Copula Analysis of Full Polarimetric Weather Radar Complex Signals

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***Abstract*—In this paper we propose the method of transition from the complex full polarimetric radar signals to univariate uniformly distributed marginals that can be used for more precise copula analysis of statistical dependence between the complex samples of signals received at orthogonally polarized sounding waveforms. The method is applied to meteorological radar signal processing.**

***Keywords*—radar signal processing; meteorological radar; radar polarimetry**

I. INTRODUCTION

Modern full polarimetric meteorological radars are capable to provide simultaneous information about co-polar and cross-polar back-scattering polarization matrix components. This gives a great advantage to obtain major polarimetric parameters, such as differential reflectivity, linear depolarization ratio, cross-correlation coefficient, etc. [1-3] that can be used to provide important weather information. Analysis of these parameters is usually based on some statistical models of reflected signals for different polarization alternates [3, 4].

In order to avoid any suppositions about the probability density functions (PDF) of signals received through the polarization channels of meteorological radar, the copula approach was introduced [5, 6]. The new copula-based cross-correlation functions proposed in [5] allow obtaining the information about statistical dependence between the signals received at orthogonally polarized sounding waveforms. The theoretical results were verified using the measurements carried out with dual-polarization radar TARA. Further adoption of copula approach was made in [6], where the copula-based cross-spectra functions were proposed, and the measurement data collected by full polarimetric PARSAX radar system [7] were used for analysis.

However, as the copula is associated with univariate marginal distributions, one needs to deal with scalar variables instead of complex, which usually provided by polarimetric measurements. In [5] the consideration was truncated to account the real parts of complex samples only, while in [6] the absolute values were used. In order to collect more precise information about dependence between the polarimetric raw data, the whole complex data sets should be considered. That is why, the method of transition from complex full polarimetric radar data to univariate marginal distributions is proposed in this paper. The measured signals received by PARSAX radar is used to illustrate the application of the method.

II. COMPLEX DATA OF FULL POLARIMETRIC RADAR

The full polarimetric radar measures the in-phase and quadrature (I/Q) components of backscattered signals for the following combinations of the linear receive and transmit polarizations: horizontal-horizontal (HH), horizontal-vertical (HV), vertical-horizontal (VH), and vertical-vertical (VV). Using the I/Q data, any of the corresponding elements of complex scattering matrix S_{HH}, S_{HV}, S_{VH}, or S_{VV}, which relates the sounding and the scattered waves, could be obtained. In case of reciprocal medium, the cross-polar components S_{HV} and S_{VH} are the same, so only three scattering elements are unique and the polarimetric data can be represented by a complex vector [8]:

$$\mathbf{u} = \begin{bmatrix} S_{HH} & S_{HV} & S_{VV} \end{bmatrix}^T .$$

The PARSAX radar system can simultaneously radiate, receive and process both vertical and horizontal polarizations with a possibility to send the signal in the same direction during many periods of observation, so the whole polarimetric information about the resolution volume is accessible at each sounding period.

III. COPULA ANALYSIS OF COMPLEX DATA

As is known, the cumulative distribution function (CDF) of a random variable with any continuous PDF has the uniform PDF. This property is used successfully in copula analysis for invariant transforming a given variable into the random quantity with the uniform distribution [9-14]. So as the polarimetric data are normally expressed by complex numbers they have bivariate CDF. That is why we suggest additional CDF transformation of initial bivariate cumulative distribution in order to achieve univariate uniformly distributed marginals, which take into account the entire complex values.

At the first step, the complex elements S_i of scattering matrix, where index i denote one of the combinations of horizontal and vertical polarizations HH, HV, or VV, are transformed using the bivariate CDF

$$S_i^{CDF} = F_i(\mathrm{Re}\{S_i\}, \mathrm{Im}\{S_i\}) ,$$

where F_i is the bivariate cumulative distribution functions of real and imaginary parts of complex polarimetric data. Now, let consider S_i^{CDF} as the scalar variable that corresponds to the initial complex one S_i. The second step is to transform the S_i^{CDF} into a uniformly distributed variable S_i^{UF}, using again CDF but just univariate for now

$$S_i^{UF} = \Phi_i\left(S_i^{CDF}\right),$$

where Φ_i is the cumulative distribution functions of S_i^{CDF}. Thus, the transition from complex polarimetric variables to univariate uniformly distributed copula marginals can be now expressed as

