Least Square Spline decomposition in time-frequency analysis of weather radar signals

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Keywords: radar meteorology, turbulence detection, spline, least square method, LSS-decomposition.

1. INTRODUCTION

Meteorology plays an important role in aviation, as it enables to predict weather conditions and detect flight dangerous meteorological phenomena [1]. Meteorological radar is used to detect the intensity and possible location of precipitation and dangerous zones in them [2]. Doppler radar systems are able to measure the speed of scatteres that constitute meteorological formations and phenomena. The tasks of measurement accuracy increasing and reliability rise of hazardous meteorological phenomena detection become much more relevant after establishing new flight control system CNS ATM adopted by ICAO – the International Civil Aviation Organization.

Atmospheric turbulence is one of the most common hazardous meteorological phenomena for flights. It may change the height, course, speed and the comfort of flight, the aircraft parts wearing out is increased [3]. It can be also a reason of an accident and even catastrophe. According to ICAO, turbulence intensity is classified on the kinetic energy of the eddy dissipation rate (Table 1).

Table 1. Turbulence classification

ε, cm^2/s^2	<0.2	0.2 – 3.4	3.4 – 42.9	42.9 - 550	>550
Intensity scale	Negligible	Light	Moderate	Heavy	Severe

Eddy dissipation rate ε can be estimated with the help of Doppler spectrum width caused by turbulence. Turbulence contribution to the Doppler spectrum variance can be derived from measured Doppler spectrum variance by subtracting a variance caused by drop fall velocity distribution according to [14]. Based on the relationship given in[2] we can use approximate expression for estimating eddy dissipation rate :

$$\varepsilon \approx \frac{0.67\sigma_{Turb}^3}{C_0^{3/2} r\theta_{0,5}} \tag{1}$$

with $C_0 \approx 1.53 - 1.68$ as dimensionless constant, $r\theta_{0,5}$ as radar antenna linear beamwidth (r is radar range, $\theta_{0,5}$ is beam width), σ_{Turb}^2 as Doppler spectrum width caused by turbulence.

For turbulence detection and estimation, the spectral analysis of reflected signal is normally used, particularly time-frequency methods are in usage. For meteorological signals the spectrograms are the most popular [2]. Some attempts of multiresolution wavelet analysis application [9, 10] are also known in meteorological signals and data analysis. Such methods are more flexible and allow varying the resolution in both time domain and frequency domain. However, they resulted in positive outcomes only at large signal-to-noise ratio (SNR). The reason of this is related with the fact that signal interpolation is the basis of the wavelet transform; that is why an efficient estimate on the background of essential white noise is impossible. As a result, the wavelet transform dissatisfies the requirements at low SNR. This paper presents a different advantageous approach to the analysis of meteorological signals using time-frequency decomposition based on least square spline method.

2. METHODOLOGY

Time-frequency decomposition, which is based on multiresolution basis splines and least-squares method – LSS (Least Square Spline) decomposition, can be an alternative to wavelet analysis. This method saves simplicity of wavelet

Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics
Experiments 2011, edited by Ryszard S. Romaniuk, Proc. of SPIE Vol. 8008, 80081T

analysis, but it is much more stable to noise. An advantage of splines is their natural adaptivity, caused by different scales and shifts of the basic functions [4]. Usage of LSS-decomposition in time-frequency analysis is similar to the wavelet decomposition, but the least square method (the base of LSS) in contrast to the interpolation is rather stable to noises.

Let's consider the procedure of LSS-decomposition. General expression for spline with knot points τh is:

$$S(t) = \sum_{\tau=0}^{R} \hat{A}_\tau B_\tau(t, h_\tau), \tag{2}$$

with $\hat{A}_\tau$ as values of spline in knot points; in LSS-decomposition, the set $\hat{A}_\tau$ is the estimation of input data with the least square method; $B_\tau(t, h_\tau)$ is a basis spline; h_τ is width of basis spline; R is number of knot points.

Estimation of parameters of least square method spline model is defined by:

$$\hat{A} = C^{-1} B, \tag{3}$$

where $B = X'Y$ is the vector, which is a scalar product of the input data and correspondent bases functions, X is a basis functions matrix, $C = X'X$.

