UDC 629.7.014.18.058.47:519.673 (045)
DOI: 10.18372/2306-1472.73.12166

Volodymyr Kharchenko[1]
Denys Matiychyk[2]

MATHEMATICAL MODEL OF UNMANNED AERIAL VEHICLE CONTROL IN SINGLE CONTROL CHANNEL

National Aviation University
1 Kosmonavta Komarova Avenue, 03680, Kyiv, Ukraine
E-mails: [1]kharch@nau.edu.ua; [2]belkaaden@gmail.com

Abstract

The main aim of this research is development of the mathematical model. Developed model will allow deeper, qualitative and faster selection of the proper values of coefficients that are necessary for optimal and effective interaction between the human and machine. ***Methods.*** *In order to develop a mathematical model of unmanned aerial vehicle control in a single control channel, the classical model of aircraft control was applied. Comparative analysis of turn execution in manual and semiautomatic control modes of unmanned aerial vehicle by remote pilot was applied.* ***Results.*** *The mathematical model "Remote pilot – Remote Control System – Unmanned Aerial Vehicle" under change of unmanned aerial vehicle heading on horizontal plane was proposed. The model takes into account the remote pilot's actions during airplane control, as well as of aircraft maneuverability when it is equipped with radio control and has a flight controller with several modes of operation.* ***Discussion.*** *The proposed model gives a possibility to analyze peculiarities of different control modes used for unmanned aerial vehicle operation. Select the proper values of coefficients that are necessary for optimal and effective interaction between the human and the machine.*

Keywords: automatic control system; control device; mathematical model; remote pilot; unmanned aerial vehicle.

1. Introduction

Today engineering systems can be made compatible with human characteristics and limitations only by means of quantitative analysis and experiment. The behavior of both man and machine can be described in comparable terms [1]. The use of Unmanned Aerial Vehicles (UAV) is urgent and has to be analyzed as interaction of the man and the machine.

During control of UAV on the distance in "semi-automatic" and "manual" control modes with the help of the video camera or Combined Flight Instrument (CFI) the problem of interaction of remote pilot and UAV arises. Its nature primarily lies in the difference of human (remote pilot) and machine properties. In our case, there is the difference of UAV and interface through which important data for the remote pilot passes. Modern flight controllers and means of receiving/transmitting commands allow fast adaptation of aircraft to automatics. But a problem of defining the proper coefficients arises when the coefficients must allow the remote pilot control the UAV effectively.

2. Analysis of the latest research and publications

Research on determination of conditions and type of control during maneuvering of aircraft is presented in work [2] which shows that when the aircraft is equipped with automatic control system it performs the tasks most effectively.

The most important task for a UAV is execution of the entire flight program in spite of all possible negative factors. Reference literature on the theory of automatic control was analyzed [3].

Tasks of UAV control are interconnected with the problem of interaction between the man and machine [4]. The latest literature about UAV application to various tasks under main flight control modes is analyzed [5].

3. Aim of research

The main aim of this research is development of the mathematical model that will allow deeper, qualitative and faster selection of the proper values of coefficients that are necessary for optimal and effective interaction between the human and machine.

The UAV has to be directed at the necessary heading when the remote pilot observes on the screen the CFI and interacts with the UAV through the remote control system. The model shows the real heading of the UAV through CFI at the Route Turning Point (RTP). Selection of proper values of coefficients is necessary for optimal and effective interaction between the human and machine.

4. Research results

The mathematical model *"Remote pilot – Remote Control System – Unmanned Aerial Vehicle"* under change of the UAV heading in horizontal plane was proposed. The model takes into account the remote pilot's actions during airplane control, as well as aircraft maneuverability that is equipped by radio control and has a flight controller with several modes of operation. A statically stable aircraft was used in the model.

