THEORY AND METHODS OF SIGNAL PROCESSING

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VERNIER ALGORITHM OF ADAPTIVE FILTRATION, SMOOTHING AND PREDICTION OF TRAJECTORIES

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Abstract—*Algorithms of signal filtering and extrapolation methods are under consideration. Brown filter as alternative for Kalman filter is proposed for noise elimination. Block diagram of adaptive filtration, smoothing and prediction of trajectory is introduced. It is shown, that proposed algorithm allows obtaining higher quality of filtration, almost without noise.*

Index terms—Extrapolation methods; filtering; prediction; smoothing.

I. INTRODUCTION

Nowadays digital signal processing is influential with such key branches as aviation, medicine, telecommunications, digital sound recording and others. Digital processing is fundamental for the most developments and applications. Noise cancellation is one of the popular areas in digital processing. Noise is a parasite signal, which appears during transmission or measurement of valid signal. Distortions and restrictions are the main limiters for obtaining of correct data. That is why elimination of noise is very important during development of aviation, telecommunications systems and etc.

Additive noise could be represented as

$$x(t) = s(t) + N(t),$$

where $x(t)$ is the noisy signal; $s(t)$ is the valid signal; $N(t)$ is the noise component, independent on valid signal, and noise elimination is necessary. There is a variety of filtering algorithms. Kalman filter is the most popular among them; it is applied in different areas of science and technology. It is simple and effective.

Filter is an algorithm of data processing, which eliminates noises and unwanted information. There is a possibility in Kalman filter to set a prior information about system character, dependence between variables and on the base of it to build more accurate estimation. But even in the simplest case (without input of a prior information) it shows great results.

Kalman algorithm consists of two repeated phases: prediction and correction. At the first stage, the prediction of the state in the next moment of time is computed (taking into account inaccuracy of their measurements). At the next stage, new information from the sensor adjusts predicted value (taking into account inaccuracy and noisiness of this information).

Disadvantage of famous Kalman filter is low carrying capacity and high sensitivity to inaccuracies of statistic description of measurements in filtering channels.

In this article Brown filer for noise elimination is proposed to use.

II. PROBLEM STATEMENT

The task is the development of noise elimination system. It is assumed, that signal $x(t)$ with the noise $N(t)$ is measured. It is known prior that precise value $x^*(t)$ of signal $x(t)$ is low-frequency function compared with noise $N(t)$. Statistic characteristics $x^*(t)$ and $N(t)$ are non-stationary and unknown. Having such constrain information it is not possible to construct optimal filter prior. It could be concluded, that filter has to be low-frequency. Following detailing is possible taking into account the main (relatively to the filter) task and choosing the structure and parameters of filter from conditions of optimality of the main task solution: identification of object – updating of missed measurements. Application of vernier principle of structure growth and adaptation simplifies the task of filters synthesis in the given statement and allows draw closer to optimal solution.

III. REVIEW OF EXISTING EXTRAPOLATION METHODS

The base of extrapolation methods is assuming respectively to the considering process. The sense of this assuming is that the process of changing of dependent variable, represented as sum of two components – regular and random one

$$y(t) = f(\mathbf{a}, t) + \eta(t).$$

It is assumed, that regular component $f(\mathbf{a}, t)$ is smooth function, which is described by fi-

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nite-dimensional vector **a**, which stores values on the period of extrapolation of prediction.

This component is called trend, level or determined base of the process. Random component $\eta(t)$ usually is non-correlated random process with mathematical expectation, equal to zero. Estimates of this component are needed for determination of prediction.

Extrapolation methods of prediction identify the best, in some sense, trend description and determine the values, which are predicted, by means of extrapolation of trend.

There are such methods – method of moving average and method of exponential smoothing [1]. In case of moving average method the smoothing is performed by means of polynomials, which approximate the groups of researched points by the least square method. The best smoothing is achieved for middle points, so desirable to choose in the group, which is smoothed, odd number of points.

