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EVALUATION OF THE QUALITY OF STABILIZATION SYSTEMS BY NORMALIZED INDIRECT INDICATORS

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Abstract*—Considered normalized indirect indicators of quality of stabilization systems of inertial control objects and methods of their calculation with the help of frequency responses.*

Index terms—Stability boundary; stability region; damping; stiffness; direct and inverse amplitude and phase frequency characteristics; stability margin; module; phase, indicator oscillatority.

I. INTRODUCTION

Stabilization system of inertial control objects allow to achieve the specified accuracy only at high quality of control processes. The problem is solved with the optimum combination of design parameters of systems and rational choice of their exploitational adjustments.

During the design and calculations of systems the quality of the control processes is usually evaluated according to their frequency characteristics. The basis for this is the relationship between the quality of the transitional process, caused by the special impact, and the frequency characteristics of the system. The transient response of the closed-loop system can be calculated on the basis of one of the Fourier integrals:

$$H(t)=\frac{A_m(0)}{2}+\frac{1}{\pi}\int_0^\infty \frac{A_m(\omega)}{\omega}\sin\left[\omega t+\varphi(\omega)\right]d\omega;$$

$$H(t)=P(0)+\frac{2}{\pi}\int_0^\infty \frac{Q(\omega)}{\omega}\cos\omega t d\omega;$$

$$H(t)=\frac{2}{\pi}\int_0^\infty \frac{P(\omega)}{\omega}\sin\omega t d\omega,$$

where $A_m(\omega)$ is the amplitude; $P(\omega)$ is the real and $Q(\omega)$ is the imaginary frequency characteristics of the closed-loop system.

For the real technical systems control signals and external disturbances have a random character. As a consequence the frequency characteristics are used only for indicative opinions about the quality of the system. Some interest, in our opinion, represents the evaluation of the quality of stabilization systems of inertial control objects by normalized indirect indicators.

II. SOLUTION OF THE PROBLEM

The indirect quality indicators can include: stability margins in modulus and phase, stability margin in frequency and indicator of oscillatority. Each of indirect quality indicators, taken separately from the others, does not allow to do definitive conclusions about the character of the control processes. Only using multiple indirect indicators, you can clearly see the properties of a closed-loop automatic control system.

Stability margins in modulus and phase are determined by the frequency characteristics of the open-loop system, which can be obtained by calculation or experimentally. Figure 1 shows the method of determining the stability margins in modulus and phase on the basis of direct amplitude-phase frequency characteristic (APFC) of the open-loop system

$$W_{\text{op}}(j\omega)=U(\omega)+jV(\omega).$$

Margin in modulus is equal

$$h=1-U(\omega)\big|_{V(\omega)=0}.$$

To determine the stability margins in phase the unit circle is conducted from the origin. The point B of intersection of this circle with $W_{\text{op}}(j\omega)$ is connected with the help of the line to the origin.

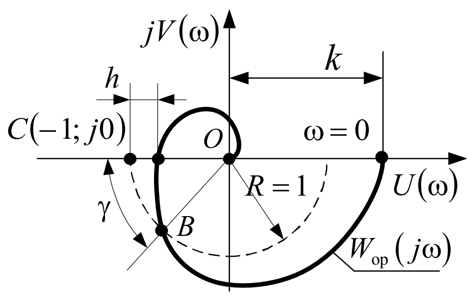

Fig. 1. Determination of the stability margins in modulus and phase on the basis of direct APFC of the open-loop system

The angle between the line OB and negative direction of the x-axis equals the angle γ of stability margin in phase. Thus, the phase margin is $\gamma=\varphi_{\text{p}}(\omega)\big|_{A_m=1}$.

Calculations and experimental research of stabilization systems of inertial control objects show that for the tracking measurement mode should be recommended values $h \geq 0,6;\ \gamma \geq 30-35^{\circ}$, and for the mode of stabilization $h \geq 0,3;\ \gamma \geq 20^{\circ}$.

Stability margin in modulus can be analytically expressed through the boundary stiffness $G_b(\omega)$ of the system. By defining according to [1] values $U(\omega)$ and $V(\omega)$, we obtain

$$U(\omega)\big|_{V(\omega)=0} = \left|\frac{G}{G_b(\omega)}\right|,$$

where G is the value of exploitational stiffness, determined by the settings of the system.

