Copula Simulation of Weather Radar Inputs

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Abstract—**This paper presents the copula based simulation of weather radar preprocessed input signals with orthogonal polarizations. The use of copulas allows taking into account their mutual dependence in a straightforward and natural way, because the copula relates exactly to the pure dependence between the marginal distributions.**

Index Terms— **Radar signal processing; meteorological radar**

I. Introduction

The modelling of input signals for polarimetric weather radars has to take into account the mutual dependence between the received polarizations for the various scattering conditions [1-3]. So, for adequate simulation results, there not only the marginal distributions of input signal should be captured, but also the structure of their influence by each other.

As shown in [4-6], one of the elegant approaches to get the mutual dependence between the sets of radar input signals is the use of copulas. Advantages of this approach include the invariance on strictly increasing transformations of input data, as well as independence on the marginal distributions. Though the most applications of copula modelling were presented in econometrics, the [7-9] show the using of copulas in hydrology for the rainfall simulations.

This paper presents the copula modeling of weather radar inputs. The preprocessed received signals of programmable Polarimetric Agile Radar in S- and X-band (PARSAX) [10] were used as the basis for simulation of different weather conditions.

II. Copula Matching

Let consider the two sample sets of weather radar preprocessed input signals with orthogonal polarizations, for example, $\mathbf{S}_{HH}$ and $\mathbf{S}_{VV}$. For the purpose of simplicity, the absolute values S_{HH} and S_{VV} of these signals will be used.

The first step in the modelling of radar inputs with the same mutual dependence as original experimental ones is the selection of proper copula and its fitting method. Thanks to the Sklar's theorem [6], one can write:

$$F(S_{HH}, S_{VV}) = C(F_{HH}(S_{HH}), F_{VV}(S_{VV})) = \\ = C(S_{HH}^{UF}, S_{VV}^{UF}),$$

where $F(S_{HH}, S_{VV})$ is bivariate cumulative distribution function (CDF) of S_{HH} and S_{VV} sets; $S_{HH}^{UF} = F_{HH}(S_{HH})$ and $S_{VV}^{UF} = F_{VV}(S_{VV})$ are transformed variables with uniform distributions, i.e. CDFs of original S_{HH} and S_{VV} signals.

There are several copula families exist, one of which is called Archimedean [11]. The advantages of these copulas are the quite simple formulation that can be given with single parameter as well possibility to easily construct multivariate distributions if needed. We will use the two popular Archimedean copulas named Clayton and Gumbel. Another widely used copula family is elliptical copulas with Gaussian copula as most famous representative. The expressions for the mentioned copulas can be written as follows [12]:

$$C_{CL}\left(S_{HH}^{UF}, S_{VV}^{UF} \middle| \theta\right) = \left(S_{HH}^{UF\,-\theta} + S_{VV}^{UF\,-\theta} - 1\right)^{-\frac{1}{\theta}},$$

$$C_{GM}\left(S_{HH}^{UF}, S_{VV}^{UF} \middle| \theta\right) = \exp\left(-\left[\left(-\log\left(S_{HH}^{UF}\right)\right)^{\theta} + \left(-\log\left(S_{VV}^{UF}\right)\right)^{\theta}\right]^{\frac{1}{\theta}}\right),$$

$$C_{GS}\left(S_{HH}^{UF}, S_{VV}^{UF} \middle| \theta\right) = \\ = \int_{-\infty}^{S_{HH}} \int_{-\infty}^{S_{VV}} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left(\frac{2\theta xy - x^2 - y^2}{2\left(1-\theta^2\right)}\right) dxdy,$$

where C_{CL}, C_{GM} and C_{GS} are the Clayton, Gumbel and Gaussian copulas, respectively. In order to match the selected copula to the experimental data, we need to make an estimation, that can be parametric or not. Among the nonparametric approaches, the Kendall's tau based estimate gives one the dependence between the value of τ and the parameter θ for the considered copulas in the following way [12]:

$$\theta_{CL} = \frac{2\tau}{1-\tau}, \quad \theta_{GM} = \frac{1}{1-\tau}, \quad \theta_{GS} = \sin\left(\frac{\pi}{2}\tau\right).$$

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In order to calculate appropriate Kendall's tau, one need to build the experimental copula, using the kernel estimates.

Although, the Kendall's tau based estimate is convenient nonparametric way, it can be implemented for only given region of τ values depending on copula type. More general nonparametric method corresponds to the maximum pseudolikelihood estimation [12].