General formula of exponential smoothing is

$$S(t_0) = \alpha y(t_0) + (1-\alpha)S(t_0 - 1),$$

where α is smoothing constant. Let's apply once more the procedure of smoothing to the values of function y, which have been smoothed. And obtain the smoothing function of second order.

$$S^{(2)}(t_0) = \alpha S(t_0) + (1-\alpha)S^{(2)}(t_0 - 1);$$

$$S^{(3)}(t_0) = \alpha S^{(2)}(t_0) + (1-\alpha)S^{(3)}(t_0 - 1);$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$S^{(n)}(t_0) = \alpha S^{n-1}(t_0) + (1-\alpha)S^{(n)}(t_0 - 1).$$

Since for all cases of exponential smoothing $0 \le \alpha \le 1$, then current estimate of function, that smoothes, at the moment t_0 is equal to linear combination of values of function y in all points of the series from 0 to t with weights, which exponentially decrease to initial points of the series.

With the purpose of implementation of exponential smoothing for prediction the description of trend in the most general case of power polynomial is applied:

$$f(\mathbf{a},t) = a_0 + a_1 t + \frac{1}{2!}a_2 t^2 + \ldots + \frac{1}{n!}a_n t^n .$$

According to Brown theorem the coefficients a_k of this polynomial could be expressed through smoothing functions of different order of initial numerical series. Then the task is closed to calculation of values of smoothing function $S(j)$, $j = \overline{1, n+1}$ and, through their linear combinations, to calculations of polynomial coefficients. According to this theorem the smoothing function of p -th order in the moment t could be expressed

$$S^{(p)}(t) = \sum_{k=0}^{n} (-1)^k \frac{y^k}{k!} \times \frac{\alpha^p}{(p-1)!}\sum_{j=0}^{\infty} j^k \beta^j \frac{(p-1+j)!}{j!}. \quad (1)$$

In matrix form it is possible to write

$$\mathbf{S}(t) = \mathbf{A}\mathbf{a}\,, \quad (2)$$

where $\mathbf{S}(t)$ is the vector of values of the smoothed process, that contains orders from 1 to p; $\mathbf{a} = \left[a_0(t), a_1(t), \ldots, a_n(t)\right]$ is the vector of unknown coefficients, which are equal to the derivatives of the process of correspondent orders; **A** is the matrix of dimension $p \times (n+1)$, elements of which are calculated according to:

$$A_{jk} = (-1)^k \frac{\alpha^j}{(j-1)(k-1)!}\sum_{i=0}^{\infty} i^k \beta^i \frac{(k-1+i)!}{i!}.$$

In practice, as usual, the order of polynomial is taken not greater than 2.

For polynomials of the first order

$$f(\mathbf{a},T) = a_0(t) + a_1(t)T$$

it is

$$\hat{a}_0(t) = 2S^{(1)}(t) - S^{(2)}(t);$$

$$\hat{a}_1(t) = \frac{\alpha}{\beta}\left[S^{(1)}(t) - S^{(2)}(t)\right].$$

For polynomials of the second order

$$f(\mathbf{a},T) = a_0(t) + a_1(t)T + a_2(t)T^2$$

it is

$$\hat{a}_0(t) = 3S^{(1)}(t) - 3S^{(2)}(t) + S^{(3)}(t);$$

$$\hat{a}_1(t) = \frac{\alpha}{2\beta^2}\left[(6-5\alpha)S^{(1)}(t) - 2(5-4\alpha)S^{(2)}(t) + (4-3\alpha)S^{(3)}(t)\right]; \quad (3)$$

$$\hat{a}_2(t) = \frac{\alpha^2}{\beta^2}\left[S^{(1)}(t) - 2S^{(2)}(t) + S^{(3)}(t)\right].$$

Values of smoothing functions could be calculated according to recurrent formula (2), but for beginning of computation it is necessary to define the initial conditions $S^{(1)}(0)$, $S^{(2)}(0)$, … In the simplest case it is $S^{(1)}(0)=S^{(2)}(0)=S^{(3)}(0)=y(0)$.