$$S_i^{UF} = \Phi_i\left[F_i(\mathrm{Re}\{S_i\}, \mathrm{Im}\{S_i\})\right] .$$

Using obtained scalar uniformly distributed polarimetric variables S_i^{UF}, the copula analysis can be conducted. The bivariate CDF of any pair of these variables give us the corresponding copula, according to the Sklar's theorem:

$$H\left(S_i^{CDF},\ S_j^{CDF}\right) = C\left(\Phi_i\left(S_i^{CDF}\right), \Phi_j\left(S_j^{CDF}\right)\right),$$

where index j denotes the polarization same way as i, and $H\left(S_i^{CDF},\ S_j^{CDF}\right)$ is bivariate CDF of corresponding pair. In explicit way, copula can be represented as:

$$C\left(S_i^{UF}, S_j^{UF}\right) = H\left(\Phi_i^{-1}\left(S_i^{UF}\right), \Phi_j^{-1}\left(S_j^{UF}\right)\right).$$

Copula density function can be written as follows:

$$c\left(S_i^{UF}, S_j^{UF}\right) = \frac{\partial C\left(S_i^{UF}, S_j^{UF}\right)}{\partial S_i^{UF} \partial S_j^{UF}} .$$

In order to process the experimental polarimetric data, we can use the kernel estimates. The bivariate cumulative distribution functions $F_i(\mathrm{Re}\{S_i\}, \mathrm{Im}\{S_i\})$ are replaced by estimates $\hat{F}_i(\mathrm{Re}\{S_i\}, \mathrm{Im}\{S_i\})$ in such a way that with increasing of samples' number the estimates are converging to CDF:

$$\hat{F}_i(\mathrm{Re}\{S_i\}, \mathrm{Im}\{S_i\}) = \frac{1}{N}\sum_{n=1}^{N} K_F\left(\mathrm{Re}\{S_i\} - \mathrm{Re}\{S_{i_n}\}, \mathrm{Im}\{S_i\} - \mathrm{Im}\{S_{i_n}\}\right),$$

where N is the samples' number, K_F is the formula, which has a form of bivariate cumulative probability distribution function. Furthermore, the cumulative distribution functions $\Phi_i\left(S_i^{CDF}\right)$ are replaced by estimates $\hat{\Phi}_i\left(S_i^{CDF}\right)$:

$$\hat{\Phi}_i\left(S_i^{CDF}\right) = \frac{1}{N}\sum_{n=1}^{N} K_\Phi\left(S_i^{CDF} - S_{i_n}^{CDF}\right),$$

where the K_Φ has a form of CDF.

The copula estimate $\hat{C}\left(S_i^{UF}, S_j^{UF}\right)$ is done by the same way with K_C in a form of bivariate CDF:

$$\hat{C}\left(S_i^{UF}, S_j^{UF}\right) = \frac{1}{N}\sum_{n=1}^{N} K_C\left(S_i^{UF} - S_{i_n}^{UF}, S_j^{UF} - S_{j_n}^{UF}\right),$$

while the copula density estimate $\hat{c}\left(S_i^{UF}, S_j^{UF}\right)$ utilize the K_c in the form of PDF of normal distribution:

$$\hat{c}\left(S_i^{UF}, S_j^{UF}\right) = \frac{1}{N}\sum_{n=1}^{N} K_c\left(S_i^{UF} - S_{i_n}^{UF}, S_j^{UF} - S_{j_n}^{UF}\right).$$

Proposed method allows taking into account the complex nature of full polarimetric radar data for copula analysis.

IV. Experiment And Data Processing

The proposed method of transition from the complex full polarimetric radar signals to univariate uniformly distributed marginals was checked with the experimental data obtained from PARSAX radar. The considering area with the regions of precipitations is shown in Fig. 1.

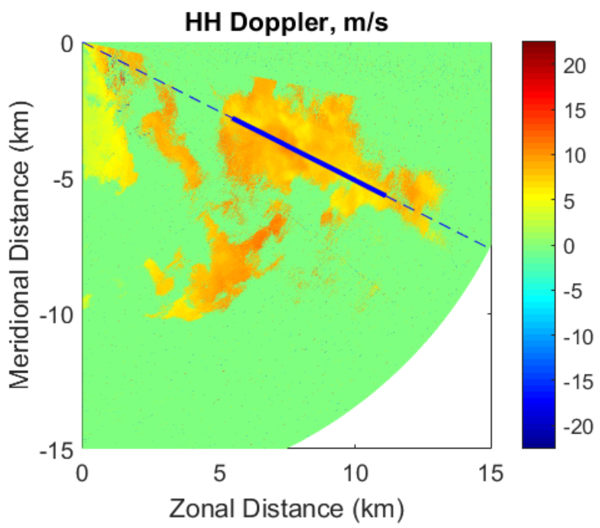

Fig. 1. The considering region with precipitations. Dashed line shows the processing direction (azimuth 117^0), while bold solid line shows the processing range bins.