Law of coordinated rudder deflection for UAV is described by equation:

$$\delta_r = \frac{m_Y^{\beta}}{m_Y^{\delta_r}}\left[\tan^{-1}\left(\frac{B_{\max}e^{-kt}\left(-K\cos\omega t-\omega\sin\omega t\right)}{V_x}\right)\right. \left. +\tan^{-1}\left(\frac{B_{\max}e^{-kt}\left[\left(\omega^2-K^2\right)-K\ \right]}{V_x}\left(\cos\omega t-3\omega\sin\omega t\right)\right)\ \right], \quad (1)$$

where δ_r is the law of rudder deflection; $B_{\max}$ is the maximum possible UAV lateral divergence from flight direction, that is given at a given distance from the next RTP (D_{RTP}) still possible entering of UAV on routing line (RL) ($B_{\max}$ depends on overload, which have (Δn_z)); V is the UAV speed; K is the exponential coefficient, the value of which is advisable to change during flights in the range $0 \le K \le 0{,}1$ to provide appropriate control at different modes of flight: take-off/cruising flight/landing; ω is the angular frequency which is limited by lateral component (V_Z) of aircraft speed and angular velocity of control surface deflection ($\omega_{\text{defl.cs}}$); t is the current time of maneuver execution; $m_Y^{\beta} / m_Y^{\delta_r}$ are derivatives of rudder moment while slide and deflection of rudder.

Dynamic properties of the remote pilot are described by non-linear, discrete, nonsteady function with random variable coefficients. The remote pilot as a link of the automatic system *"Remote pilot – Remote Control System – UAV"* can be presented with a transfer function [6] as follows:

$$W_P(P) = \frac{K_0 K_N (T_1 P + 1)}{(T_2 P + 1)(T_N P + 1)} e^{P\tau}, \quad (2)$$

where K_0 is the link amplifying coefficient; K_N is the amplifying coefficient that takes into account neuromuscular reaction of remote pilot; T_1 is the time constant of remote pilot adjustment, as forcing link of automatic system; T_2 is the time constant of remote pilot inertness; T_N is the time constant of neuromuscular reaction of remote pilot; τ is the time constant, that characterizes remote pilot reaction latency on external factors; P is the Laplace transform.

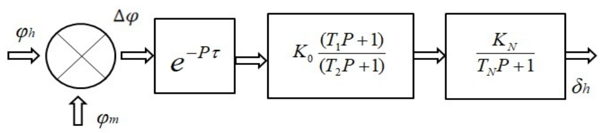

Fig. 1. Block diagram of transfer function implementation.

In Fig. 1, the first element with a transfer function describes the delay of the remote pilot's reaction on the difference of CFI readings which is in front of the eyes of the remote pilot on GCS display (variant: difference between the desirable and available heading which is defined visually when the UAV is controlled "from side") and the needed heading on the RTP which is described in the task.

The second element describes the remote pilot's inertance and his attempts to compensate for it with more energetic actions of UAV control device.

The third element, inertial link of the first order, describes the muscle influence of the remote pilot on the UAV control device.

The block diagram of UAV control contour in UAV targeting process on RTP on inclined surface is presented in Fig. 2.

Unmanned aerial vehicle movement in horizontal projection is regarded as the most complex. While targeting remote pilot has to control UAV in such a way for condition fulfilled:

$$\Delta\varphi_h = \varphi_h(t) - \varphi_{mh}(t) \quad (3)$$

where

$\varphi_h(t)$ – sight angle in horizontal plane;

$\varphi_{mh}(t)$ – heading angle of marker position;

$\Delta\varphi_h(t)$ – targeting error (control parameter).