III. Copula Simulation and Dependence Representation

After the fitting of selected copula to the experimental data one can use the conditional distribution $\frac{\partial}{\partial S_{HH}^{UF}} C\left(S_{HH}^{UF}, S_{VV}^{UF}\right)$ in order to build another data set with the same dependence structure.

The convenient way to visualize the mutual dependence between two data sets is so-called Chi-plots, that was proposed to build using ranks [7]. As we use the kernel estimates for the CDFs of original data sets and their copulas, the corresponding Chi-plot can be based on the calculated CDFs, as follows:

$$H_i = \frac{n\hat{C}\left(S_{HH}^{UF}, S_{VV}^{UF}\right) - 1}{n-1},$$

$$F_i = \frac{(n+1)\hat{S}_{HH}^{UF} - 1}{n-1}, \; G_i = \frac{(n+1)\hat{S}_{VV}^{UF} - 1}{n-1},$$

where n is the number of samples in set, the cap $\hat{}$ means the kernel estimate. H_i, F_i, and G_i are used to calculate the pairs of (χ_i, λ_i) for Chi-plot in a usual way [13, 14]:

$$\chi_i = \frac{H_i - F_i G_i}{\sqrt{F_i(1-F_i)G_i(1-G_i)}},$$

$$\lambda_i = 4\,\mathrm{sign}\left(\tilde{F}_i \tilde{G}_i\right)\max\left(\tilde{F}_i^2, \tilde{G}_i^2\right),$$

$$\tilde{F}_i = F_i - 1/2, \; \tilde{G}_i = G_i - 1/2.$$

The values of χ_i that correspond to independent initial signals are close to zero, while substantial non-zero values show the mutual dependence.

IV. Simulation Results

The experimental data from PARSAX correspond to the wet snow weather condition. The number of time-series samples is 512 for the each of considered polarizations (HH and VV). The region of consideration with precipitations is shown in Fig. 1.

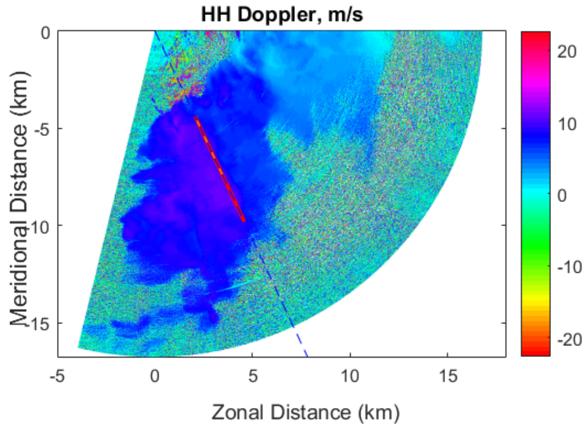
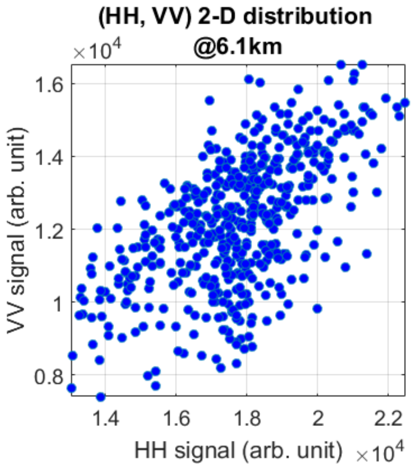
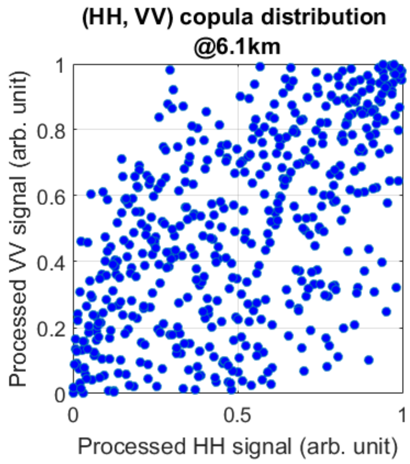


Figure 1. Area of consideration with wet snow precipitations. Pink line shows the direction of observing

The Fig. 2-a shows the mutual distribution of input signals with HH and VV polarizations, while Fig. 2-b shows the mutual distribution of transformed with CDF signals.