Consequently, the stability margin in modulus is equal

$$h = 1 - \frac{G}{G_b(\omega)} = \frac{G_b(\omega) - G}{G_b(\omega)}. \qquad (1)$$

The last relation defines the stability margin for any adjustment of the stabilization system, that is for any operating point inside the region of sustainability. Using equation (1), it is possible to build lines of equal stability margin in modulus (Fig. 2) inside the stability region of the system.

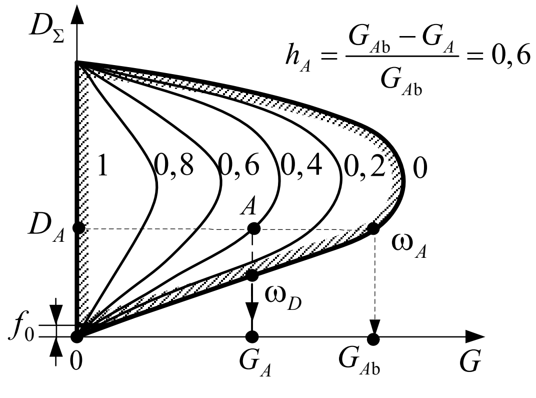

Fig. 2. The stability region with lines of equal value of stability margin in modulus

Figure 3 shows the method of determining the stability margins in modulus and phase on the basis of inverse APFC of the open-loop system

$$E_{op}(j\omega) = \frac{1}{W_p(j\omega)} = U_E(\omega) + jV_E(\omega).$$

In this case, the stability margin in modulus accepted to estimate with the help of value

$$h_E = U_E(\omega)\big|_{V_E(\omega)=0} - 1.$$

Since $U_E(\omega)\big|_{V_E(\omega)=0} = \frac{1}{U(\omega)\big|_{V(\omega)=0}} = \frac{1}{1-h}$, then we have

$$h_E = \frac{h}{1-h} = \frac{G_b(\omega) - G}{G}. \qquad (2)$$

Equation (2) defines the relationship between the values of stability margins in modulus h and h_E.

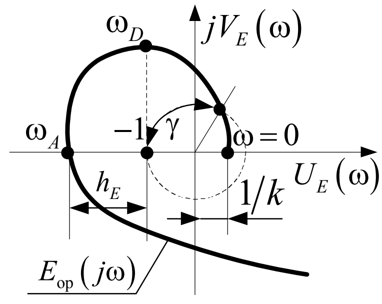

Fig. 3. Determination of the stability margins in modulus and phase on the basis of inverse APFC of the open-loop system

Determination of stability margin in phase on the basis of inverse $E_{op}(j\omega)$ APFC is performed fundamentally as well as on the basis of direct $W_{op}(j\omega)$.

Stability margins in modulus and phase can be calculated by using the direct logarithmic characteristics of an open-loop system [2].

Figure 4 shows the stability region of stabilization system of inertial control object, in which identified six possible operating points. For these points were calculated and plotted inverse frequency characteristics $E_{op}(j\omega)$ of open-loop systems. Characteristics are shown in Fig. 5.

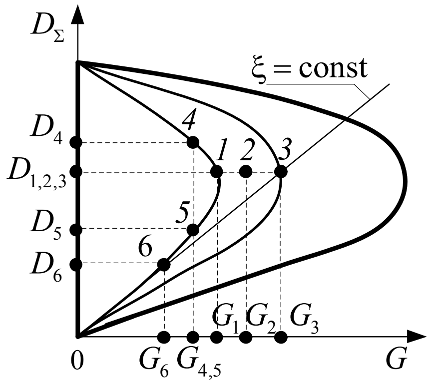

Fig. 4. The stability region

Comparison of characteristics *1*, *2* and *3*, obtained when damping is constant $D_{1,2,3}$ and stiffness is variable $G_3 > G_2 > G_1$, make it possible to conclude that increasing the stiffness of the system decreases its stability margins in modulus and phase.

Analysis of the characteristics of *4*, *1*, *5* and *6*, constructed for different combinations of stiffness and damping, but with a constant value of stability margin in modulus, shows, that with increasing damping stability margin in phase increases, reaching a maximum value near high frequency stability border (point *4*).

Considering the characteristics of the system, that correspond to operating points *2* and *6*, which provide the same value of a tracking error, come to the conclusion, that increasing stiffness reduces the stability margin of the system in modulus and increases its phase margin.