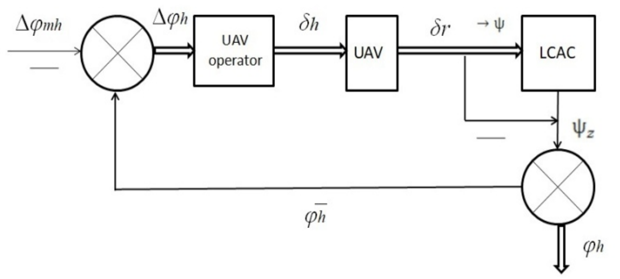

Fig. 2. Block diagram of UAV control contour

The element of the block diagram that deals with horizontal projection is described by equation (4), which links control δ_h (remote pilot's reaction on control parameter) with input signal $\Delta\varphi_h$:

$$\delta_r(P)=\left(K_{0h}\frac{T_{1h}P+1}{T_{2h}P+1}\times\frac{K_{Nh}}{T_{Nh}P+1}\times e^{-P\tau}\right)\Delta\varphi_h(P) \qquad (4)$$

Input and output signals of the links which characterize the remote pilot's reaction latency on the discrepancy parameter in general is linked by correlation:

$$Z(t)=KX(t-\tau)\,, \qquad (5)$$

where,

$Z(t)$ – output signal of control device of directional axis;

$K = K_0K_N$ – remote pilot amplifying coefficient;

$K(t-\tau)=e^{-P\tau}$ – parameter that characterizes remote pilot inertness;

$X=\Delta\varphi$ – control parameter.

This correlation gives the possibility to consider the signal latency which comes to remote pilot's model input on time (τ). Then given signal without distortions comes to input of the next link.

Taking into account (5) we express (4) in the next form:

$$\ddot{\delta}(t)=a_1\Delta\dot{\varphi}_h(t-\tau)+a_2\Delta\varphi_h(t-\tau)-a_3\times \Delta\dot{\varphi}_h(t-\tau)-a_4\varphi_h(t) \qquad (6)$$

where,

$$a_1=\frac{K_hT_{1h}}{T_{2h}T_{Nh}};\ a_2=\frac{K_h}{T_{2h}T_{Nh}};\ a_3=\frac{T_{2h}+T_{Nh}}{T_{2h}T_{Nh}};$$

$$a_4=\frac{1}{T_{2h}T_{Nh}}; \qquad (7)$$

$$K_h=K_{0h}K_{Nh} \qquad (8)$$

$$\Delta\dot{\varphi}_h(t-\tau)=\dot{\varphi}_h(t-\tau)-\varphi_{mh}(t-\tau) \qquad (9)$$

For convenience of taking into account of the remote pilot's influence on law of control devices deflection (1) equation (6) takes the following form:

$$\Delta\delta(t)=\frac{a_1\Delta\dot{\varphi}_h(t-\tau)+a_2\Delta\varphi_h(t-\tau)-a_3\Delta\dot{\delta}_h(t)-\ddot{\delta}_h(t)}{a_4}\ \text{or,} \qquad (10)$$

$$\Delta\delta(t)=K_hT_{1h}\Delta\dot{\varphi}(t-\tau)+K_h\Delta\varphi_h(t-\tau)- (T_{2h}+T_{Nh})\times\Delta\dot{\delta}_h(t)-\ddot{\delta}_h(t) \qquad (11)$$

where,

$\Delta\delta_h(t)$ – parameter of UAV control device position change during the influence of the remote pilot on UAV control in directional axis.

Conformity of UAV control device deflection $\Delta\delta_h(t)$ and rudder $\Delta\delta_r(t)$ established by selection of the amplifying coefficients values of control devices damper UAV (exponential curve, dual-rate and travel limit) in directional axis and has next form:

$$\Delta\delta_r(t)=\Delta\delta_h(t)K_\psi^r \qquad (12)$$

Model conformity in real cycle is established by the selection of such values of the coefficients at which the average quadratic deviation of the total dissipation of optical vision center of the video camera on ground (total dissipation of aviation means) of targeting corresponds to the given value. Initial values of coefficients were taken according to the recommendations [6, 7] that are equal to the following values:

$$K_h=3; K_\varphi^r=0,25; T_{1h}=1; T_{Nh}=1; \qquad (13)$$

The values of coefficients are taken from the radio transmitter Futaba used in the field experiment [8].