The matrix C^{-1} has the definite diagonal preference, which is a result of locality of the basis functions. Taking into account the fact that the spline has nonzero values only on the four intervals and is a local function, a significant correlation exists between the knots that are not farther than nine parameters; practically negligible are those elements, which are farther than four elements from the main diagonal. That is why:

$$a_j = d_{i,j} b_j + \sum_{i=1}^{4} d_{i,j+i} \left(b_{j-i} + b_{j+i} \right), \tag{4}$$

with $d_{i,j}$ as the element of the matrix C^{-1}.

From the formulas (2) and (3):

$$S = X\hat{A} = X\left(X'X\right)^{-1} X'Y = LY \tag{5}$$

where L is the matrix with the rows as the pulse response characteristic of the finite impulse response digital low-pass filters. The number of nonzero elements in the rows of the matrix C^{-1} can be considered as seven, therefore for the calculation seven data segments is needed. To increase the accuracy of calculation it is possible to increase the number of data segments.

Therefore the algorithm can be considered as decimation digital filter and is performed in the real time mode.

Basis spline consists of four segments $x \in [-1,0]$, $x \in [0,1]$, $x \in [1,2]$, $x \in [2,3]$, that are shown in figure 1.

If the Fourier transform is applied to the spline, the parameters of the spline (that are the value in knot points) will characterize the weight of the certain frequency domain that is set by frequency pattern of the basis spline. The basis spline spectra of the same scale shifted in time differ only by a phase. The change of basis width υ influences onto the width of basis spline spectrum. Thus, every coefficient of a spline that characterizes the certain frequency band in the certain time refers to the certain knot point of the spline

$$B_{\tau,\upsilon}(t) = B_0\left(\frac{t-\tau}{\upsilon}\right),\ \upsilon \in R,\ \tau = 0, \pm 1, \pm 2, \ldots \tag{3}$$

with τ as a shift of the spline relative to the initial basis spline, υ as a scale coefficient of a spline relative to the basis spline.

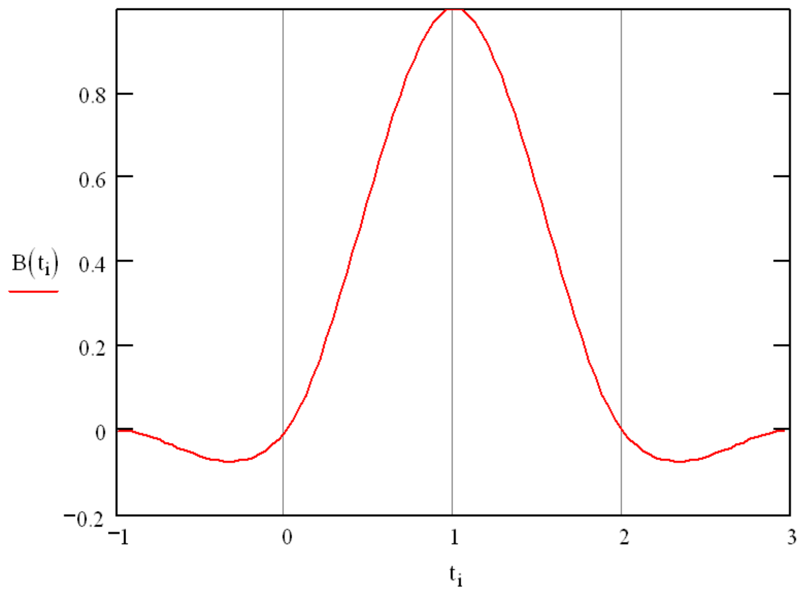

Figure 1. Four segments of the basis spline.

Spectra of basis splines of different width are shown in figure 2 (width change is multiple of 2).