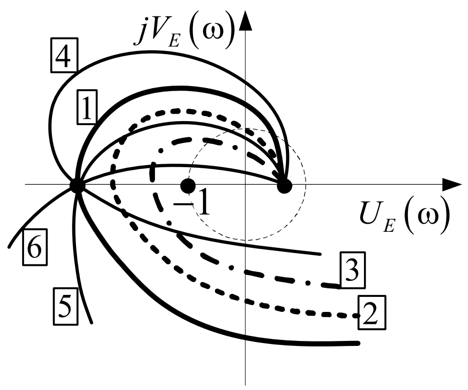

Fig. 5. Inverse APFC of the system for different operating points

Conclusions should be considered while choosing the operational adjustments of stabilization systems.

It should be noted that, for the stabilization system, the transfer function of which has in the numerator polynomial of the operator, it is impossible to make conclusions about the quality of control only on the values of stability margins in modulus and phase. Perhaps such a combination of parameters of the system in which quality evaluation may lead to incorrect conclusions. That is why the evaluation of the quality should always be carried out simultaneously by several indicators. As additional indicators can be a stability margin in frequency and an indicator of oscillatority.

Let us denote as ω_D the frequency, which corresponds to the value $U_E(\omega) = -1$ of the real part of the feedback APFC of the open loop system and denote as ω_A the frequency which corresponds to the point of intersection the inverse APFC with abscissa axis (see Fig. 3).

Ratio

$$\nu = \frac{\omega_D}{\omega_A}$$

will be called the stability margin of the system in frequency.

If we shall put on the stability border (Fig. 2) the frequencies of fluctuations occurring in the system, then the intersection of the line AG_A with the stability border determines the frequency ω_D, and the intersection of the continuation the line D_AA with the stability border gives the value of the frequency ω_A.

Thus, each of the ratios $\nu = \omega_D/\omega_A$ determines one operating point in the stability region and its corresponding inverse APFC (Fig. 5). Experience in analysis and design of automatic control systems shows that the margin of stability in frequency should be chosen equal to $\nu = 0,4-0,6$.

An indicator of oscillatority M is the ratio of the maximum oscillation amplitude the output quantity of the closed-loop system to the amplitude of the output signal when $\omega = 0$

$$M = \frac{A_m(\omega)_{\max}}{A_m(0)}.$$

When $M < 1$ transient processes occur in the system without overshoot. When $M > 1$ there are oscillatory processes, and overshoot and regulation time with increasing the indicator of oscillatority are increased. Amplitudes $A_m(\omega)_{\max}$ and $A_m(0)$ are usually determined by the amplitude-frequency characteristics of a closed-loop system.

If the stabilization system is under the influence of the control signal $r(t)$, then the indicator of oscillatority can be determined graphically with the help of APFC of the open-loop system. Taking into account that in the case considered $A_{mr}(0) = 1$, we have

$$M_r = A_{mr}(\omega) = \left| \frac{U(\omega) + jV(\omega)}{1 + [U(\omega) + jV(\omega)]} \right| = \sqrt{\frac{U^2(\omega) + V^2(\omega)}{[1 + U(\omega)] + V^2(\omega)}}.$$

The last expression can be written as the equation of the circle

$$\left[U(\omega) - \frac{M_r^2}{1 - M_r^2} \right]^2 + V^2(\omega) = \frac{M_r^2}{(1 - M_r^2)^2}.$$

The radius of this circle is equal $\left| \frac{M_r}{1 - M_r^2} \right|$ and the center is located on the real axis of the complex plane $U(\omega) \to jV(\omega)$ at a distance $\frac{M_r^2}{1 - M_r^2}$ from the origin. Consequently, the locus of points on a plane $U(\omega) \to jV(\omega)$, corresponding to a constant value of indicator of oscillatority is a circle.

Specifying different values of indicator of oscillatority, we can construct a family of circles $M_r = \text{const}$. When $M > 1$ the centers of the circles are located on the left, while $M < 1$ – on the right from axis of ordinates. When $M = 1$ the circle degenerates into a straight line parallel to the imaginary axis $jV(\omega)$ and located on the left from it in the distance $\frac{1}{2}$. Family of circles is shown in Fig. 6.