The coefficients were refined in the simulation process. Data analysis that carried out in work [4] indicates that the remote pilot's amplification coefficient K_h influences the established error. With increasing K_h we need to increase K_φ^r accordingly and vice versa.

The value of transfer process significantly is influenced by coefficient T_{1h} (time constant of remote pilot's preparation as the forming link of the automatic system). This is explained by the fact that under decreasing T_{1h} the remote pilot compensates his reaction latency faster on divergence of positions' alignment of CFI and the desired UAV heading.

The value of T_{2h} and T_{Nh} in the given mode $\Delta\varphi_h(t)$ transfers an insignificant influence.

For the lateral channel, there are the following coefficient values:

$$K_h \geq 3,5;\ 0,6 \leq T_1 \leq 1,5\ \text{s};$$

$$T_2 \leq 0,8\ \text{s};\ T_N \leq 0,08\ \text{s};\ \tau = 0,3\ \text{s}. \qquad (14)$$

These coefficients support the values of the remote pilot's model.

While targeting the UAV by ground markers in horizontal projection, equation (12) with equations that describe the UAV control model (1) form a closed system of differential equations: mathematical model *"Remote pilot – Remote Control System – UAV"*. Solving the given system of equations we can obtain the control devices deflection law considering the remote pilot's actions:

– for rudder:

$$\delta_r = \frac{m_Y^{\beta}}{m_Y^{\delta_r}}\left[\tan^{-1}\left(\frac{B_{\max}e^{-kt}\left(-K\cos\omega t-\omega\sin\omega t\right)}{V_x}\right)+\tan^{-1}\left(\frac{B_{\max}e^{-kt}\left[\left(\omega^2-K^2\right)-K\right]}{V_x}\left(\cos\omega t-3\omega\sin\omega t\right)\right)\right]+\Delta\delta_r(t) \quad (15)$$

or:

$$\delta_r = \frac{m_Y^{\beta}}{m_Y^{\delta_r}}\left[\tan^{-1}\left(\frac{B_{\max}e^{-kt}\left(-K\cos\omega t-\omega\sin\omega t\right)}{V_x}\right)+\tan^{-1}\left(\frac{B_{\max}e^{-kt}\left[\left(\omega^2-K^2\right)-K\right]}{V_x}\left(\cos\omega t-3\omega\sin\omega t\right)\right)\right]+\left[K_hT_{1h}K_{\psi}^{r}(\dot{\varphi}_h(t-\tau)-\varphi_{mh}(t-\tau)\right]+\left[K_1K_{\psi}^{r}(\varphi_h(t-\tau)-\varphi_{mh}(t-\tau))\right]-\left[(T_{2h}+T_{Nh})K_{\psi}^{r}\Delta\dot{\delta}_h(t)+K_{\psi}^{r}\Delta\ddot{\delta}_h(t)\right] \quad (16)$$

In order to check the developed mathematical model test flights were conducted on the UAV airplane type equipped with RC equipment and flight controller [9]. UAV control modes used during flights include: *manual* control mode when the UAV executes commands directly through radio channel, flight controller stabilizers are not engaged.

The second control mode was *semiautomatic* control mode when all stabilizers were engaged (on three axes) [10].

The task of test flights was in UAV's heading change in horizontal plane from 0^0 to 24^0 (from RTP 1 to RTP 2) without the change of flight altitude.

Actually, the UAV reacts on stick deflection according to linear law. When the remote pilot switches to semiautomatic control mode, a turn is executed similarly to exponential law. This indicates that the stabilization units of the UAV counteract its turn, i.e. the stabilization units are trying to immediately return the aircraft to its initial position. The airplane reacts with inertance on control devices so the turn itself is delayed. Therefore, the UAV requires more distance and time to execute a turn.

