AUTOMATIC CONTROL SYSTEMS

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ESTIMATION ALGORITHM OF ARBITRARY DYNAMICS OBJECT STATE UNDER RANDOM ACTIONS AND INCOMPLETE MEASURING BY UNSTABLE SYSTEM

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Abstract**—***The article proposes estimation algorithm of linear time-invariant systems with arbitrary dynamic behavior of control object and measuring system subject to real operating conditions.*

Index Terms—Algorithm; dynamics model; estimation system; vector; matrix; random process; spectral density; stabilization object.

I. INTRODUCTION

A characteristic feature of contemporary aerospace technology development is continuous complication of newly created products and need of operation of these products in optimum modes under the impact of various parametric and signal disturbing factors and interference caused by both external and internal effects. As it is known, the action of disturbing factors has stochastic nature that defines the need of probabilistic approach to the problems of aircraft creation and operation.

It is necessary to upgrade existing algorithms and to develop new algorithms of synthesis of flight control optimum systems, state estimations and identification of dynamics of linear time-invariant systems due to the following conditions. Firstly, vital needs of practice force to consider real operating conditions more comprehensively in stating the tasks of stochastic optimum flight control of contemporary aircraft. Secondly, developers of complicated controlled complexes need comparatively simple algorithms of synthesis which result directly in construction of optimum structures of flight control systems, guarantees of successful and efficient solution of the tasks set for the flight.

Reliable knowledge of signal dynamics models defining stochastic state of complicate linear stabilizable object, and also knowledge of dynamics models of the object itself and affecting random disturbances in normal operating modes is extremely important for subsequent successful development of science-based draft proposals for creation of optimum systems of object stabilization, i.e. for the stages of dynamic design of mentioned systems.

II. PROBLEM STATEMENT

Same as in publication [1], it is possible to sate and solve the task of optimum estimation of arbitrary dynamics linear object state, though when stating the task consider probable correlation of interference, impacts and signals in the system. For the purpose of simplification of resultant algorithms, elimination of the problem of random matrix selection required at free-variable function generation, and disuse of Frobenius transformations [2], the statement is changed essentially and new way of task solution is proposed therein.

III. DESCRIPTION OF THE OPTIMUM CONTROL

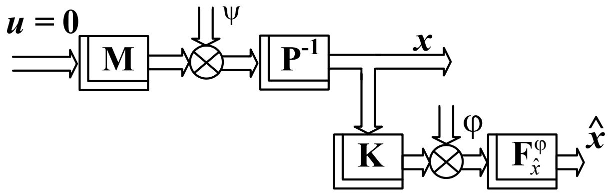
Let the motion of the object which state is estimated with the purpose of generating closed optimum control is described by the system of ordinary differential equations

$$Px(t)=\psi(t), \quad (1)$$

where x is n-dimensional vector of output responses of the object, ψ is disturbance vector that is n-dimensional centered stationary random process with known matrices of spectral densities $S_{\psi\psi}$ and reciprocal spectral densities $S_{\psi\varphi}$, $S_{\varphi\psi}$; P is dimension matrix $n\times n$ which elements are argument polynomials $\rho=d/dt$, at that its determinant can have zeroes with positive real component. It is assumed that output responses of the object are measured by a certain system, measuring is accompanied by interference φ that is ν-dimensional centered stationary random process with known matrix of spectral densities $S_{\varphi\varphi}$. Measuring results are defined by the equation

$$y(t)=Kx(t)+\varphi(t), \quad (2)$$

where y is ν-dimensional vector of measuring, φ is measuring interference that is ν-dimensional centered random process with known matrices of spectral densities $S_{\varphi\varphi}$ and reciprocal spectral densities $S_{x\varphi}, S_{\varphi x}$, at that the matrix elements of measuring system transfer functions K with dimension $\upsilon \times n$ can have poles with positive real component (Figure).



Flow chart of estimation system for unstable control object

Same as in publication [1], there is denoted by $\hat{x}(t/\delta)$ the best estimation of the vector x in terms of error dispersion minimum $\varepsilon(t/\delta) = \hat{x}(t/\delta) - \hat{x}(t)$ in the moment t by the results of measuring prior to moment δ. The task of vector x optimum estimation is defined as follows: estimate $\hat{x}(t) = x(t/t)$ vector $x(t)$, the best in terms of error dispersion minimum

$$e_0 = M\left[\varepsilon'(t) R \varepsilon(t)\right] = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left(S'_{\varepsilon\varepsilon} R\right) ds, \quad (3)$$

assuming that admissible estimations are linear with respect to the vector y.