With respect to formula (2) for predictive function it is necessary to calculate the coefficients $a_k, k=0,1,2$. This procedure has the following steps:

1. $$\begin{aligned} &S^{(1)}(0)=S^{(2)}(0)=S^{(3)}(0)=y(0);\\ &S^{(1)}(1)=\alpha y(1)+(1-\alpha)S^{(1)}(0);\\ &S^{(2)}(1)=\alpha S^{(1)}(1)+(1-\alpha)S^{(2)}(0);\\ &S^{(3)}(1)=\alpha S^{(2)}(1)+(1-\alpha)S^{(3)}(0). \end{aligned}$$

2. Coefficients $\hat{a}_0(t)$, $\hat{a}_1(t)$, $\hat{a}_2(t)$ are calculated by formulas (3).
3. Steps 1, 2 are repeated for 2-nd point of output series, then – for 3-rd point, till the last n-th point, that corresponds to t_0-th moment of the latest data receiving.
4. The description of polynomial with the last values of coefficients $\hat{a}_0(t_0)$, $\hat{a}_1(t_0)$, $\hat{a}_2(t_0)$ is formulated, and the required time of prediction t is substituted.

As result, we obtain the prediction

$$y(t_0+t)=a_0(t_0)+a_1(t_0)t+a_2(t_0)t^2.$$

Notice, that the application of moving average and exponential smoothing provides satisfactory results in the case of relatively small random variations of output series. At significant random disturbances the confidence intervals of predictions in these procedures will grow essentially.

IV. VERNIER ALGORITHM OF ADAPTIVE FILTRATION, SMOOTHING AND PREDICTION OF TRAJECTORIES OF PLANES

As first approximation let's apply such filter:

$$W_\varphi(p)=\frac{\alpha_1}{p+\alpha_1}. \quad (4)$$

Discrete analog is:

$$\dot{\hat{x}}(t)+\alpha_1\hat{x}(t)=\alpha_1 x(t). \quad (5)$$

The formula of recurrent method of Brown exponential smoothing is:

$$\hat{x}(k)=\alpha x(k)+(1-\alpha)\hat{x}(k-1), \quad (6)$$

where $\alpha=\alpha_1\cdot\Delta t, \Delta t=t_k-t_{k-1}=\text{const}$, k is the number of counting t_k of time t.

More accurate dependence is α and α_1:

$$\alpha=1-\exp[-\Delta t\cdot\alpha_1],$$

and $0\le\alpha\le 1$.

Excluding intermediate variables $\hat{x}(k)$ and applying (6) n times, expression for exponentially averaged estimate in point n could be obtained:

$$\hat{x}(n)=\alpha\sum_{j=0}^{n}(1-\alpha)^j x(n-j).$$

According to Brown the smoothed predicted value $\hat{x}(t+T)$ is calculated on base of expanding of $\hat{x}(t)$ in Taylor series in current point t:

$$\begin{aligned}\hat{x}(t+T)=\hat{x}(t)+T\dot{\hat{x}}(t)+\frac{T^2}{2!}\ddot{\hat{x}}(t)+\ldots\\ +\frac{T^{n-1}}{(n-1)!}\hat{x}^{(n-1)}(t)+R_n(t),\end{aligned} \quad (7)$$

where $R_n(t)$ is the residual number.

Limiting by three members of series (7), let's build filter-predicator on the base of equation (1). In Fig. 1 the filter is represented, where $\dot{\hat{x}}$ and $\ddot{\hat{x}}$ are defined by linear combination of signals $\hat{x}(t), y_1(t), y_2(t)$.

Then

$$\hat{x}(t+T)=\hat{x}(t)+T\dot{\hat{x}}(t)+0{,}5T^2\ddot{\hat{x}}(t). \quad (8)$$

In correspondence with Fig. 1

$$\dot{\hat{x}}(t)=\left[\frac{k_1\alpha^3}{(p+\alpha)^3}+\frac{k_1\alpha^2}{(p+\alpha)^2}\right]x(t)=\frac{\alpha^3(k_1+k_2+k_2\alpha^{-1}p)}{(p+\alpha)^3}x(t),$$

$$\ddot{\hat{x}}(t)=\left[\frac{k_4\alpha^3}{(p+\alpha)^3}+\frac{k_5\alpha^2}{(p+\alpha)^2}+\frac{k_3\alpha}{p+\alpha}\right]x(t)=\frac{\alpha^3\left[(k_4+k_5+k_3)+(k_5\alpha^{-1}+k_3\cdot 2\alpha^{-1})p+k_3\alpha^{-2}p^2\right]}{(p+\alpha)^3}x(t);$$

