THEORY AND METHODS OF SIGNAL PROCESSING

UDC 629.735.051:681.513.5(045)

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OPTIMAL FILTRATION OF DETERMINISTIC INFORMATION IN MULTIVARIATE MEASURING ROUTE

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Abstract. *The new methodology of optimal filtering is represented for deterministic components of measured information in on-board multivariate measuring route. Two variants of the mentioned problem are considered, where first is for measuring route with physically realizable (stable) elements, and the second one takes into account possibility of unstable elements in a system.*

Keywords: optimal filtering; measuring route; Wiener-Kolmogorov method; factorization; separation.

Introduction

Nowadays requirements to quality (accuracy) of on-board measuring equipment, used on moving objects of different purposes, are raised sharply. In fact, competitive ability of such complexes is determined by filtration quality of useful stochastic information, obtained by them.

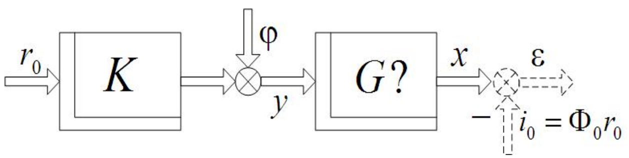
Ways for optimal filtration of random ergodic information of measuring routes, based on Wiener-Kolmogorov's method, are well known [1, 2, 6]. But for certain operating modes of routes, stated above, the optimal filtration of deterministic components of information flowing there is also very important. Till now this fact wasn't given significant attention in scientific literature. To provide competitiveness of on-board measuring systems and complexes [5] it is necessary to fill in mentioned blanks in processing of measured information.

The problem statement

In the present paper two variants of optimal processing of deterministic information in multivariate measuring route and several modernization aspects of the Wiener-Kolmogorov method for successful optimal filtration of deterministic information are presented.

The block diagram of the multivariate measuring route is shown in figure.

Here **K** is known ν×n matrix of transfer functions of a multivariate measuring system. This matrix is obtained after appropriate processing of HIL (Hardware-in-the-loop) simulation results of an on-board measuring complex.



Block diagram of measuring route

Also we have vectors of input signals $\mathbf{r}_0$ and $\boldsymbol{\varphi}$ with dimensions n×1 and ν×1 accordingly. Their characteristics are known as results of HIL simulation of the system in long-term mode of its operation as well. The vector of observation signals **y** goes through multivariate optimal filter-observer, which structure (transfer function) **G** is actually needed to be determined. As a result of that the n×1 estimation vector **x** appears on output of the measurement route. Comparing this vector with vector $\mathbf{i}_0$ of required output signals of the measurement route (here $\boldsymbol{\Phi}_0$ is known matrix of required transformations of program vector of signals $\mathbf{r}_0$) allows determining vector of filtration error signals **ε**, necessary to further estimation of measurement quality factor and synthesis of the desired structure of **G** .

In two stated below problems of optimal filtering of deterministic information the condition $n \geq \nu$ is assumed to be valid. The dynamic characteristics of investigated vectors of signals are presented in the following way:

$$\begin{aligned}
&\mathbf{r}_0 = \boldsymbol{\Theta}_r \cdot \mathbf{L}_{n\times1},\ \mathbf{r}_{0*} = \mathbf{L}_{1\times n}\cdot\boldsymbol{\Theta}_{r*};\\
&\varphi = \boldsymbol{\Theta}_\varphi \cdot \mathbf{L}_{\nu\times1},\ \varphi_* = \mathbf{L}_{1\times\nu}\cdot\boldsymbol{\Theta}_{\varphi*};\\
&\varepsilon = \boldsymbol{\Theta}_\varepsilon \cdot \mathbf{L}_{n\times1},\ \varepsilon_* = \mathbf{L}_{1\times n}\cdot\boldsymbol{\Theta}_{\varepsilon*};\\
&\mathbf{L}_{n\times1}\cdot\mathbf{L}_{1\times n} = \mathbf{L}_n;\\
&\mathbf{L}_{\nu\times1} = \mathbf{B}_1\cdot\mathbf{L}_{n\times1},\ \ \mathbf{L}_{1\times\nu} = \mathbf{L}_{1\times n}\cdot\mathbf{B}_2,
\end{aligned} \qquad (1)$$

where

$$\mathbf{B}_1 = \begin{bmatrix} E_\nu\, 0_{\nu\times(n-\nu)} \\ 0_{(n-\nu)\times n} \end{bmatrix},\ \ \mathbf{B}_2 = \begin{bmatrix} E_\nu \\ 0_{(n-\nu)\times\nu} \end{bmatrix}.$$

Here sign «*» symbolizes Hermitian conjugation, $\mathbf{L}_{n\times1}$ and $\mathbf{L}_{1\times n}$ are unit vectors (column and row) with length n; $\boldsymbol{\Theta}_r$, $\boldsymbol{\Theta}_\phi$, $\boldsymbol{\Theta}_\varepsilon$ are diagonal matrices of appropriate dimensions.