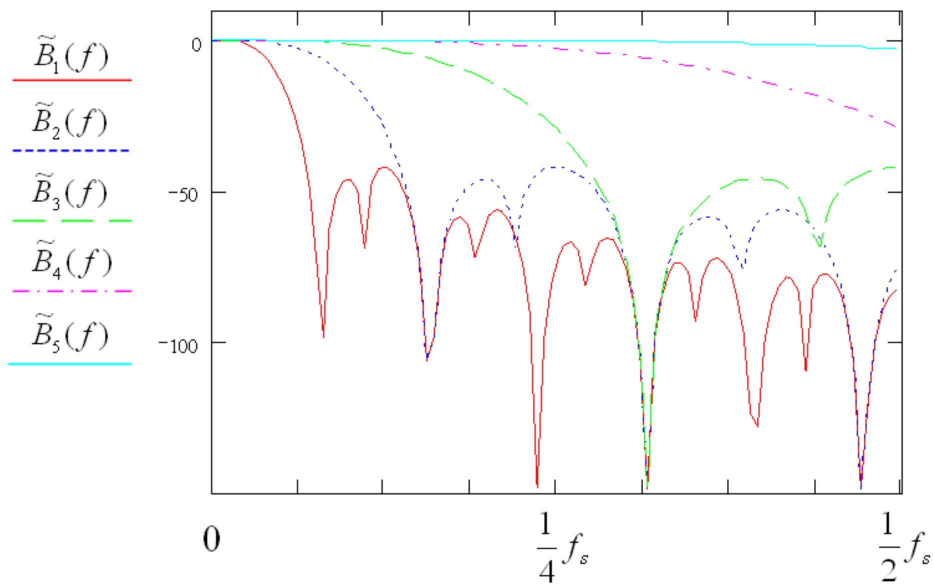

Figure 2. Spectra of basis splines.

Thus, spline parameters of each scale are able to characterize the frequency domain in certain moments of time. And estimations of spline parameters are obtained by least square method. Let's use these peculiarities for frequency-time analysis.

Schematic model of one stage of the LSS is shown at Fig.3 right part, with y_i (dots at upper part of the figure) as input signal, $S(t_i, \hat{A})$ as a spline, constructed by least square method, e_i as the difference between input signal and constructed spline [5].

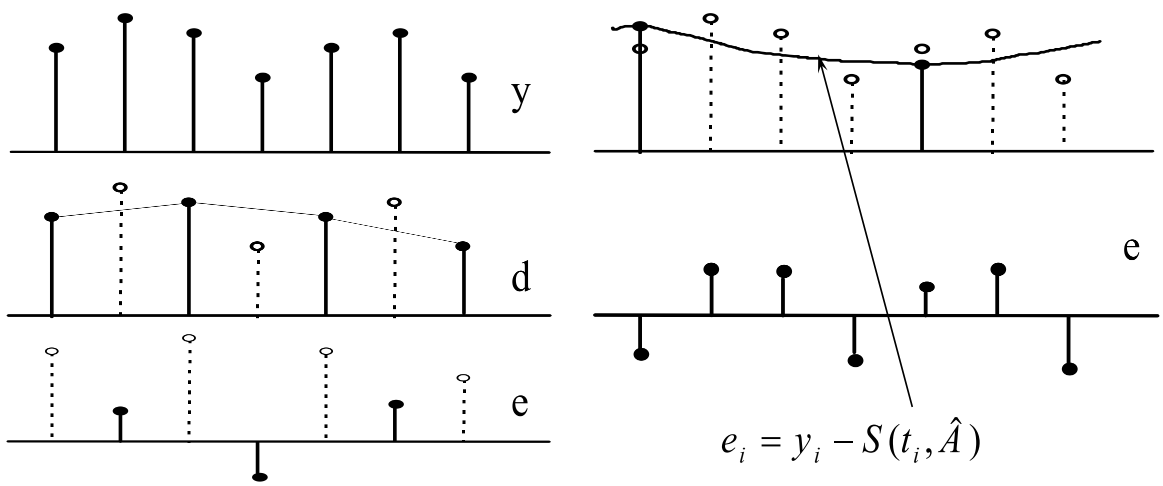

Figure.3. Schematic model of LSS stage in comparison with wavelet decomposition.

One can conclude that the proposed LSS decomposition is rather similar to the wavelet decomposition (fig.3, left part). But instead of interpolation, which is applied at wavelet decomposition, the least square method is used in case of LSS decomposition.

The proposed decomposition is performed in several stages. Spline coefficients characterizing frequency domain, which remain of each stage are input data for the next stage of the decomposition. The basis spline width decreases with every next stage. The width is set to be the multiple of sampling period.

We shall see below that for the decomposition of meteorological reflected signal at the first stage the width of basis spline is taken of 64 samples, at the second stage it is 32 samples, then 16 samples, etc. The narrower the basis spline, the higher the time (range) resolution and the lower frequency resolution. In general, a spline width at each stage can be changed according to certain signal depending on a part of the spectrum that is a subject of investigation.

Each decomposition stage divides the signal on low-frequency band and high-frequency band with the help of couple of mirror filters. At each stage, the spline extracts its frequency band from high-frequency band. As a result we get decomposition of the signal on frequency bands as it is shown in figure 4.

At the first stage the low-frequency band is extracted. At the second stage the correspondent frequencies are absent, because earlier the spline obtained by LSM was subtracted from the signal. At each stage, a frequency band extracted by spline redoubles.