Let the vector of object output responses is estimated by the vector of observations y using estimation system with the matrix of transfer functions $F^{\varphi}_{\varepsilon} = F^{\varphi}_{\hat{x}}$. Then the following equation is valid

$$\hat{x} = F^{\varphi}_{\varepsilon} y,$$

wherein $\hat{x}$ and y are Fourier transforms of vectors $\hat{x}$ and y.

It is necessary to modify equations (1) and (2) by Laplace and introduce certain designations. The matrix of estimation system transfer functions from interference input φ to the vector $\hat{x}$ can be designated as $F^{\varphi}_{\hat{x}}$, and the matrix of transfer functions from disturbance input ψ to the vector $\hat{x}$ as $F^{\psi}_{\hat{x}}$. Then Laplace transformation of estimation error will be written as follows

$$\varepsilon = F^{\psi}_{\varepsilon}\psi + F^{\varphi}_{\varepsilon}\varphi = \left(F^{\varphi}_{\varepsilon} K - E_n\right) P^{-1}\psi + F^{\varphi}_{\varepsilon}\varphi = F^{\varphi}_{\varepsilon}\left(KP^{-1}\psi + \varphi\right) - P^{-1}\psi. \quad (4)$$

To be able to solve the task of estimation of arbitrary dynamics object state at unstable measuring system, and also incomplete measuring, it is necessary to define $F^{\varphi}_{\hat{x}}$ as variable function in estimation quality functional (3). The basis F of such function can be obtained from the following equation

$$F^{\varphi}_{\varepsilon} = F\mathbf{B} + \mathbf{AC}, \quad (5)$$

where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are polynomial matrices which structure and parameters are defined based on the following considerations. For estimation stability, the function $F^{\varphi}_{\hat{x}}$ shall be physically feasible, i.e. its poles shall be in only left half plane of complex variable. If physical feasibility conditions of function F are provided, fulfillment of previous condition is guaranteed under appropriate selection of polynomial $\mathbf{A}$ and $\mathbf{B}$. Selection of structures of matrices $\mathbf{A}$ and $\mathbf{B}$ is connected with providing physical feasibility of the matrix $F^{\psi}_{\varepsilon} = \left(F^{\varphi}_{\hat{x}} K - E_n\right) P^{-1}$. Matrices $\mathbf{B}$ and $\mathbf{C}$ in the equation (5) are the result of the following transformation

$$\mathbf{B}^{-1}\theta = \mathbf{KP^{-1}}; \quad \mathbf{K} = \mathbf{C}^{-1}\mathbf{K_0}, \quad (6)$$

consisting in performing operations [3] of unilateral removal of only those poles of matrices $\mathbf{KP^{-1}}$ and $\mathbf{K}$ located in right half plane. As the function F^{ψ}_{ε} shall be physically feasible separating [4] the expression of this matrix, it shall be written as follows

$$F^{\psi}_{\varepsilon} = \left[F^{\psi}_{\varepsilon}\right]_{+} = \left[(F\mathbf{B} + \mathbf{AC})(\mathbf{KP^{-1}}\psi + \varphi) - P^{-1}\psi\right]_{+}; \quad \left(\mathbf{AK_0}P^{-1} - P^{-1}\right)_{-} = 0. \quad (7)$$

The last equation will be rewritten as follows

$$\left(\mathbf{AK_0}P^{-1}\right)_{-} = P^{-1}_{-}, \quad (8)$$

where P^{-1}_{-}, «+» and «-» are separation signs.

Thus, solutions of equations (6) and (8) define selection of matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ for functions (5) yet prior to solution of estimation optimization task. After this stage, estimation error equation (4) can be written as follows

$$\varepsilon = \left(F\mathbf{B} + \mathbf{AC}\right)\psi_0 - P^{-1}\psi,$$

$$\psi_0 = KP^{-1}\psi + \varphi,$$

and spectral density of estimation error as follows

$$S'_{\varepsilon\varepsilon} = FBS'_{\psi_0\psi_0} B_* F_* + FB\left(S'_{\psi_0\psi_0} C_* A_* - S'_{\psi\psi_0} P^{-1}_*\right) + \left(ACS'_{\psi_0\psi_0} - P^{-1}S'_{\psi_0\psi}\right)BF + \Sigma, \quad (9)$$

where $\sum = ACS'_{\psi_0\psi_0} C_* A_* - ACS'_{\psi\psi_0} P_*^{-1}$

$$- P^{-1} S'_{\psi_0\psi} C_* A_* + P^{-1} S'_{\psi\psi} P_*^{-1};$$

$$S'_{\psi_0\psi_0} = KP^{-1} S'_{\psi\psi} P_*^{-1} K_* + KP^{-1} S'_{\varphi\psi}$$

$$+ S'_{\psi\varphi} P_*^{-1} K_* + S'_{\varphi\varphi};$$

$$S'_{\psi\psi_0} = KP^{-1} S'_{\psi\psi} + S'_{\psi\phi};$$

$$S'_{\psi_0\psi} = S'_{\psi\psi} P_*^{-1} K_* + S'_{\phi\psi}.$$

Herein S_{ij} are matrices of spectral and reciprocal spectral densities of the signs named in indices, « ′ » is transposition sign.