The processing complex signals consist of the 512 observation periods for each of range bins. The area of the radar observation contains the thunderstorm in this example. Meteorological interpretation of the processed data is not a goal of this paper and is the subject of additional research. The absolute values of initial complex polarimetric signals for HH, HV, and VV polarizations are presented in Fig. 2.

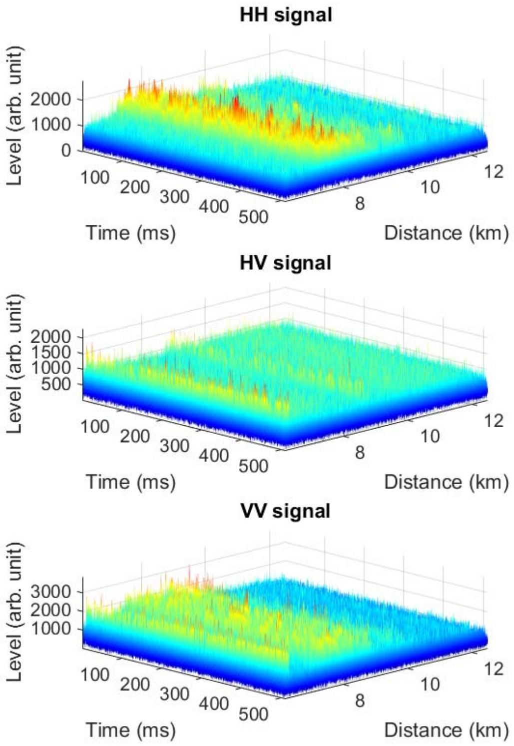

Fig. 2. The absolute values of initial polarimetric signals for HH, HV, and VV polarizations corresponding to the considering region with precipitations.

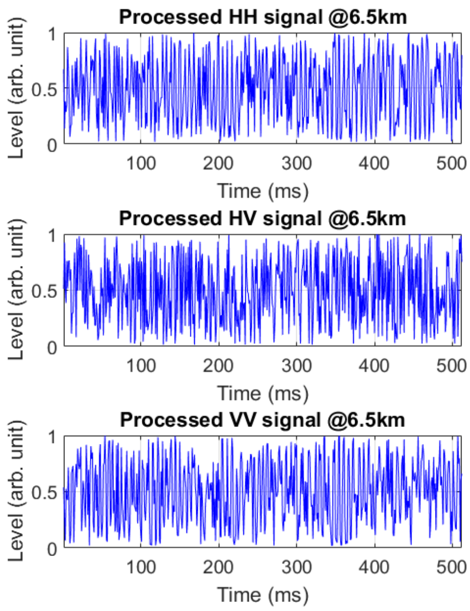

Fig. 3. The processed uniformly distributed signals.

The processed scalar uniformly distributed variables are depicted in Fig. 3. They are used to compose the copula bivariate distributions shown in Fig. 4. The Kendall's tau correlation coefficients are equal to 0.28, -0.06 and -0.03 for (HH, VV), (HV, HH) and (HV, VV) pairs respectively.

The corresponding copulas and copula densities are presented in Figs. 5, 6.