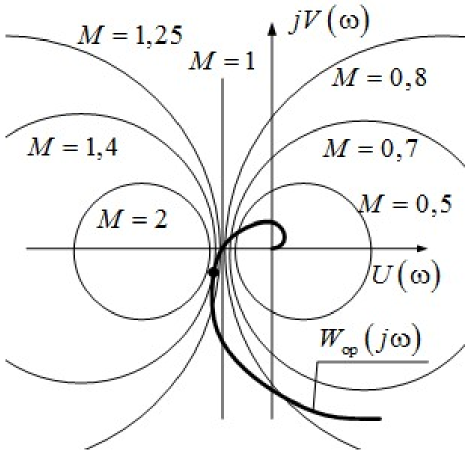

Fig. 6. Determination of indicator of oscillatority with the help of direct APFC

If we shall lay the characteristic $W_{op}(j\omega)$ of the open-loop system on the received nomogram, then the circle $M_{ri} = \text{const}$ tangent to the APFC, will show the value of the indicator of oscillatority of the corresponding closed-loop system, that is working under the influence of the control signal.

Nomogram $M_{ri} = \text{const}$ allows by APFC of the open-loop system to determine the amplitude frequency (AFC) characteristic of the corresponding closed-loop system that is working under the influence of the control signal. Knowing the frequencies of intersection points APFC of the open-loop system with the circles $M_{ri} = \text{const}$ and considering that $M_r = A_{mr}(\omega)$, we can construct characteristic $A_{mr}(\omega)$.

Indicator of oscillatority M_r is quite simple determined and by reverse APFC of the open-loop system. In this case, the locus of points corresponding to the inverse number of the indicator of oscillatority $\frac{1}{M_r} = \text{const}$ in the complex plane $U_E(\omega) \to \to jV_E(\omega)$ is a circle which has equation

$$\left[U_E(\omega) - 1\right]^2 + V_E^2(\omega) = \left(\frac{1}{M_r}\right)^2.$$

The radius of this circle is $\frac{1}{M_r}$, and the center has coordinates $(-1; j0)$, i. e. coincides with the beginning of the coordinate plane $P_E(\omega) \to jQ_E(\omega)$.

Consequently, for determining the oscillatority indicator with the help of inverse frequency characteristic $E_{op}(j\omega)$ enough to hold from the point $(-1; j0)$ a circle tangent to the hodograph $E_{op}(j\omega)$. Quantity inverse the radius of the circle will be equal M_r.

Method of determining the indicator of oscillatority with the help of inverse APFC is shown in Fig. 7.

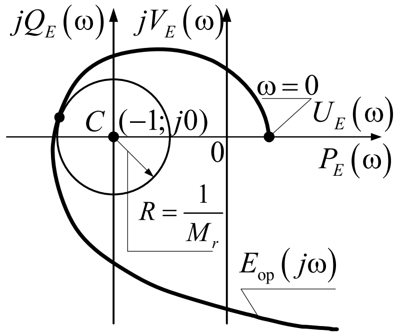

Fig. 7. Determination of indicator of oscillatority with the help of inverse APFC

For providing good quality of the transitional process the oscillatority indicator should be selected in the range $M = 1{,}2 - 1{,}5$.

CONCLUSIONS

For the most complete preliminary evaluation of quality the control in the stabilization systems of inertial objects should be used simultaneously several indirect indicators. But even in this case of assessment will be approximate and should be checked by calculation of the transient response or with the help of modeling the system.

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О. К. Аблесімов, Н.І. Кутова. Оцінка якості систем стабілізації за нормованими непрямими показниками

Розглянуто нормовані непрямі показники якості систем стабілізації інерційних об’єктів керування та методики їх розрахунку за частотними характеристиками систем.

Ключові слова: межа стійкості; область стійкості; демпфірування; жорсткість; прямі й зворотні амплітудно-фазові частотні характеристики; запас стійкості; модуль; фаза; показник коливальності.

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Напрям наукової діяльності: автоматичні системи керування.

А. К. Аблесимов, Н. И. Кутовая. Оценка качества систем стабилизации по нормированным косвенным показателям

Рассмотрены нормированные косвенные показатели качества систем стабилизации инерционных объектов управления и методики их расчета по частотным характеристикам систем.

Ключевые слова: граница устойчивости; область устойчивости; демпфирование; жесткость; прямые и обратные амплитудно-фазовые частотных характеристики; запас устойчивости; модуль; фаза; показатель колебательности.

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