Taking into account notations (1) and scheme of the measuring route an estimation of the route output vector should be written as

$$\begin{aligned}&\mathbf{x}=\mathbf{G}\mathbf{y}=\mathbf{G}(\mathbf{K}\mathbf{r}+\varphi)=\mathbf{G}(\mathbf{K}\mathbf{\Theta}_r\mathbf{L}_{n\times1}+\mathbf{\Theta}_\varphi\mathbf{L}_{\nu\times1})\\&=\mathbf{G}(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1)\mathbf{L}_{n\times1};\\&\mathbf{x}_*=\mathbf{L}_{n\times1}(\mathbf{\Theta}_{r*}\mathbf{K}_*+\mathbf{B}_2\mathbf{\Theta}_{\varphi*})\mathbf{G}_*,\end{aligned}\tag{2}$$

and estimation vector of measurement errors ε can be described in the following view

$$\begin{aligned}&\boldsymbol{\varepsilon}=\mathbf{x}-\mathbf{i}_0=\left[\mathbf{G}(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1)-\mathbf{\Phi}_0\mathbf{\Theta}_r\right]\mathbf{L}_{n\times1};\\&\boldsymbol{\varepsilon}_*=\mathbf{L}_{1\times n}\left[(\mathbf{\Theta}_{r_*}\mathbf{K}_*+\mathbf{B}_2\mathbf{\Theta}_{\varphi_*})\mathbf{G}_*-\mathbf{\Theta}_{r_*}\mathbf{\Phi}_{0*}\right].\end{aligned}\tag{3}$$

Algorithms

It is worthwhile to consider two variants of the optimal filtration task. The first variant is possible when all of the functions used in the task, namely $\mathbf{\Theta}_r$, $\mathbf{\Theta}_\varphi$, $\mathbf{K}$ and $\mathbf{\Phi}_0$, are physically realizable, i. e. stable. In the second variant of the task it is taken into account that all or some of the functions, listed above, can be unstable. In both variants of the task the proper consideration must be given to the fact that direct choose of filter-observer structure is based at first on calculated (basic) functions of the route, which are marked by index «$_0$», but these functions can change for each mode of the investigated measuring route operation.

Problem 1. The essence of the problem of optimal filtering of deterministic information in a measuring route with only stable units and signals is as follows. Let a block diagram of the measuring route to look like in figure 1, units and signals of the route are physically realizable (stable). Notions (1) and expressions (2) and (3) are considered to be true in the task under study. If structural synthesis of the filter-observer is implemented at calculated (basic) elements of measuring route, then expressions for vector of measuring errors (3) should be represented as:

$$\begin{aligned}&\boldsymbol{\varepsilon}_0=\left[\mathbf{G}_0(\mathbf{K}_0\Theta_r+\Theta_{\varphi0}\mathbf{B}_1)-\mathbf{\Phi}_0\Theta_r\right]\mathbf{L}_{n\times1}\\&=\mathbf{\Theta}_{\varepsilon0}\mathbf{L}_{n\times1};\\&\boldsymbol{\varepsilon}_{0*}=\mathbf{L}_{1\times n}\left[(\mathbf{\Theta}_{r*}\mathbf{K}_{0*}+\mathbf{B}_{1*}\mathbf{\Theta}_{\varphi0*})\mathbf{G}_{0*}-\mathbf{\Theta}_{r*}\mathbf{\Phi}_{0*}\right]\\&=\mathbf{L}_{1\times n}\mathbf{\Theta}_{\varepsilon0*}.\end{aligned}\tag{4}$$

