Method of measurement of stress in a loaded structures

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Abstract
The method of penalty functions is analyzed. Methodology of finding of optimal parameters of complex loaded components is used. In this report also there described how to determine the directions of principal strain tensor axes, major strains and stresses by measuring deformations in complex loaded details.
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Introduction

Strains and stresses during full-scale trials of various machines, construction vehicles and other products (springs, brackets, levers) are measured through a widely used method based on using discrete metal and semiconductor strain gauges. A specific feature of complex details testing is the presence of a large number of strain-gaging points. Therefore, for testing purposes we use the multichannel strain-gauge station, information and measuring system. This will enable the accuracy of the testing.

It should be noted that practical calculations on optimization of parameters of complex loaded components require a number of different algorithms. In the tasks where many local solutions are expected you can use different combinations of calculation methods for finding the global minimum functional. The feasibility of using such combinations is dictated by the nonlinearity of the problem matter [1-3].

The purpose is to develop measuring method for strain and stress in difficult parts of loaded cars and aircraft based on the penalty function method.

Statement of basic material

It is established that in case where the objective function is continuous and the acceptance region forms a closed set then determining of optimal design parameters of the parts is not sufficient. All settings of loaded complex parts presented in the form of restrictions are divided into intervals. Each interval corresponds to a segment. The penalty function method is used for nonlinear system limits. In the penalty function method task to constrained minimum is replaced by a sequence of tasks to the absolute minimum.

It is necessary to minimize the function $f(k_i)$, $i=1,\dots,n$, in the presence of constraints $g(x)=0$. We will minimize this function without conditions. However on every attempt where restrictions arise we will pay a fine. The easiest method of penalty functions is as follows [3]. We construct a sequence of points x^k, k/1 satisfying the equation:

$$F(X,R)=f\left(X^k\right)\pm\lambda_k g\left(x^k\right)=\min\left\lfloor f(X)\pm\lambda_k g^2(X)\right\rfloor<\varepsilon \quad (1)$$

where $\lambda_k \to \infty$ at $k \to \infty$; $\lambda_k g^2(X)$ - is the simplest penalty function; ε - required accuracy of calculations.

The values λ_k can be interpreted as relative unit fines for violation $(g(X)=1)$ of restrictions, i.e. is the barrier function. When minimizing this penalty we prevent a deviation from the border. This border allows a deviation to the both sides in the region: a (+) and a (-). We also use all restrictions, accuracy of solution and the procedure calculation of functional $F(X,R)$.

The proposed method of finding the optimal parameters of complex loaded parts may be carried out in 4 stages:

Step 1. Call up the subroutine for calculating the penalty function.

Step 2. Start an iterative process that is aimed to construct a set of penalty functions and penalty parameters.

Step 3. Solve linear equations to determine the current value of variable functional. In case of nonlinear effect, one should linearize.

Step 4. Check the used constraints where the values of variables fully fall into the limitation. If the value of functional deviates from previous calculations by the minimum value ΔKi the solution is found. If not, then go to the next iteration (successive approximation). All subsequent processes are to be repeated.

The next step will be to calculate new coefficients. Then we construct the penalty factors and the finally we construct the penalty functions. This provides the appropriate level of problem relaxation by using a particular algorithm [4-6].

To calculate the amplitude-response characteristics we assume an idealized road, considering its microprofile and harmonic current. We also assume that distribution coefficient ξy=1.

The relative displacement values are defined via the formula [5, 6]

$$\frac{Z_a}{q_0}=1+Z_\nu\cdot\sin(\nu\cdot t+\varphi_\nu), \quad (2)$$

where ν - frequency of operations exciting force.

The relative amplitude of oscillation at the component test

$$Z_\nu=\omega_0\cdot\sqrt{\frac{4\psi_0^2\cdot\nu^2+\omega_0^2}{\left(\omega_k^2-\nu^2\right)^2+4\psi_0^2\cdot\omega_0^2\cdot\nu^2}}\cdot\omega_k^2\cdot\sqrt{\left(\omega_k^2-\nu^2\right)^2+4\psi_k^2\cdot\omega_k^2\cdot\nu^2}, \quad (3)$$

where ω0, ωk – are partial frequencies of element.

$$\omega_0=\sqrt{\frac{C_{\text{пр}}}{m}}, \quad (4)$$

where $C_{ïð}=\frac{C_ð\cdot C_ø}{C_ð+C_ø}$ –element stiffness; C_p, $C_ш$ – spring stiffness; m - mass of element; $\psi_0=\frac{h_0}{2\cdot\omega_0}$ and $\psi_k=\frac{h_k}{2\cdot\omega_k}$ - relative damping factor of oscillations when parts testing respectively; $h_0=\frac{k_0}{m_{ïä}}$, $h_k=\frac{k}{m_{ïä}}$ - suspension factor of resistance;

k - factor of resistance of inelastic suspension.

