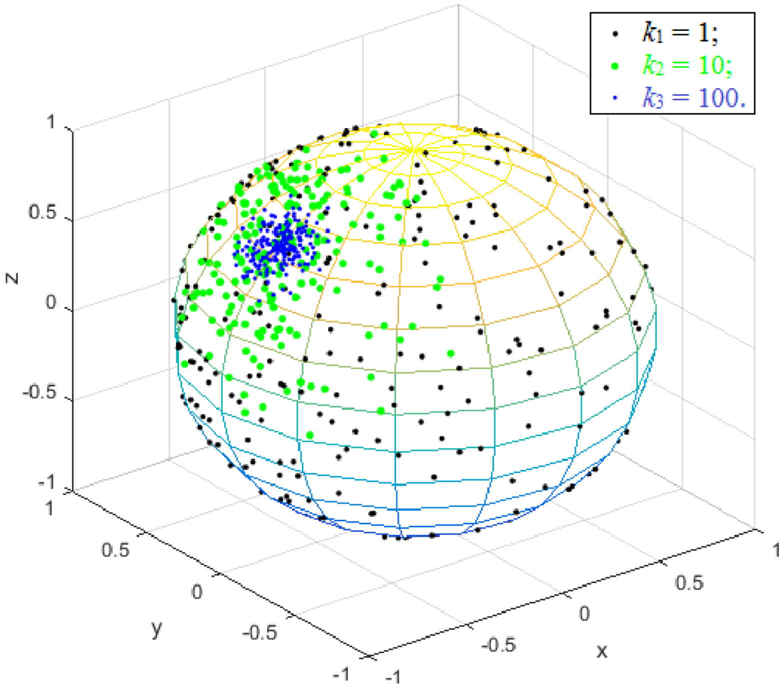

Figure 1 – Points sampled from three Fisher distributions on the sphere

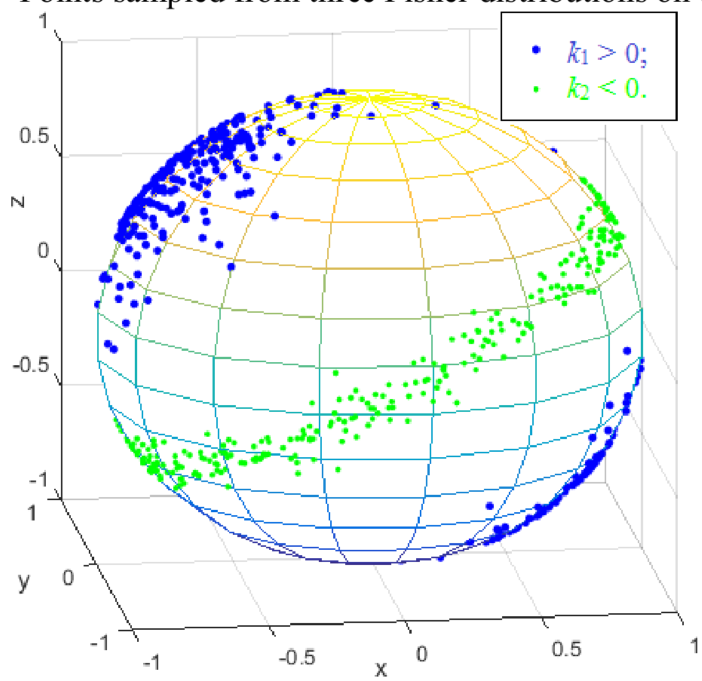

Figure 2 – Points sampled from two Watson distributions on the sphere

The probability density function of the Fisher distribution is [1]:

$$f(\theta,\varphi) = C_F e^{k(\sin\theta\sin\alpha\cos(\varphi-\beta)+\cos\theta\cos\alpha)}\sin\theta\ ,\ 0 \le \theta \le \pi,\ 0 \le \varphi < 2\pi, \qquad (2)$$

where α and β are location parameters, the distribution having rotational symmetry about the direction (α, β); $k \geq 0$ – a shape parameter called the concentration parameter, since the large value of k the more the distribution concentrated towards the direction (α, β), when $k = 0$ the distribution is uniform on the sphere; C_F – the normalization constant: $C_F(k) = \dfrac{k}{4\pi \sinh k} = \dfrac{k}{2\pi(e^k - e^{-k})}$.

The probability density function of the Watson distribution is [1]:

$$f(\theta,\varphi) = C_W e^{k(\sin\theta \sin\alpha \cos(\varphi-\beta) - \cos\theta\cos\alpha)^2} \sin\theta,\ 0 \leq \theta \leq \pi,\ 0 \leq \varphi < 2\pi. \quad (3)$$

The distribution has a rotational symmetry about the direction (α, β); the normalization constant is $C_W(k) = \left(4\pi \int_0^1 e^{ku^2} du\right)^{-1}$. When the concentration parameter $k > 0$, the distribution is bipolar, when $k < 0$ – girdle or equatorial. The large value of |k| the more the distribution is concentrated in the bipolar case round the poles (α, β) and (π - α, β ± π), the more it is concentrated in the girdle case round the great circle in the plane normal to (α, β).

Conclusions

1. The basic features and the methods of simulation of spherical data are described.
2. The results of spherical data modeling from the Fisher and the Watsons distributions are presented on the unit sphere.
3. The perspectivity of the spherical data modeling method development by the probability density function of the arbitrary vector application is justified.

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