From expression (4) and condition $\mathbf{\Theta}_{\varepsilon_0}=\mathbf{0}_n$ it is easy to determine the required structure $\mathbf{G}_0$ of filter-observer in the following view

$$\begin{aligned}&\mathbf{G}_0=\mathbf{\Phi}_0\mathbf{\Theta}_r(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi_0}\mathbf{B}_1)^{-1};\\&\mathbf{G}_{0*}=(\mathbf{\Theta}_{r_*}\mathbf{K}_{0_*}+\mathbf{B}_2\mathbf{\Theta}_{\varphi_{0*}})^{-1}\mathbf{\Theta}_{r_*}\mathbf{\Phi}_{0_*},\end{aligned}\tag{5}$$

and measurement error estimation vector for investigated variant of the optimal filtering task could be obtained from the expression (3) using equations (5) as follows

$$\begin{aligned}&\overline{\boldsymbol{\varepsilon}}=\Big[\mathbf{\Phi}_0\mathbf{\Theta}_r(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi_0}\mathbf{B}_1)^{-1}(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1)\\&-\mathbf{\Phi}_0\mathbf{\Theta}_r\Big]\mathbf{L}_{n\times1}=-\mathbf{\Phi}_0\mathbf{\Theta}_r\Big[\mathbf{E}_n-(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi_0}\mathbf{B}_1)^{-1}\\&\times(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1)\Big]\mathbf{L}_{n\times1};\\&\overline{\boldsymbol{\varepsilon}}_*=-\mathbf{L}_{1\times n}\Big[\mathbf{E}_n-(\mathbf{\Theta}_{r_*}\mathbf{K}_*+\mathbf{B}_2\mathbf{\Theta}_{\varphi_*})\\&\times(\mathbf{\Theta}_{r_*}\mathbf{K}_{0*}+\mathbf{B}_2\mathbf{\Theta}_{\varphi_{0*}})^{-1}\Big]\mathbf{\Theta}_{r_*}\mathbf{\Phi}_{0_*}.\end{aligned}\tag{6}$$

Taking into account estimation (6), it is possible to determine, for example as in [7], the minimum value of quality index of measurement performed by the route:

$$\mathbf{I}_{0\,\min}=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\mathrm{tr}(\overline{\boldsymbol{\varepsilon}}\overline{\boldsymbol{\varepsilon}}_*\mathbf{R})ds,\tag{7}$$

where $\mathbf{R}$ is weighting positively definite n-th order matrix.

Substituting expressions (6) into index (7), there is no difficulty to estimate the value of $\mathbf{I}_{0\,\min}$ with the help of known [7] tables of dispersion integrals as follows

$$\begin{aligned}&\mathbf{I}_{0\,\min}=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\mathrm{tr}\Big(\{\mathbf{\Phi}_0\mathbf{\Theta}_r\Big[\mathbf{E}_n-(\mathbf{\Theta}_{r*}\mathbf{K}_*+\mathbf{B}_2\mathbf{\Theta}_{\varphi*})\\&\times(\mathbf{\Theta}_{r*}\mathbf{K}_{0*}+\mathbf{B}_2\mathbf{\Theta}_{\varphi_0*})^{-1}-(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi_0}\mathbf{B}_1)^{-1}\\&\times(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1)+\Big|(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi_0}\mathbf{B}_1)^{-1}\\&\times(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1)\Big|^2\Big]\mathbf{\Theta}_{r*}\mathbf{\Phi}_{0*}\}\,\mathbf{R}\Big)ds.\end{aligned}\tag{8}$$

Certain values of the quality index (8) and their changes in a function of the route variable parameters may be presented in a view of appropriate surfaces of reachable quality of measurement for each set of input data. Also it allows making specific conclusions about effectiveness of introduced synthesis procedure.

Problem 2. The second task of optimal filtering of deterministic information in multivariate measuring route with unstable elements differs a bit from the first one in statement part. Here expressions (1), (2), (4) and (5) are also true (as in the first task), but the new conditions are introduced:

$$\begin{aligned}&\mathbf{G}=\mathbf{G}_0+\hat{\mathbf{G}};\\&\mathbf{K}=\mathbf{K}_0+\hat{\mathbf{K}};\\&\mathbf{\Theta}_\varphi=\mathbf{\Theta}_{\varphi 0}+\hat{\mathbf{\Theta}}_\varphi,\end{aligned}\qquad(9)$$

where the sign “ ^ ” marks variable parts of investigated functions. Besides, considering the conditions (9) the new notion is introduced – an estimation of additional vector of measuring error, caused by parameters variation of the measuring route’s elements