Phase angle is calculated

$$\varphi_k=arctg\left(\frac{\omega_0\left(\omega_0^2-\nu^2+4\psi_0\nu^2\right)^2}{2\psi_0\nu^3}\right). \quad (5)$$

We know that $t=\frac{Z\pi}{\nu}$ then the equation (1) can be presented in the following form:

$$\frac{Z_a}{q_0}=1+Z_\nu\cdot\sin(2\cdot\pi+\varphi_\nu). \quad (6)$$

The vibration speed and vibration acceleration can be found by differentiation of equation (6):

$$\frac{\dot{Z}_a}{q_0}=Z_\nu\cdot\nu\cdot\cos(2\cdot\pi+\varphi_\nu), \quad (7)$$

$$\frac{\ddot{Z}_a}{q_0}=-Z_\nu\cdot\nu^2\cdot\sin(2\cdot\pi+\varphi_\nu). \quad (8)$$

In order to ensure the minimum weight of complex details' an experimental verification of the structures is required.

The verification can identify real stresses in the load-bearing parts in various types of loads. Strain gauges were created for this purpose. However, direct stress measurement is impossible. During experiments the tension in research facilities can only be determined by a direct measurement of deformations occurring in these facilities. Stresses are indirectly identified by measuring deformations using the elasticity theory, which links components of strain and stress [4-7].

The easiest way to achieve this result is to determine stresses in uniaxially (linearly) stressed element of the research object, where the dependence of normal stresses σ from the normal strain ε is described by Hooke's law

$$\sigma = \varepsilon E, \tag{9}$$

where E - spring constant of the material studied element object.

The results of the stress and strain measurements in deformed loaded elements are found using generalized Hooke's law, which has the following form in the main axes of stress tensor:

$$\left.\begin{aligned} \varsigma_I &= [\sigma_I - \mu(\sigma_{II} + \sigma_{III})]/E \\ \varsigma_{II} &= [\sigma_{II} - \mu(\sigma_{III} + \sigma_I)]/E \\ \varsigma_{III} &= [\sigma_{III} - \mu(\sigma_I + \sigma_{II})]/E \end{aligned}\right\} \tag{10}$$

where μ – Poisson's ratio of the material element; σ_1, σ_2, σ_3 - the major stress in element subject of research; ς_1, ς_2, ς_3 - major strain in element [8-9].

The main tensions take the following form when main changes in the tension of the main strains are altered:

$$\left.\begin{aligned} \sigma_I &= \frac{E}{1-\mu^2}\left[\left(\varsigma_I + \mu\varsigma_{II}\right)\right] \\ \sigma_{II} &= \frac{E}{1-\mu^2}\left[\left(\varsigma_{II} + \mu\varsigma_I\right)\right] \end{aligned}\right\}. \tag{11}$$

Value (11) shows that the problem of determining the normal stresses is reduced to finding the major strains in them. In general, in the load-bearing elements neither the directions of the principal axes of the tensor strain, nor the main strain are unknown. In deformation theory it is believed that the main axis of strain and stress tensor coincide. To determine the directions of the principal axes of the deformations tensor and the main strain on the surface of the object we install a device ("tenzorozetka") consisting of three strain gauges. The axes of the gauges are located at 45° [10].

If we accept that the unspecified axes X and Y are the main axes of the strain tensor and that there is an analogy between the theories of stresses and strains, then we can determine the strain on sloping sites using the formula

$$\varsigma_\alpha = \varsigma_x \cos^2\alpha + \varsigma_y \sin^2\alpha, \tag{12}$$

where ς_x, ς_y – are major strains on the surface of the subject of research.

Based on the formula (12) we may find the strains in directions of installed strain gauges on the surface of the socket. Experimental studies are conducted for the lower arm front suspension of the car

$$\left.\begin{aligned} \varsigma_1 &= \varsigma_\alpha = \varsigma_x \cos^2\alpha + \varsigma_y \sin^2\alpha \\ \varsigma_2 &= \varsigma_{(\alpha+45^\circ)} = \varsigma_x \cos^2(\alpha+45^\circ) + \varsigma_y \sin^2(\alpha+45^\circ) \\ \varsigma_3 &= \varsigma_{(\alpha+90^\circ)} = \varsigma_x \cos^2(\alpha+90^\circ) + \varsigma_y \sin^2(\alpha+90^\circ) \end{aligned}\right\}. \tag{13}$$

After conversion into (13) we obtain

$$\left.\begin{aligned} \varsigma_1 &= \varsigma_x \cos^2\alpha + \varsigma_y \sin^2\alpha \\ \varsigma_2 &= \frac{\varsigma_x + \varsigma_y}{2} - \frac{\varsigma_x - \varsigma_y}{2}\sin 2\alpha \\ \varsigma_3 &= \varsigma_x \sin^2\alpha + \varsigma_y \cos^2\alpha \end{aligned}\right\}. \tag{14}$$

