

# PHYSICS



EDUCATION IN ENGLISH

Module 1

# Mechanics

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# PHYSICS

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**Module 1**

**Mechanics**

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Ukraine's joining Bologna process requires creating new books in physics (in English in particular). The book is developed for all forms of studying physics on the Credit-based Modular System basis in higher school.

«Physics. Module 1. Mechanics» presents Newtonian mechanics and Special theory of relativity. It contains eight Study Units which include theoretical information, test questions, sample problems, laboratory works and individual home tasks.

It is designed for students of engineering specialities.

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## P R E F A C E

### Aim of Studying Physics

The discipline «Physics» has for an object the basic physical phenomena and laws study, mastering fundamental concepts and theories of classical and modern physics, up-to-date research methods and eventually formation of scientific outlook. So, the *tasks* are the following:

- learning objective laws of the world and connections between physical phenomena;
- mastering ways and methods of problem solving;
- acquaintance with experimental equipment, formation of practical skills of experiment performance;
- formation of skill of seeing the concrete physical maintenance in the future profession application.

As a result of studying the discipline «Physics» students are to *know*:

- physical meaning and definition of every used term and concept;
- basic physical phenomena, laws, theories, and spheres of their practical application in engineering;
- major methods of physical researches;
- the system of units of physical magnitudes.

Students ought to get the following *skills*:

- to apply physical laws for practical problem-solving;
- to use physical laws and research methods during studying engineering and special disciplines;
- to carry out physical measurements;
- to estimate errors.

Physics is the basis for studying all other special disciplines in engineering higher educational institutions. Physics explains and gives comprehension of physical phenomena, familiarizes with concepts, models and laws. The aim of studying physics and higher mathematics is to give students a thorough grounding for further mastering engineering subjects.

## Foreword to Module M1 «Mechanics»

The discipline «Physics» is studied during two or three terms and consists of several modules (M). The following modules are studied during the first term: M1 — Mechanics; M2 — Molecular-kinetic theory and thermodynamics; M3 — Electrodynamics.

This manual represents the first module «M1 Mechanics». It helps master essential principles of classical mechanics and theory of relativity.

**Mechanics** is the part of physics which covers the elementary form of motion of matter (mechanical motion). The Greek word «mechanics» means the art to construct machines. It is one of antique sciences which development is closely related with human practical activities. The authors of the first mechanics treatises were Aristotle (IV century B.C.) and Archimedes (III century B.C.). This science got its further development in the works of Leonardo Da Vinci, Galileo, etc. Formation of bases of classical mechanics was completed with the creation of Newtonian dynamics by bringing into being Newton's «Mathematical Principles of Natural Philosophy» (1687).

With further development of physics, it was found out that approaches to studying mechanics can be absolutely different depending on the size of bodies and their speed. Therefore, we distinguish today three directions of mechanics: Classical mechanics (Newtonian mechanics), Relativistic mechanics (Einstein theory of relativity) and Quantum mechanics. All of them study mechanical motion of bodies but have the certain limits of application. *Classical mechanics studies motion of macroscopic bodies at low speeds in comparison with the speed of light. Motion of bodies at speeds close to the speed of light is studied by Relativistic mechanics. Motion of microscopic particles is studied by Quantum mechanics.*

Classical and Relativistic mechanics are studied in this manual. For the description of mechanical phenomena and laws, it is convenient to use the idealized models of real bodies. For example, a body is named **material point (MP)** if its geometrical size can be disregarded. A body is named **perfectly rigid body** or **rigid body (RB)** if its dimensions never change. So, it is possible to disregard deformation of any RB.

*As a result of studying the module*, one ought to know: basic kinematical and dynamic characteristics; laws of motion; the essence of such concepts as work, power, kinetic and potential energy; laws of conservation; postulates of special theory of relativity; Lorentz transformation of coordinates and their consequences; the basic equation of relativistic dynamics and interrelation of mass and energy.

One is to be able to solve direct and inverse problems of kinematics and dynamics, sketch graphs, define errors of physical measurements, use methods of experimental researches in mechanics. It is necessary to understand the limits of application of classical mechanics, the difference between inertial frame and noninertial one, interrelations and interdependence between mechanical values.

Remember that the differential and integral calculus is widely used in the module. So, you should start with revising principal mathematical notion.