$$\begin{aligned}\hat{\boldsymbol{\varepsilon}}&=\hat{\mathbf{G}}\left(\hat{\mathbf{K}}\mathbf{\Theta}_r+\hat{\mathbf{\Theta}}_\varphi\mathbf{B}_1\right)\mathbf{L}_{n\times 1}\\&=\hat{\mathbf{G}}\left[\left(\mathbf{K}-\mathbf{K}_0\right)\mathbf{\Theta}_r+\left(\mathbf{\Theta}_\varphi-\mathbf{\Theta}_{\varphi 0}\right)\mathbf{B}_1\right]\mathbf{L}_{n\times 1},\\\hat{\boldsymbol{\varepsilon}}_*&=\mathbf{L}_{1\times n}\left[\mathbf{\Theta}_{r*}\left(\mathbf{K}_*-\mathbf{K}_{0*}\right)\right.\\&\left.+\mathbf{B}_2\left(\mathbf{\Theta}_{\varphi *}-\mathbf{\Theta}_{\varphi 0*}\right)\right]\hat{\mathbf{G}}_*.\end{aligned}\qquad(10)$$

As in the first task, the expressions (5) and (6) are considered valid there. These equations characterize, as it was mentioned above, the vector of measuring errors caused by possible difference between real parameters of the route and basic (calculated) ones in various modes of its operation.

Taking into account expressions (10) and (6) it is possible to estimate full vector of errors of the measuring route in the following way

$$\begin{aligned}\boldsymbol{\varepsilon}&=\hat{\boldsymbol{\varepsilon}}+\bar{\boldsymbol{\varepsilon}}=\left\{\hat{\mathbf{G}}\left[\left(\mathbf{K}-\mathbf{K}_0\right)\mathbf{\Theta}_r+\left(\mathbf{\Theta}_\varphi-\mathbf{\Theta}_{\varphi 0}\right)\mathbf{B}_1\right]\right.\\&-\mathbf{\Phi}_0\mathbf{\Theta}_r\left[\mathbf{E}_n-\left(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi 0}\mathbf{B}_1\right)^{-1}\right.\\&\left.\left.\times\left(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1\right)\right]\right\}\mathbf{L}_{n\times 1};\\\boldsymbol{\varepsilon}_*&=\mathbf{L}_{1\times n}\left\{\left[\mathbf{\Theta}_{r*}\left(\mathbf{K}_*-\mathbf{K}_{0*}\right)+\mathbf{B}_2\left(\mathbf{\Theta}_{\varphi *}-\mathbf{\Theta}_{\varphi 0_*}\right)\right]\right.\\&\times\hat{\mathbf{G}}_*-\left[\mathbf{E}_n-\left(\mathbf{\Theta}_{r*}\mathbf{K}_*+\mathbf{B}_2\mathbf{\Theta}_{\varphi *}\right)\right.\\&\left.\left.\times\left(\mathbf{\Theta}_{r*}\mathbf{K}_{0*}+\mathbf{B}_2\mathbf{\Theta}_{\varphi 0*}\right)^{-1}\right]\mathbf{\Theta}_{r*}\mathbf{\Phi}_{0*}\right\}.\end{aligned}\qquad(11)$$

Optimal structure synthesis of $\hat{\mathbf{G}}$ is reasonable to perform basing on Wiener-Kolmogorov method [1, 2, 4], but modernized [3] due to processing problem of exactly deterministic information. As quality functional of measurement in such cases the following expression may be used

$$\mathbf{I}_1=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\operatorname{tr}\left(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}_*\mathbf{R}\right)ds.\qquad(12)$$

By substituting estimated vector (11) of total measurement errors in the route into the functional (12) such formula is obtained:

$$\begin{aligned}\mathbf{I}_1&=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left(\left(\left\{\hat{\mathbf{G}}\left[\left(\mathbf{K}-\mathbf{K}_0\right)\mathbf{\Theta}_r+\left(\mathbf{\Theta}_\varphi-\mathbf{\Theta}_{\varphi 0}\right)\right.\right.\right.\right.\\&\left.\times\mathbf{B}_1\right]-\mathbf{\Phi}_0\mathbf{\Theta}_r\left[\mathbf{E}_n-\left(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi 0}\mathbf{B}_1\right)^{-1}\right.\\&\left.\left.\times\left(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1\right)\right]\right\}\mathbf{L}_n\left\{\left[\mathbf{\Theta}_{r_*}\left(\mathbf{K}_*-\mathbf{K}_{0*}\right)\right.\right.\\&\left.+\mathbf{B}_2\left(\mathbf{\Theta}_{\varphi_*}-\mathbf{\Theta}_{\varphi 0_*}\right)\right]\hat{\mathbf{G}}_*-\left[\mathbf{E}_n-\left(\mathbf{\Theta}_{r_*}\mathbf{K}_*\right.\right.\\&\left.\left.\left.\left.\left.+\mathbf{B}_2\mathbf{\Theta}_{\varphi_*}\right)\left(\mathbf{\Theta}_{r_*}\mathbf{K}_{0_*}+\mathbf{B}_2\mathbf{\Theta}_{\varphi 0_*}\right)^{-1}\right]\mathbf{\Theta}_{r_*}\mathbf{\Phi}_{0_*}\right\}\mathbf{R}\right)\right)ds.\end{aligned}\qquad(13)$$