To obtain accurate derivatives

$$\dot{\hat{x}}(t)=\frac{\alpha^3 p}{(p+\alpha)^3}x(t), \qquad \ddot{\hat{x}}(t)=\frac{\alpha^3 p^2}{(p+\alpha)^3}x(t),$$

it is necessary to fulfill such conditions:

$$k_2=\alpha,\ k_1=-\alpha,\ k_4+k_5+k_3=0,\ k_5+2k_3=0,$$
$$k_3\alpha=\alpha^3,\ k_3=\alpha^2,\ k_5=-2\alpha^2,\ k_4=-\alpha^2+2\alpha^2=\alpha^2.$$

Going from (4) to (5), Brown filter-predicator could be obtained:

$$\hat{x}(k+m)=\hat{x}(k)+T\dot{\hat{x}}(k)+T^2\frac{\ddot{\hat{x}}(k)}{2},$$

where

$$\hat{x}(k)=3\phi^{'}(k)-3\phi^{''}(k)+\phi^{''}(k),$$

$$\dot{\hat{x}}(k)=\frac{\alpha}{2\Delta t(1-\alpha)^2}\big[(6-5\alpha)\phi^{'}(k)-(10-8\alpha)\phi^{''}(k)+(4-3\alpha)\phi^{'''}(k)\big],$$

$$\ddot{\hat{x}}(k)=\frac{2\alpha}{\Delta t^2(1-\alpha)^2}\big[\phi^{'}(k)-2\phi^{''}(k)+\phi^{'''}(k)\big],$$

$$\phi^{'}(k)=\alpha x(k)+(1-\alpha)\phi^{'}(k),$$

$$\phi^{''}(k)=\alpha\phi^{'}(k)+(1-\alpha)\phi^{''}(k-1),$$

$$\phi^{'''}(k)=\alpha\phi^{''}(k)+(1-\alpha)\phi^{'''}(k-1).$$

Analysis of filter-predicator (8) has contradictions between requirements of filtration and accuracy of prediction. The larger the level of disturbances is, the smaller α will be and the greater the speed of sluggishness will be. But steady-state error $\varepsilon(\infty)$ during observation of valid component of the signal $x^*(t)$ is also larger:

$$\varepsilon(\infty)=\lim_{s\to\infty} sW_{\varepsilon}(s)x^*(s),$$

where $W_{\varepsilon}(s)=1-\dfrac{\alpha^3}{(s+\alpha)^3}$; s is the Laplace variable.

So, for $x^*(t)=at: x^*(s)=\dfrac{a}{s^2}$,

$$\varepsilon(\infty)=\lim_{s\to\infty} s\frac{(s+\alpha)^3-\alpha^3}{(s+\alpha)^3}\frac{a}{s^2}=\frac{3a}{\alpha};$$

for $x^*(t)=\dfrac{at^2}{2}: \varepsilon(\infty)=\infty.$

Result of developed block diagram on the base of Brown filter it is shown in Fig. 2.