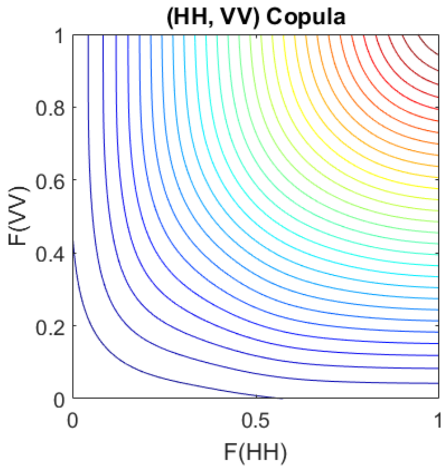
(a)



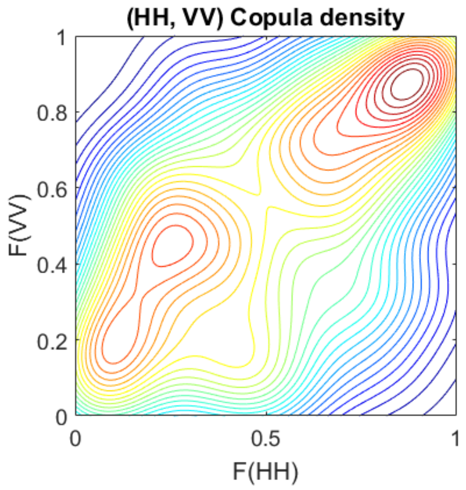
(b)

Figure 2. Mutual distribution of orthogonally polarized input signals (a) and transformed with CDF signals (b) for the range bin, that corresponds to the 6.1km radial distance from the PARSAX

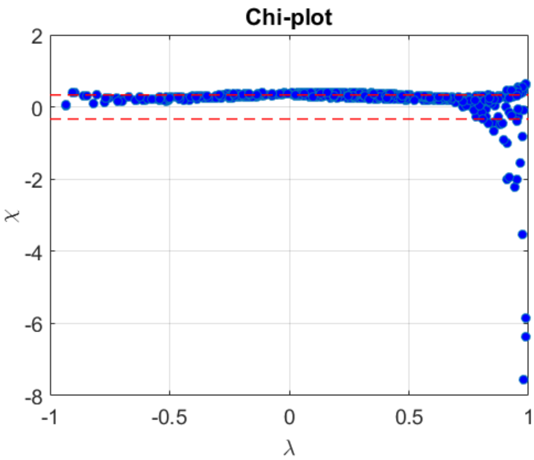
The experimental copula and copula density contour plots as well as Chi-plot are shown in Fig. 3. It can be seen, that Chi-plot is more convenient in order to graphically represent the dependence.



(a)



(b)



(c)

Figure 3. Experimental data plots: copula (a), copula density (b), Chi-plot (c)

The value of Kendall's tau is equal to 0.409 for the considered experimental copula. The simulated Clayton, Gumbel and Gaussian copulas, copula densities and Chi-plots are shown in Figs. 4 - 6.