Signal contribution in each band is characterized by knot-point values of the spline at each definite scale. Thereby frequency-time analysis is performed.

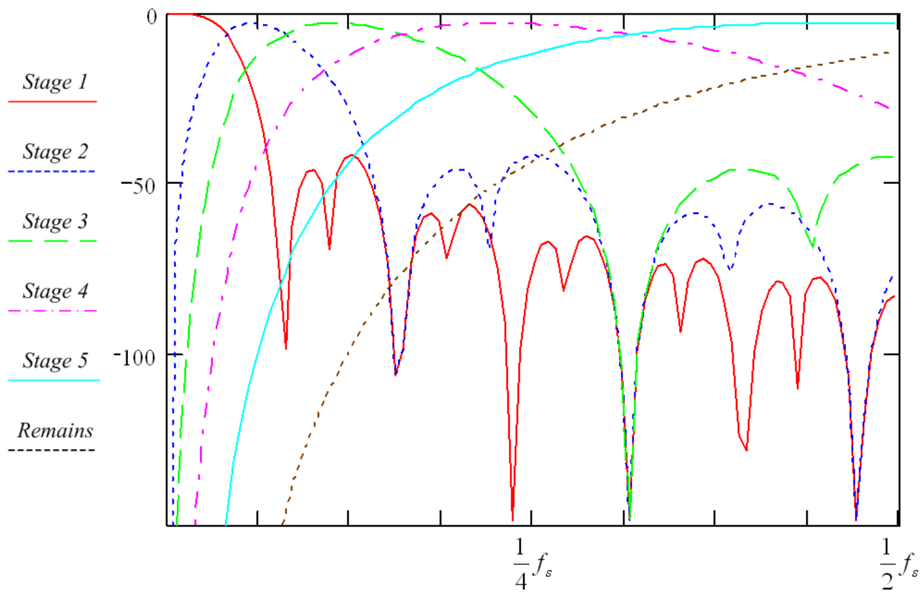

Figure 4. Decomposition of the signal on frequency.

3. LSS-DECOMPOSITION OF METEOROLOGICAL DATA

For frequency-time analysis the data of Transportable Atmospheric Radar (TARA) [6, 7] were used. TARA is an S-band FMCW Doppler polarimetric radar. The data were obtained during the rain sounding campaign at the Delft University of Technology (The Netherlands). These data were provided to the National Aviation University for processing in the framework of the joint project.

TARA frequency modulated signal has a saw tooth shape, 512 "sweeps" in one profile, each with 1024 range rates. Range resolution is 15 meters, so sampling frequency is 20MHz.

In order to find the Doppler spectrum, all 512 sweeps (of same range bin) need to be processed. The LSS – decomposition significantly reduces the amount of information required for calculations.

As an example, the main component signal of vertical polarization (VV) at antenna tilt of 45 degrees was selected for the processing and decomposed. The first profile was used for LSS – decomposition in this example.

For the implementation of LSS-decomposition the data of the 250 sweep were selected. Decomposition was performed in six stages, as is shown in Fig.5.

At the first stage of decomposition, the spline fragments according to LSM (least square method) are constructed every 64 samples. The second stage is the difference between the signal and the spline that approximates by fragments constructed every 32 samples; and so on, increasing the number of spline fragments.

The seventh graph (below) shows the remains of decomposition, that is, the difference between signal on the sixth graph and approximation spline (Fig.5).