The ratio for major strains is found via

$$\left.\begin{aligned} \varsigma_x &= \frac{\varsigma_1 + \varsigma_3}{2} + \sqrt{\left(\varsigma_1 - \varsigma_3\right)^2 + \left(\varsigma_1 + \varsigma_3 - 2\varsigma_2\right)^2}\,/\,2 \\ \varsigma_y &= \frac{\varsigma_1 + \varsigma_3}{2} + \sqrt{\left(\varsigma_1 - \varsigma_3\right)^2 + \left(\varsigma_1 + \varsigma_3 - 2\varsigma_2\right)^2}\,/\,2 \end{aligned}\right\}. \tag{15}$$

The main stress on the surface of the object of study:

$$\left.\begin{aligned} \sigma_x &= \frac{E}{1-\mu^2}\left(\varsigma_x + \mu\varsigma_y\right) \\ \sigma_y &= \frac{E}{1-\mu^2}\left[\left(\varsigma_y + \mu\varsigma_x\right)\right] \end{aligned}\right\}. \tag{16}$$

Therefore "tenzorozetka" is used for many purposes:

- determination of the direction of the tensor strain principal axes;
- determination of principal strain;
- determination of the main stress measured by strain method.

Using the determinations stated above we get to a formula of identification the direction of the main strain tensor axes and a formula for major strains [11, 12]

$$\varsigma_{x,y} = \frac{\varsigma_1 + \varsigma_3}{2} \pm \frac{1}{\sqrt{2}}\sqrt{\left(\varsigma_1 - \varsigma_2\right)^2 + \left(\varsigma_3 - \right.} \tag{17}$$

Studies show that formulas (15) and (17) are not equivalent. However, the accuracy of the solution is doubted when we identify the main strain using (15) and (17). Note that (12) is taken under condition of complete analogy between the theories of stress and strain. When we have an uniaxial loading element the formula takes the form presented below:

$$\varsigma_\alpha = \varsigma_x \cos^2\alpha. \tag{18}$$

According to the stresses theory, normal stresses for uniaxial loaded elements on sloping sites is defined as

$$\sigma_\alpha = \sigma_x \cos^2 \alpha, \quad (19)$$

The results of this formula are confirmed experi-mentally.

Formula (18) when used in the strains theory to determine strains for uniaxial loaded elements on sloping sites does not produce the results that can be experimentally confirmed. In this case we should use:

$$\varsigma_\alpha = \varsigma_x \cos^2 \alpha. \quad (20)$$

However, uniaxial load element strain tensor can be represented by the formula

$$\varsigma_\alpha = \varsigma_x \cos^2 \alpha - \mu\varsigma_x \sin^2 \alpha = \varsigma_x \left(\cos^2 \alpha - \mu \sin^2 \alpha\right), \quad (21)$$

The formula corresponds to the boundary conditions. A strain of a complex loaded element on sloping sites is determined by formula (22) rather than (12)

$$\varsigma_\alpha = \varsigma_x \left(\cos^2 \alpha - \mu \sin^2 \alpha\right) + \varsigma_y \left(\sin^2 \alpha - \mu \cos^2 \alpha\right). \quad (22)$$

Conclusion

In the biaxial loaded elements the geometric surface model of stresses and strains is the sum of uniaxial surface models of stresses and strains. It has been found that in the directions of the tensor strain axes we get tensions and accompanying strains:

$$\varsigma_x = \varsigma_I - \mu\varsigma_{II},$$
$$\varsigma_y = \varsigma_{II} - \mu\varsigma_I. \quad (23)$$

There are two types of strain tensor: mixed tensor with major strains ς_x, ς_y containing the main tensed strain ς_I, ς_{II} and related strains $\mu\varsigma_I$, $\mu\varsigma_{II}$; as well as stress tensor strain containing only the main strains ς_I, ς_{II}.

Stress tensor strain is determined using formula (20), if $\mu = 0$ and if we replace mixed main strain ςx, ς_y by tensed strains ς_I, ς_{II}

$$\varsigma_{I,II} = \frac{(\varsigma_1 + \varsigma_3)}{2} \pm \frac{1}{\sqrt{2}} \sqrt{\left(\varsigma_1 + \varsigma_3\right)^2 + \left(\varsigma_1 + \varsigma_3 - 2\varsigma_2\right)^2}. \quad (24)$$

It must be emphasized that this major tension will be put as follows:

$$\left.\begin{aligned} \sigma_I &= \varsigma_I E \\ \sigma_{II} &= \varsigma_{II} E \end{aligned}\right\}. \quad (25)$$

Thus, in practice, the directions of the principal the tensor strain axes are often known in advance. It has been proven that in such cases the major strain ς_x and ς_y can be found by setting up gauges in these directions or by using “tenzorozetka”. The main stresses are found via formulas (15) at $\sigma_I = \sigma_X$ and $\sigma_{II} = \sigma_Y$..

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