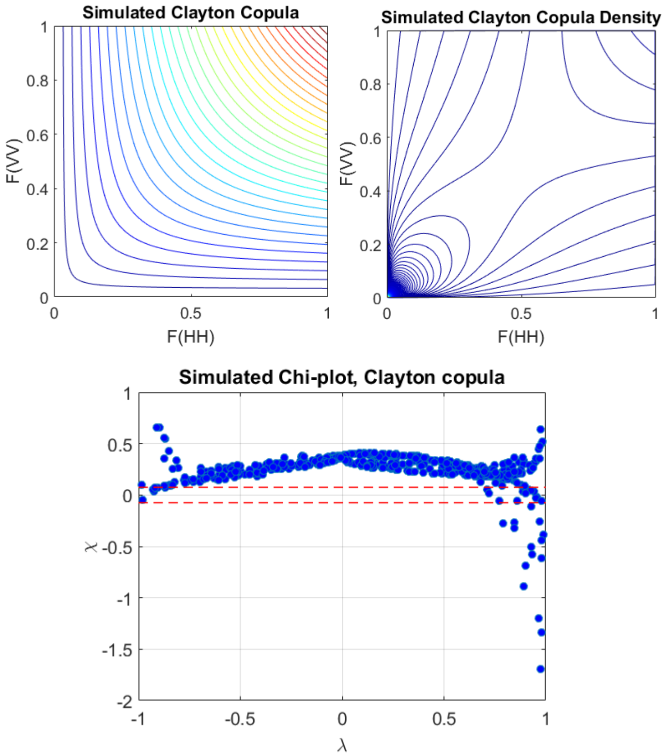


Figure 4. Clayton copula simulation results

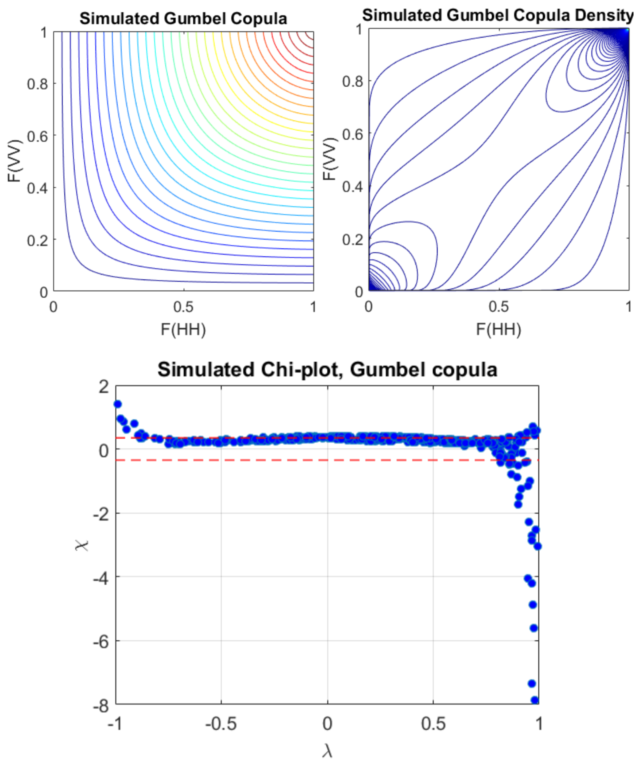


Figure 5. Gumbel copula simulation results

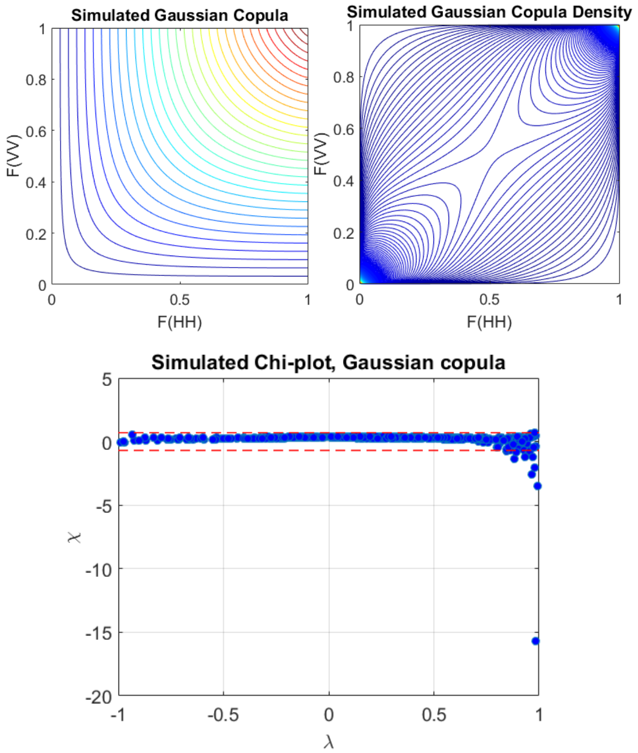


Figure 6. Gaussian copula simulation results

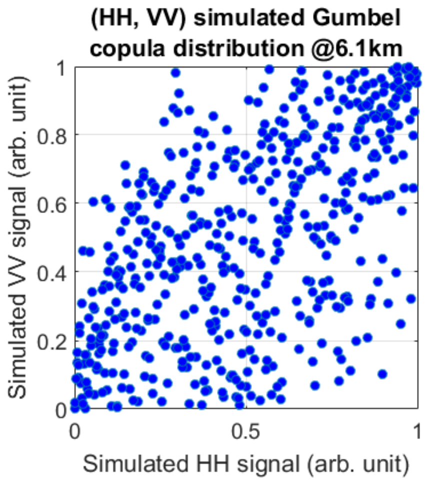


Figure 7. Simulated orthogonally polarized signals mutual distribution appropriate to the Gumbel copula

As can be seen, the best fitting for the experimental signals that appropriate to wet snow weather conditions is achieved for the Gumbel copula. The mutual distribution of simulated signals for this copula is shown in Fig. 7.

V. Conclusion

The proposed copula simulation of polarimetric weather radar input signals allows to take into account not only the marginal statistical distributions of orthogonally polarized components, but also their mutual distribution. This point is very important exactly for polarimetric measurements and simulations because the relations between the horizontally and vertically polarized signals are the basis for many useful polarimetric parameters.

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