As follows, to execute a more intensive turn the manual control mode is more beneficial. It is quite problematic to change control mode for the remote pilot (this may increase latency on time parameter for decision-making process of the UAV remote pilot). So we can recommend deactivation of the heading stabilizer while roll and pitch stabilizers are enabled for semiautomatic control mode in flight controller.

5. Conclusions

1. It was determined that the remote pilot should use of manual control mode in rudder channel for quick UAV setting on the desired heading. At the same moment, pitch and roll channels should be set on semi-automatic control mode.

2. Results of mathematical modeling and flight tests performance show that finding of coefficients that reflect the law of control surfaces deflection is an important factor in adjustment process of contour "*Remote pilot – Remote Control System – UAV*". Incorrect adjustment of channel transfer coefficient, in particular of the lateral channel, leads to excess/decrease of desired deflection values. This significantly complicates the hold of the UAV on heading and putting of the UAV on the necessary RTP.

3. To solve problem of modeling of the interaction of remote pilot and unmanned aerial vehicle a mathematical model was developed. The model considers the features of the human-remote pilot contour and features of machine contour, i.e. the UAV.

4. The UAV contour takes into account the main geometrical and dynamical characteristics of the aircraft: wing area, overload on channel, airspeed, angular velocity, time for maneuver execution, derivatives of rudder moment and air density.

5. Contour of the human-remote pilot describes latency of the remote pilot's reactions, remote pilot's inertance and his attempts to compensate for this by more energetic actions of the UAV control devices as well as inertial link of the first order, i.e. muscular influence of the remote pilot on the UAV control devices.

References:

[1] T.B. Sheridan, W.R. Ferrell. (1975) *Man-Machine Systems: Information, Control, and Decision Models of Human Performance*, The MIT Press, 468 p.

[2] *Sposob upravleniya letatel'nym apparatom* [Aircraft control method]. – M.: VVIA im. N.Ye. Zhukovskogo, avtor izobret. Yu.P. Belovodski i dr., zayavl. 7.03.85, № 3109220/40-23. (In Russian)

[3] T. Lombaerts, (2012) Automatic Flight Control Systems - Latest Developments, Publisher: InTech, Chapters published January 18, 214 p.

[4] Greg A. Jamieson, (2006) *Developing Human-Machine Interfaces to Support Monitoring of UAV Automation*, DRDC Toronto CR 2006-251, 45 p.

[5] Kenzo Nonami; Muljowidodo Kartidjo; Kwang-Joon Yoon; Agus Budiyono. (2013) *Autonomous Control Systems and Vehicles: Intelligent Unmanned Systems*. Springer, 250 p.

[6] Pashkovskiy I.M. (1975) *Ustoychivost' i upravlyayemost' samoleta*. [Aircraft Stability and Control] – M.: Mashinostroyeniye, – 328 p. (In Russian)

[7] Bodner V.A. (1978) *Aviatsionnyye trenazhery* [Aviation simulators]. – M.: Mashinostroyeniye, – 192 p. (In Russian)

[8] Futaba RC systems. Available at: http://www.futabarc.com/systems/futk9410-14sg/

[9] Pratt, R. W. (2000) *Flight Control Systems: Practical issues in design and implementation*. Edited by IEE Control Engineering Series 57, *IEEE Publ.*, Padstowe, Cornwall, 382 p.

[10] McLean, D. (1990) *Automatic Flight Control Systems*. Englewood: Prentice Hall Inc., 593 p.