**Module «Mechanics»** consists of the following study units (SU):

**Preliminary SU** — Brief mathematical data, Glossary;

**SU 1** — Kinematics;

**SU 2** — Dynamics;

**SU 3** — Laws of conservation;

**SU 4** — Fundamentals of the special theory of relativity (STR);

**SU 5** — Laboratory works;

**SU 6** — Individual home tasks;

**Supplementary SU** — Key words, Help tables.

The Preliminary unit contains the test on mathematic to check if a student is ready to study the module, brief math data and the glossary with explanations of terminology in mathematics and physics.

Study units 1 — 4 include theoretical material, test questions, sample problems as well as problems to solve in classes. Study unit 5 has the theory of errors and instructions to perform laboratory works. Study unit 6 contains problems to be solved by students on their own. Supplementary units lighten the work with the module. The block diagram of the module is represented on Fig. 1.

For proper studying the module, we advise using the self-check questions. Each question is supplied with the reference where to find an answer in case of difficulty.

Concepts which are studied in the module are the basis for all engineering specialities; they are used in theoretical mechanics, resistance of materials, aerodynamics and so on.

## KINEMATICS

*Having worked through the Study Unit students will know main linear and angular kinematics characteristics of motion and their definitions, be able to find these characteristics by different kinds of motion descriptions (vectorial, trajectorial and coordinate). Examples of direct and inverse kinematics problem solutions will help students to analyze and describe mathematically body motion.*

### 1.1. Mechanical Motion. Reference Frame

Kinematics is a part of mechanics that studies motion of bodies regardless the causes of this motion. This term was derived from Greek word «kineta», which means motion. According to the laws and the dependences, established in kinematics, flying object moving parameters are determined; calculations of motion transmission in various aviation mechanisms are done and so on.

Mechanical motion is change of a moving body position in space relative to other bodies. Consequently it follows that mechanical motion is relative. Indeed, any body can be motionless relative to some bodies and moving relative to others. But on the other hand, this motion is absolute, because the body, relative to which a given «motionless» body moves, can always be indicated, i.e. there are no absolutely immovable bodies in nature. Thus, at the beginning of any body motion investigation, one should determine relative to what the other body this motion will be regarded. The body, relative to which motion is regarded, is called reference body. For mathematical description of motion, coordinate system should be connected with the reference body. There exist a lot of different coordinate systems (polar, cylindrical, spherical etc.). The right-angle Cartesian coordinate system is the most widely used.

We should take into account that there exist two types of Cartesian systems: right-hand and left-hand, which differ according the cork-screw rule. If the handle of a screw is rotated from the positive end of OX axis to the positive end of OY axis, then in the right-hand coordinate system the screw will move translationally in the positive direction of OZ axis, and in the left-hand system- in the negative direction. In physics the right-hand system is applied in most cases (Fig. 1.1).

The displacement of bodies takes place in time that is why a clock is also needed for motion description. A reference body, a coordinate system connected with it, and a clock constitute a reference frame.

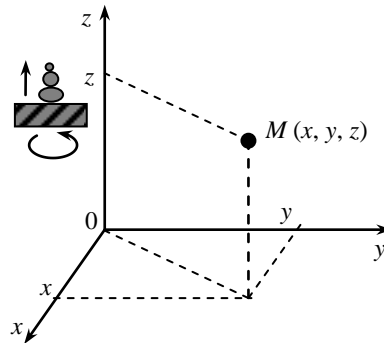


Fig. 1.1

It should be noted that in classical mechanics the *conception of space and time* developed by Newton is generally recognized. According to this conception space and time are regarded as such that are not connected between themselves or with the motion of bodies. In other words, in classical mechanics space and time are thought to be absolute and to exist independently of each other. That is why the time flow does not depend on a reference frame and is the same everywhere.

**1.2. Methods of Material Point Motion Description.  
Direct Problem of Kinematics**

First of all let us regard motion of the simplest object-material point. Let define some notions, which are used in description of such a motion.



Fig. 1.2

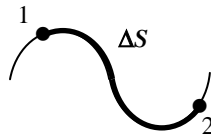


Fig. 1.3

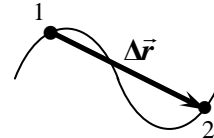


Fig. 1.4

**Trajectory** is an imaginary line a material point travels along (Fig. 1.2).  
**Distance, path** ( $\Delta S$  or  $S$ ) is the length of a trajectory (Fig. 1.3). Distance is a scalar value, and is measured in meters (m) in SI.



**Displacement** ( $\Delta \vec{r}$ ) is the shortest distance between the initial and the final points of a trajectory (Fig. 1.4). Displacement is a vector value, and its direction starts at the initial point and ends at the final point of the trajectory. It is measured in meters (m) in SI.