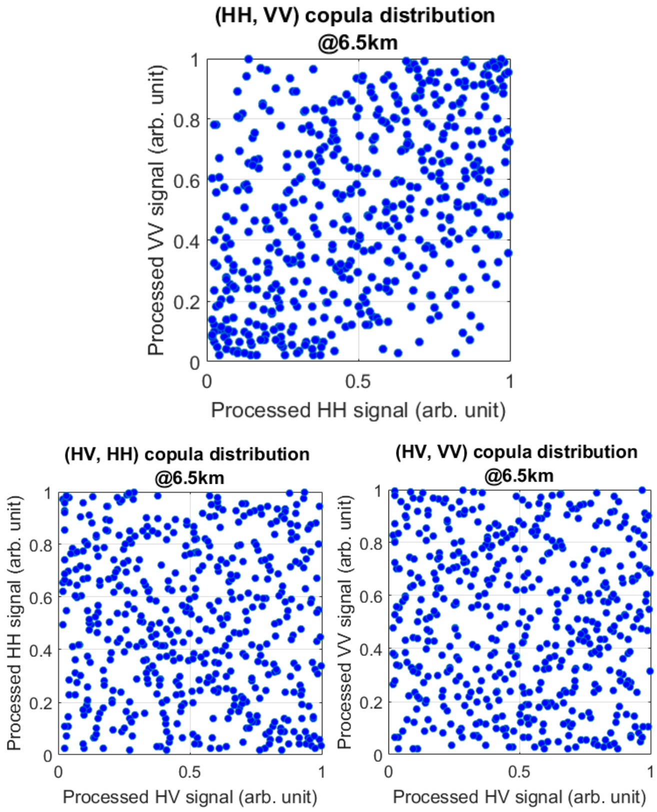

Fig. 4. Copula distributions of the processed HH, HV, and VV polarimetric pairs.

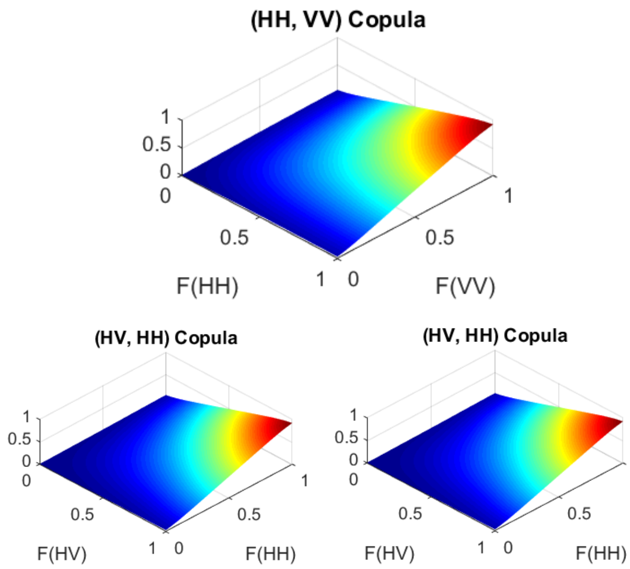

Fig. 5. The copula functions based on the proposed method

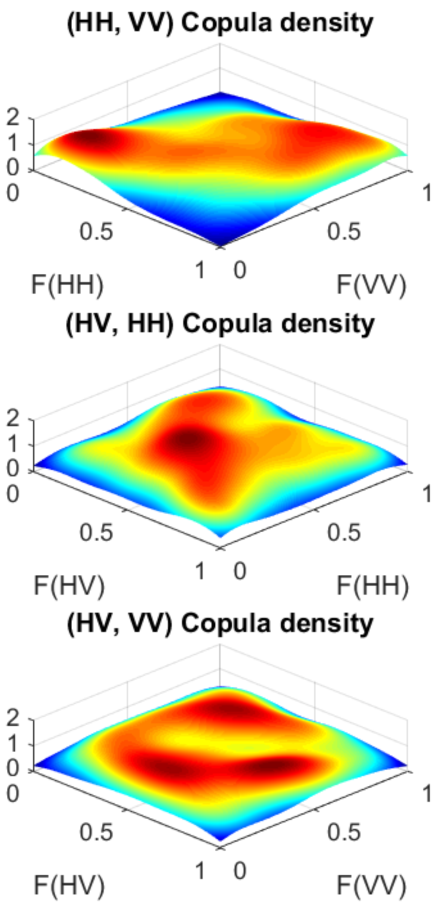

Fig. 6. Copula density functions.

It should be noted that the Kendall's tau value calculated for mutual distribution of processed HH and VV components for the same resolution volume (azimuth 117^0, range 6.5km) but taking into account only one scalar value of the complex signal shows much poorer sensitivity. For either real or imaginary parts or angle or absolute value, the Kendall's tau was equal to or smaller then 0.08.

V. Conclusion

Accounting the complex character of full polarimetric weather radar signals in copula-based signal processing gives one the sensitive tool to investigation of inhomogeneity regions in clouds and precipitations, related with high or low level of mutual correlation. Proposed method of transition from bivariate complex signals to the univariate copula marginals with uniform distribution can be used to further development of the copula signal processing algorithms, such as introduced in [5, 6] copula-based spectral-polarimetric measures, or the copula-based polarimetric classification.

The estimated by authors copula PDF can be used as a basis for constructing statistical classification and detection algorithms for polarimetric radars. The use of copula transform gives us a possibility to develop classification and detection algorithms based on pure relations between polarimetric parameters independently upon the shape of probability density function of complex signal parameters.

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