В.П. Харченко[1], Д.М. Матійчик[2]

Математична модель керування безпілотним повітряним судном в одному каналі керування

Національний авіаційний університет, просп. Космонавта Комарова, 1, Київ, Україна, 03680
E-mails: [1]kharch@nau.edu.ua; [2] belkaaden@gmail.com;

Мета: Головна мета дослідження – створення математичної моделі модель керування безпілотним повітряним судном в одному каналі керування. Створена модель дозволить більш глибоко, якісно і швидше підібрати потрібні величини коефіцієнтів, необхідних для оптимальної і ефективної взаємодії людини і машини. **Методи:** Для створення математичної моделі керування безпілотним повітряним судном в одному каналі керування було застосовано класичну модель керування повітряним судном. Було застосовано порівняльний аналіз виконання повороту оператором безпілотного повітряного судна в ручному та напівавтоматичному режимі керування. **Результати:** Запропонована математична модель «Оператор – система дистанційного керування – безпілотне повітряне судно» під час зміни напрямку по курсу в горизонтальній площині. Модель враховує дії оператора під час керування безпілотним повітряним судном, маневреність повітряного судна обладнаним системою дистанційного керування та польотним контролером із можливістю вибору декількох режимів керування. **Обговорення:** Запропонована модель дає можливість проаналізувати особливості різних режимів керування безпілотним повітряним судном, які застосовуються при функціонуванні безпілотного повітряного судна. Вибір відповідних коефіцієнтів на різних режимах керування безпілотним повітряним судном дозволяє оптимально та ефективно взаємодіяти людині з машиною.

Ключові слова: автоматична система керування; безпілотне повітряне судно; математична модель; оператор; орган керування.

В.П. Харченко[1], Д.М. Матийчик[2]

Математическая модель управления беспилотным воздушным судном в одном канале управления

Национальный авиационный университет, просп. Космонавта Комарова, 1, Киев, Украина, 03680
E-mails: [1]kharch@nau.edu.ua; [2] belkaaden@gmail.com;

Цель: Главная цель исследования - создание математической модели модель управления беспилотным воздушным судном в одном канале управления. Созданная модель позволит более глубоко, качественно и быстрее подобрать нужные величины коэффициентов, необходимых для оптимальной и эффективной взаимодействия человека и машины. **Методы:** Для создания математической модели управления беспилотным воздушным судном в одном канале управления было применено классическую модель управления воздушным судном. Был применен сравнительный анализ выполнения поворота оператором беспилотного воздушного судна в ручном и полуавтоматическом режиме управления. **Результаты:** Предложенная математическая модель «Оператор – система дистанционного управления – беспилотное воздушное судно» во время изменения направления по курсу в горизонтальной плоскости. Модель учитывает действия оператора при управлении беспилотным воздушным судном, маневренность воздушного судна оборудованным системой дистанционного управления и полетным контроллером с возможностью выбора нескольких режимов управления. **Обсуждение:** Предложенная модель позволяет проанализировать особенности различных режимов управления беспилотным воздушным судном, применяемых при функционировании беспилотного воздушного судна. Выбор соответствующих коэффициентов на различных режимах управления беспилотным воздушным судном позволяет оптимально и эффективно взаимодействовать человеку с машиной.

Ключевые слова: автоматическая система управления; беспилотное воздушное судно; математическая модель; оператор; орган управления.

Volodymyr Kharchenko. Doctor of Engineering. Professor.
Vice-Rector on Scientific Work of the National Aviation University, Kyiv, Ukraine.
Editor-in-Chief of the scientific journal Proceedings of the National Aviation University.
Winner of the State Prize of Ukraine in Science and Technology, Honored Worker of Science and Technology of Ukraine.
Education: Kyiv Institute of Civil Aviation Engineers, Kyiv, Ukraine.
Research area: management of complex socio-technical systems, air navigation systems and automatic decision-making systems aimed at avoidance conflict situations, space information technology design, air navigation services in Ukraine provided by CNS/ATM systems.
Publications: 520.
E-mail: knarch@nau.edu.ua

Denys Matiychyk (1991). Postgraduate student.
Department of Air Navigation Systems, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine (2014).
Research area: Unmanned Aerial Systems.
Publications: 7.
E-mail: belkaaden@gmail.com