In Cartesian coordinate system the position of a material point  $M$  can be set either by three coordinates  $(x, y, z)$  or with the help of a position vector  $\vec{r}$ .

**Position vector**  $\vec{r}$  of a point is a vector that starts at the origin and ends at the given point (Fig. 1.5).

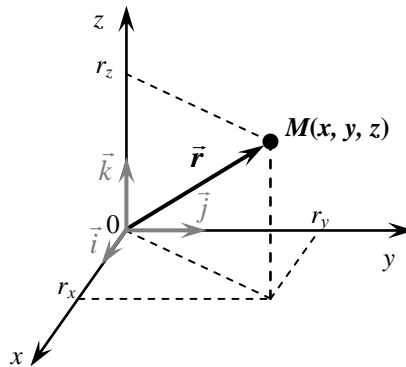


Fig. 1.5

A position vector may be written with the help of its projections  $r_x$ ,  $r_y$ ,  $r_z$  on the corresponding coordinate axes:

$$\vec{r} = r_x \vec{i} + r_y \vec{j} + r_z \vec{k} \quad (1.1)$$

and for its absolute value:

$$|\vec{r}| = r = \sqrt{r_x^2 + r_y^2 + r_z^2}, \quad (1.2)$$

where  $\vec{i}$ ,  $\vec{j}$ ,  $\vec{k}$  are unit vectors of the corresponding coordinate axes:

$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1.$$

Whereas  $r_x = x$ ,  $r_y = y$ ,  $r_z = z$  formulas (1.1) and (1.2) may be written also as:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}, \quad (1.3)$$

$$r = \sqrt{x^2 + y^2 + z^2}. \quad (1.4)$$

In physics the absolute value of any vector  $|\vec{a}|$  is signed as  $a$ .

It is clear, that during material point motion its position vector, distance and coordinates change with time. Correspondingly three kinds of motion law description are used in kinematics:

➤ *vectorial*, when the equation of a point position vector dependence on time is known

$$\vec{r} = \vec{r}(t); \tag{1.5}$$

➤ *trajectorial*, when the equation of a point motion along the trajectory is known

$$S = S(t); \tag{1.6}$$

➤ *coordinate*, when the equations of a point motion in Cartesian coordinates are known

$$x = x(t), y = y(t), z = z(t). \tag{1.7}$$

Equations (1.5), (1.6), (1.7) are called kinematics equations of motion.

*Direct problem of kinematics* lies in finding a material point position at any moment of time if its kinematics equations of motion are known.

### 1.3. Kinematics Characteristics of Material Point Translational Motion

Kinematics characteristics of translational motion are: displacement, velocity and acceleration.

#### 1.3.1. Displacement

A material point at the moment of time  $t$  is in position 1, and after a time interval  $\Delta t$  — it is in position 2 (Fig. 1.6, a).

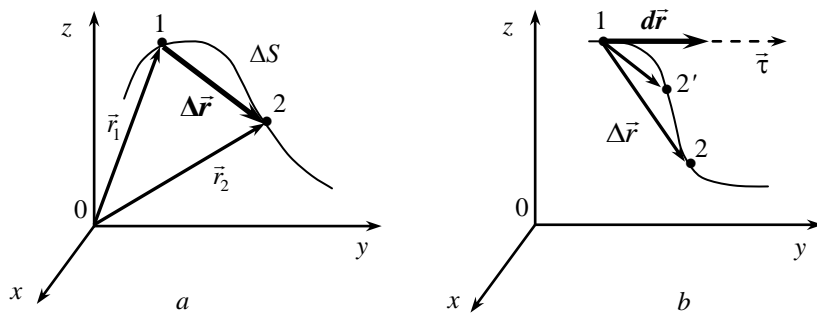


Fig. 1.6

Let us draw the position vectors  $\vec{r}_1$  and  $\vec{r}_2$  at the points 1 and 2. Then the displacement vector will be defined as the following:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1, \quad (1.8)$$

i.e. the **displacement vector** is the change (increment) of the position vector with time. Taking into account expression (1.3), vector  $\Delta\vec{r}$  may be written as the increment of the material point coordinates ( $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ) during time  $\Delta t$ :

$$\Delta\vec{r} = \Delta x \vec{i} + \Delta y \vec{j} + \Delta z \vec{k} \quad (1.9)$$

and its absolute value is:

$$|\Delta\vec{r}| = r = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}. \quad (1.10)$$