To solve the set task we have to substitute the matrix (9) into the functional (3) and considering the designations

$$BS'_{\psi_0\psi_0} B_* = DD_*; \quad \Gamma_* \Gamma = R; \tag{10}$$

$$F_0 = \Gamma F D, \tag{11}$$

rewrite it as follows

$$e_0 = \frac{1}{j} \int_{-j\infty}^{j\infty} tr\Big[F_0 F_{0*} + F_0 D^{-1} B \times \left(S'_{\psi_0\psi_0} C_* A_* - S'_{\psi\psi_0} P_*^{-1} \right) \Gamma_* + \Gamma \left(ACS'_{\psi_0\psi_0} - P^{-1} S'_{\psi_0\psi} \right) B_* D_*^{-1} F_{0*} + R\Sigma \Big] ds. \tag{12}$$

Matrices **D** and **Γ** in the equation (10) are the result of factorization analytic in the right half plane; F_0 is supplementary free-variable fractional rational function which shall be analytic in the right half plane together with variation δF_0, at that the last one if $s \to \infty$ has asymptotics $1/s^{\mu}$, $\mu \geq 1$.

The task of optimization of compound object state estimation is reduced to the task of minimizing the functional (12) in the class of functions F_0. Minimization of the functional is done as per procedure of Viner-Kolmogorov method. The condition minimizing the functional will be as follows

$$F_0 = -(T_0 + T_+), \tag{13}$$

where $T = T_0 + T_+ + T_- = \Gamma\left(ACS'_{\psi_0\psi_0} - P^{-1} S'_{\psi_0\psi} \right) B_* D_*^{-1}$; «$_0$», «+», «-» are separation signs. The algorithm for function F definition considering matrices (11) and (13) shall be written as follows

$$F = -\Gamma^{-1} (T_0 + T_+) D^{-1}. \tag{14}$$

By substituting the function (13) into the functional (12), we can define minimum dispersion of estimation error. By substituting the algorithm (14) into the equation (5), we can obtain the matrix of optimum transfer functions of closed estimation system $F_{\hat{x}}^{\varphi}$ which is subject to implementation in future. It is easy to make certain that in particular case when the object and measuring system are stable, optimum structures of functions $F_{\hat{x}}^{\varphi}$ and V coincide.

CONCLUSION

Thereby, resultant algorithms of the task can be used for dynamic design of optimum structures of estimation systems of linear time-invariant systems with arbitrary dynamic behavior of control object and measuring system subject to real operating conditions. The algorithms are relatively simple, transparent, easily reconstructed for particular cases of estimation tasks and can be generalized for situations arising when solving the tasks of aircraft dynamics identification in operating (non-special) modes. They can be useful for creating and upgrading flight control systems, navigation, etc.

REFERENCES

[1]. Naumenko, K. I. (1984). Observation and controlling the movement of dynamic systems. Kyiv, Naukova dumka Publ., 208 p. (in Russian).

[2]. Gantmacher, F.F. (1967). Theory of Matrices. Moscow, Nauka Publ., 575 p. (in Russian).

[3]. Blochin, L. N. (1982). Optimal stabilization systems. Kyiv, Technika Publ., 143 p. (in Russian).

[4]. Davis, M. C. (1963).“Factoring the Spectral Matrix”. *IEEE Trans. Auto. Cont.*, AG-8, no. 4. pp. 296–305.

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В. М. Азарсков, О. Ю. Курганський, Г. І. Рудюк. Алгоритм оцінювання стану об'єкта з довільною динамікою у разі випадкових впливів і неповних вимірювань нестійкою системою

Запропоновано алгоритм оцінювання стану лінійних інваріантних у часі систем з довільними динамічними характеристиками об'єкта управління і системи вимірювання з урахуванням реальних експлуатаційних умов.
Ключові слова: алгоритм; модель динаміки; система оцінювання; вектор; матриця; випадковий процес; спектральна щільність; об'єкт стабілізації.

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В. Н. Азарсков, А. Ю. Курганский, Г. И. Рудюк. Алгоритм оценивания состояния объекта с произвольной динамикой при случайных воздействиях и неполных измерениях неустойчивой системой

Предложен алгоритм оценивания состояния линейных инвариантных во времени систем с произвольными динамическими характеристиками объекта управления и системы измерения с учетом реальных эксплуатационных условий.
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