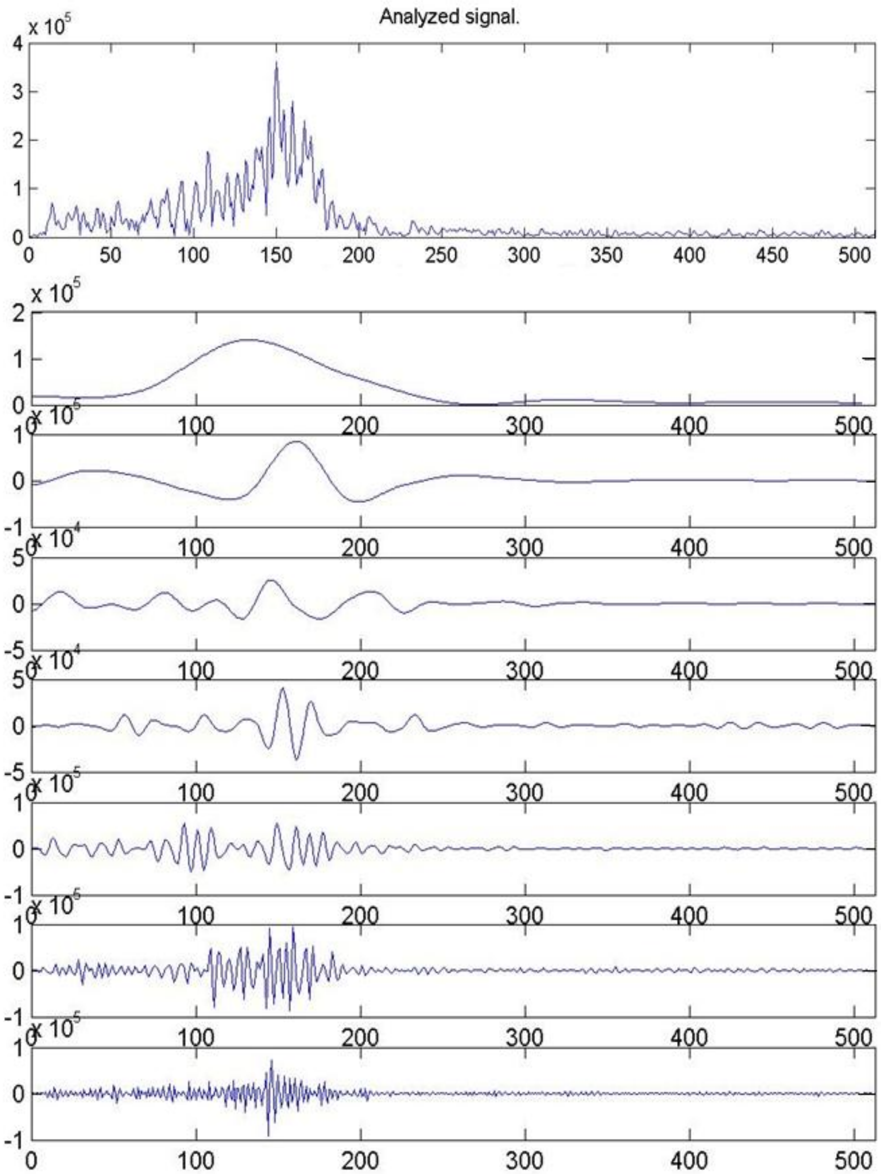

Figure 5. Stages of the decomposition.

Thus, we can see that in contrast to the wavelets, high-frequency but not low-frequency component is further decomposed.

In figure 6 the final result of LSS decomposition is given, where the coefficients at each stage (Fig. 5) are represented by gray scale (intensity of color).

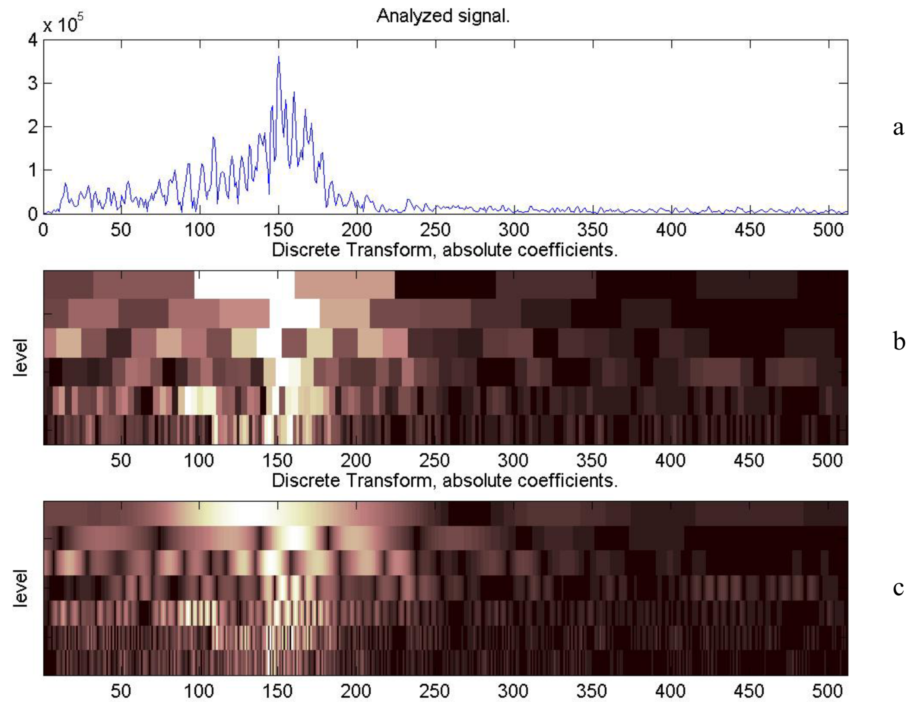

Figure 6. Result of LSS decomposition: a) analyzed signal; b) set of spline knot points; c) set of splines.