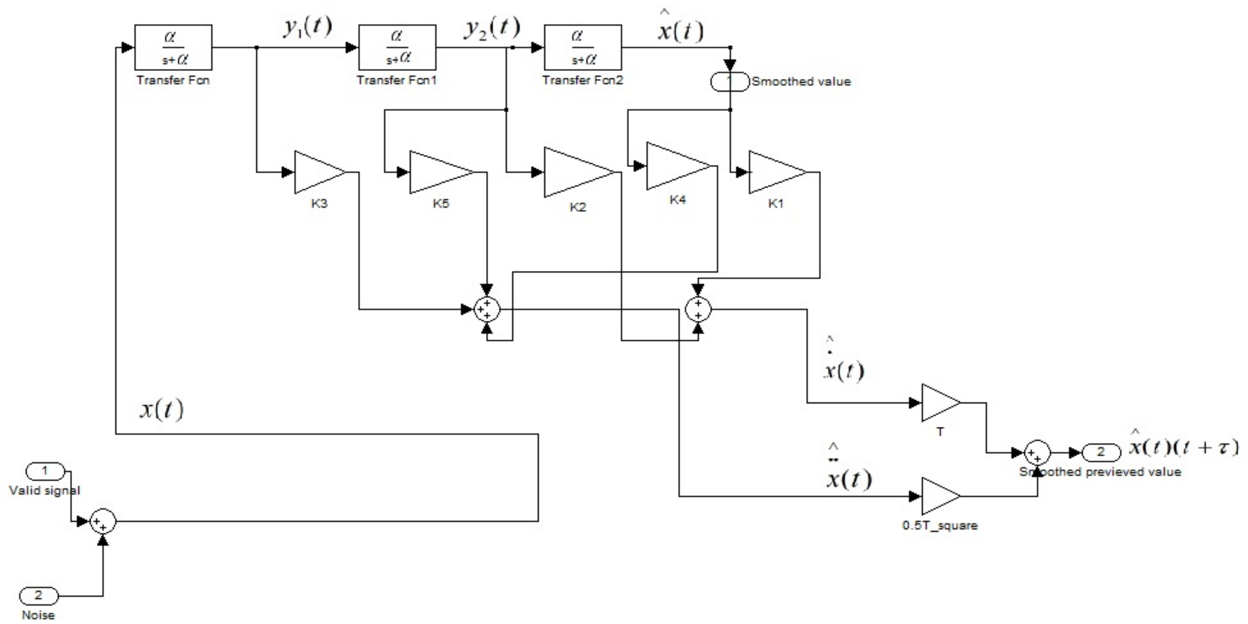
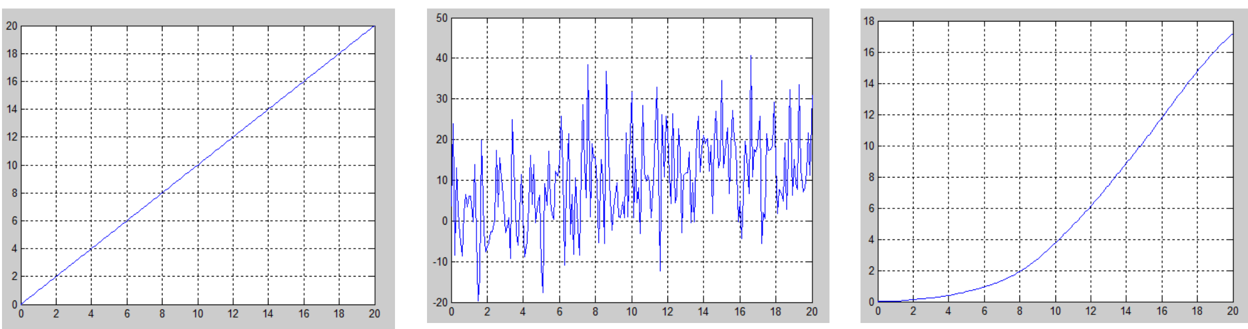


Fig. 1. Filter-predicator



a b c

Fig. 2. Result of developed block diagram on the base of Brown filter
(a) is the valid signal; (b) is the noisy signal; (c) is the smoothed signal

Conclusion

Research of developed vernier algorithm of adaptive filtration, smoothing and prediction of trajectories shows its sufficient high accuracy. Proposed method of filtering and developed filter-extrapolator allows improvement of navigation accuracy and correspondently safety of UAV flights.

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А. М. Сильвестров, М. П. Мухіна, І. В. Седень. Ноніусні алгоритми адаптивної фільтрації, згладжування та прогнозування траєкторії

Розглянуто алгоритми фільтрації сигналів та методи екстраполяції. Запропоновано застосовувати фільтр Брауна для усунення шумів, як альтернативу до фільтра Калмана. Представлено структурну схему адаптивної фільтрації, згладжування та прогнозування. Показано, що запропонований алгоритм дозволяє отримувати більш чистий сигнал, майже без шумів.

Ключові слова: методи екстраполяції; фільтрація; прогнозування; згладжування.

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А. Н. Сильвестров, М. П. Мухина, И. В. Седень. Нониусные алгоритмы адаптивной фильтрации, сглаживания и прогнозирования траекторий

Рассмотрено алгоритмы фильтрации сигналов, методы экстраполяции. Предложено применять фильтр Брауна для устранения шумов, как альтернативу фильтра Калмана. Представлена структурная схема адаптивной фильтрации, сглаживания и прогнозирования. Показано, что представленный алгоритм позволяет получать более чистый сигнал, почти без шумов.

Ключевые слова: методы экстраполяции; фильтрация; прогнозирование; сглаживание.

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