From Fig. 1.6, *b* we can see that the displacement vector coincides with the chord, which ties up the corresponding part of the trajectory. That's why in all cases, except rectilinear motion, the absolute value of the displacement vector is less than the distance passed within the same time interval:

$$|\Delta\vec{r}| < \Delta S. \quad (1.11)$$

Now let us shorten the time interval  $\Delta t$  to a quite small value, called *elementary time interval* and denote it as  $dt$ . Thus, during  $dt$  rather a small displacement will take place, and it will be referred to as an *elementary displacement*  $d\vec{r}$ . A material point will pass rather small, i.e. *elementary distance*  $dS$ . When  $\Delta t$  becomes shorter, the value of  $\Delta r$  is drawing closer  $\Delta S$ , i.e. when  $\Delta t \rightarrow 0$  we may count that

$$dr = dS. \quad (1.12)$$

The vector  $d\vec{r}$  is directed along the tangent to the trajectory (Fig. 1.6, *b*). If we denote a unit vector of tangent as  $\vec{\tau}$ , in a vector form we may have (when  $\Delta t \rightarrow 0$ ):

$$d\vec{r} = dS \cdot \vec{\tau}. \quad (1.12, a)$$

### 1.3.2. Velocity

There are average and instantaneous velocities.

**Average velocity** of a displacement  $\langle \vec{v} \rangle$  during time interval  $\Delta t$  is called vectorial value, which equals the ratio of a displacement vector to this time interval:

## CONTENTS

<b>PREFACE</b> .....	3
Aim of studying Physics .....	3
Foreword to module M 1 «Mechanics» .....	4
<b>PRELIMINARY STUDY UNIT</b> .....	7
1. Math self-control questions.....	7
2. Brief mathematical data .....	7
<b>GLOSSARY</b> .....	17
<b>Study Unit 1. KINEMATICS</b> .....	21
1.1. Mechanical motion. Reference frame .....	21
1.2. Methods of material point motion description. Direct problem of kinematics .....	22
1.3. Kinematics characteristics of material point translational motion ...	24
1.4. Inverse problem of kinematics .....	31
1.5. Circular motion of material point.....	32
1.6. Fundamental principles of kinematics of rigid body motion .....	36
1.7. Test questions.....	38
1.8. Sample problems .....	40
1.9. Problems .....	42
<b>Study Unit 2. DYNAMICS</b> .....	44
2.1. Dynamic characteristics of translational motion.....	44
2.2. Newton's Laws.....	46
2.3. Types of forces .....	50
2.4. Dynamic characteristics of rotational motion of rigid body.....	51
2.5. Basic equation of dynamics of rotary motion of rigid body.....	57
2.6. Work, power, efficiency .....	59
2.7. Energy. Mechanical energy .....	63
2.8. Kinetic energy .....	63
2.9. Potential energy .....	65
2.10. Non-inertial reference frames.....	67
2.11. Inertia forces in rotating systems .....	68
2.12. Test questions.....	71
2.13. Sample problems .....	73
2.14. Problems.....	78

<b>Study Unit 3. LAWS OF CONSERVATION</b> .....	80
3.1. Laws of conservation in mechanics .....	80
3.2. Laws of conservation and symmetry of space and time.....	82
3.3. Reactive motion .....	82
3.4. Collision .....	84
3.5. Test questions.....	87
3.6. Sample problems.....	87
3.7. Problems .....	91
<b>Study Unit 4. ELEMENTS OF SPECIAL THEORY OF RELATIVITY</b> .....	93
4.1. Galilean transformations .....	93
4.2. Postulates of special theory of relativity .....	95
4.3. Lorentz transformations and consequences from them.....	95
4.4. Notion of relativistic dynamics .....	99
4.5. Basic equation of relativistic dynamics .....	99
4.6. Kinetic energy of relativistic particle.....	101
4.7. Interconnection of mass and energy .....	103
4.8. Test questions.....	104
<b>Study Unit 5. LABORATORY WORKS</b> .....	106
5.1. Instructions of laboratory works arrangement .....	106
5.2. Theory of Errors.....	106
5.3. Laboratory Works .....	113
<b>Laboratory work 1:</b> Physical measurement errors calculation and solid body density determination.....	113
<b>Laboratory work 2:</b> Random distribution study and determination of gravity acceleration using mathematical pendulum.....	116
<b>Laboratory work 3:</b> Rotational motion laws study and moment of inertia determination .....	121
<b>Study Unit 6. INDIVIDUAL HOME TASKS</b> .....	127
6.1. Order of performing individual home tasks .....	127
6.2. Individual Home Tasks .....	129
6.3. Answers .....	142
<b>SUPPLEMENTARY STUDY UNIT</b> .....	144
Key words .....	144
Help tables.....	147
Literature list .....	150