During every stage of the decomposition, the time (range) resolution increases and the frequency resolution decreases. That is why the result of decomposition has the form shown in figure 7.

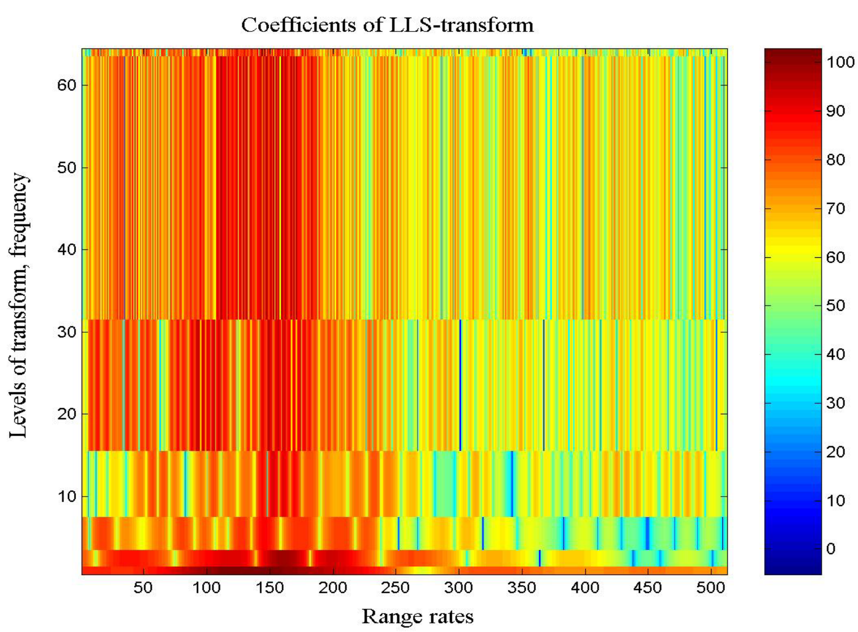

Figure 7. The result of LSS-decomposition taking into account frequency and range resolution.

As far as the definition of hazardous turbulence is connected with Doppler spectrum width caused by turbulence contribution σ_{Turb} (see formula 1), analyzing the results, the level of danger can be determined. Frequency extraction scheme can be changed by changing the width of the basis splines. Thus, one can see that different time and frequency resolution can be used depending on the frequency band, which is a subject of the investigation in particular case.

4. CONCLUSIONS

In this paper a new approach for meteorological Doppler radar signal processing has been proposed. Least Square Spline decomposition has been analyzed and applied for real data processing.

Qualitative comparison of LSS-decomposition with wavelet decomposition has been done. The advantage of LSS-decomposition is the less input data that are needed that makes possible to use this method for airborne radar signal processing.

The Method is rather stable to noise due to the features of the least square method applied, and it has smooth spectral functions. This property is especially important for implementation of new sophisticated methods, related with spectral distributions of polarimetric parameters [11] like spectral different reflectivity [12] and differential Doppler velocity [13].

The least square method in LSS-decomposition allows obtaining the evaluation of reliability for the coefficients of the decomposition. This fact can be used as the ground for further development of new turbulence detecting algorithms, based on testing statistical hypotheses about decomposition coefficients.

Prospects of further research are related with choosing the frequency division scheme according to the type of meteorological phenomenon that has to be detected, constructing the confidence intervals, and then a hardware implementation of the developed algorithms.

Further research will be related with development of the criterion and fulfilling a qualitative comparison of considered approaches. The limits of SNR when the method is suitable for operation should be investigated.

Another important task that will be the aim of future research is application LSS-decomposition to more data taking into account polarimetric features of the signals and development of LSS equivalents to Doppler-polarimetric signal processing [14] in real time.

5. ACKNOWLEDGEMENT

The raw data of rain remote sensing that were processed in this work were provided by the International Research Center for Telecommunications and Radar at the Delft University of Technology, Delft, The Netherlands.

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