The first variation of the functional (13) will be of the following form

$$\begin{aligned}\delta\mathbf{I}_1&=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left(\left(\mathbf{R}\left\{\hat{\mathbf{G}}\left[\left(\mathbf{K}-\mathbf{K}_0\right)\mathbf{\Theta}_r\right.\right.\right.\right.\\&\left.+\left(\mathbf{\Theta}_\varphi-\mathbf{\Theta}_{\varphi 0}\right)B_1\right]-\mathbf{\Phi}_0\mathbf{\Theta}_r\left[\mathbf{E}_n\right.\\&\left.\left.-\left(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi 0}\mathbf{B}_1\right)^{-1}\left(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1\right)\right]\right\}\\&\times\mathbf{L}_n\left[\mathbf{\Theta}_{r*}\left(\mathbf{K}_*-\mathbf{K}_{0*}\right)+\mathbf{B}_2\left(\mathbf{\Theta}_{\varphi *}-\mathbf{\Theta}_{\varphi 0*}\right)\right]\\&\times\delta\hat{\mathbf{G}}_*+\delta\hat{\mathbf{G}}\left[\left(\mathbf{K}-\mathbf{K}_0\right)\mathbf{\Theta}_r+\left(\mathbf{\Theta}_\varphi-\mathbf{\Theta}_{\varphi 0}\right)\mathbf{B}_1\right]\\&\times\mathbf{L}_n\left\{\left[\mathbf{\Theta}_{r*}\left(\mathbf{K}_*-\mathbf{K}_{0*}\right)+\mathbf{B}_2\left(\mathbf{\Theta}_{\varphi *}-\mathbf{\Theta}_{\varphi 0*}\right)\right]\hat{\mathbf{G}}_*\right.\\&-\left[\mathbf{E}_n-\left(\mathbf{\Theta}_{r*}\mathbf{K}_*+\mathbf{B}_2\mathbf{\Theta}_{\varphi *}\right)\left(\mathbf{\Theta}_{r*}\mathbf{K}_{0*}\right.\right.\\&\left.\left.\left.\left.+\mathbf{B}_2\mathbf{\Theta}_{\varphi 0*}\right)^{-1}\right]\mathbf{\Theta}_{r*}\mathbf{\Phi}_{0*}\right\}\mathbf{R}\right)ds.\end{aligned}\qquad(14)$$

It is necessary to introduce the following notations:

$$\begin{aligned}&\mathbf{R}=\mathbf{\Gamma}_*\mathbf{\Gamma};\\&\mathbf{D}\mathbf{L}_n\mathbf{D}_*=\left[\left(\mathbf{K}-\mathbf{K}_0\right)\mathbf{\Theta}_r+\left(\mathbf{\Theta}_\varphi-\mathbf{\Theta}_{\varphi 0}\right)\mathbf{B}_1\right]\\&\times\mathbf{L}_n\left[\mathbf{\Theta}_{r*}\left(\mathbf{K}_*-\mathbf{K}_{0*}\right)+\mathbf{B}_2\left(\mathbf{\Theta}_{\varphi *}-\mathbf{\Theta}_{\varphi 0*}\right)\right];\\&\mathbf{T}=\mathbf{T}_0+\mathbf{T}_++\mathbf{T}_-=\mathbf{\Gamma}\mathbf{\Phi}_0\mathbf{\Theta}_r\left\{\mathbf{E}_n\right.\\&-\left(\mathbf{K}_0\mathbf{\Theta}_r+\mathbf{\Theta}_{\varphi 0}\mathbf{B}_1\right)^{-1}\left(\mathbf{K}\mathbf{\Theta}_r+\mathbf{\Theta}_\varphi\mathbf{B}_1\right)\\&\left.\times\left[\left(\mathbf{K}-\mathbf{K}_0\right)\mathbf{\Theta}_r+\left(\mathbf{\Theta}_\varphi-\mathbf{\Theta}_{\varphi 0}\right)\mathbf{B}_1\right]^{-1}\right\}\mathbf{D}.\end{aligned}\qquad(15)$$

Here functions $\mathbf{\Gamma}_*$, $\mathbf{\Gamma}$, $\mathbf{D}$, $\mathbf{D}_*$ are signs of Wiener factorization procedure, and functions $\mathbf{T}_0$, $\mathbf{T}_+$, $\mathbf{T}_-$ mean results of matrix separation process.

With regard for designations (15) the variation (14) of quality functional (13) of measurement should be rewritten as follows:

$$\begin{aligned}\delta\mathbf{I}_1&=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\operatorname{tr}\left(\left(\mathbf{\Gamma}_*\left(\mathbf{\Gamma}\hat{\mathbf{G}}\mathbf{D}-\mathbf{T}\right)\mathbf{L}_n\mathbf{D}_*\delta\mathbf{G}_*\right.\right.\\&\left.\left.+\delta\mathbf{G}\mathbf{D}\mathbf{L}_n\left(\mathbf{D}_*\hat{\mathbf{G}}_*\mathbf{\Gamma}_*-\mathbf{T}_*\right)\mathbf{\Gamma}\right)\right)ds.\end{aligned}\qquad(16)$$

On the basis of modernized Wiener-Kolmogorov method the condition of variation (16) nulling or, in other words, algorithm of optimal structure synthesis of $\hat{\mathbf{G}}$ should be prescribed so:

$$\hat{\mathbf{G}} = \mathbf{\Gamma}^{-1}\left(\mathbf{T}_0 + \mathbf{T}_+\right)\mathbf{D}^{-1}. \qquad (17)$$

Substituting algorithm (17) into expression (13) and with regards to notations (15) it is possible to write down quality functional of measurement, or, to be more precise, its minimum value by the following formula

$$\mathbf{I}_{1\min} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \mathrm{tr}\Big[\left(\mathbf{\Gamma}\hat{\mathbf{G}}\mathbf{D} - \mathbf{T}\right)\mathbf{D}^{-1}\mathbf{D}\mathbf{L}_n\mathbf{D}_*\mathbf{D}_*^{-1}$$

$$\times\left(\mathbf{D}_*\hat{\mathbf{G}}_*\mathbf{\Gamma}_* - \mathbf{T}_*\right)\Big]ds = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \mathrm{tr}\left[\left(\mathbf{T}_-\right)\mathbf{L}_n\left(\mathbf{T}_-\right)_*\right]ds,$$

and determine this minimum value by known [7] table of dispersion integrals.

Further it is possible to estimate and investigate changes of value of $\mathbf{I}_{1\min}$ in the variable parameters function of the measuring route for each of the required modes of the route operation with the help of appropriate software. Also the introduced procedure allows making certain conclusions about providing effective functioning of the measuring route.

Conclusions

In the paper the methodology is presented for solving two variants of important practical problem of optimal filtering of deterministic information in multivariate measuring routes, which are parts of basic on-board equipment of moving objects.

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Received 20 October 2013.

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Л. М. Блохін, Н. Д. Новіцька, І. Ю. Прокоф'єва. Оптимальна фільтрація детермінованої інформації у багатовимірному вимірювальному тракті
Запропоновано нову методологію оптимальної фільтрації детермінованих складових вимірюваної інформації у бортовому багатовимірному вимірювальному тракті. Розглянуто два варіанти поставленої задачі, перший з яких розв'язується для вимірювального тракту з фізично реалізованими (стійкими) елементами, а другий враховує можливість наявності нестійких елементів в структурі системи.
Ключові слова: оптимальна фільтрація; вимірювальний тракт; метод Вінера-Колмогорова; факторизація; сепарація.

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Л. Н. Блохин, Н. Д. Новицкая, И. Ю. Прокофьева. Оптимальная фильтрация детерминированной информации в многомерном измерительном тракте
Предложена новая методология оптимальной фильтрации детерминированных составляющих измеренной информации в бортовом многомерном измерительном тракте. Рассмотрены два варианта поставленной задачи, первый из которых рассчитывается для измерительного тракта с физически реализуемыми (устойчивыми) элементами, а другой учитывает возможность наличия неустойчивых элементов в структуре системы.
Ключевые слова: оптимальная фильтрация; измерительный тракт; метод Винера-Колмогорова; факторизация; сепарация.

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