3.3 PREFERENCES AND VARIATIONAL PRINCIPLES IN THE SIMPLEST MODELS OF "LIGHT-SHADOW" ECONOMY

Let us continue a consideration of the two-component economy problems from positions of subjective analysis and postulated variational principle stating optimality of the players' individual preferences distributions. Dynamical properties of the "light-shadow" economy system's behavior in the model with the proportional taxation come into sight at compiling the functional of the type of the simplest problem of calculus of variations:

$$I_T[\alpha]=\int_{t_0}^{t_1} V_T\cdot dt=\int_{t_0}^{t_1} T\cdot V_p\cdot\frac{\xi^\alpha}{1+\xi^\alpha}\cdot dt, \tag{19}$$

where all designations are kept as those introduced above except α – with which it is symbolized the power index at ξ, that is $\frac{V_p^{(1)}}{V_p^{(2)}}=\left(\frac{C}{T}\right)^\alpha=\xi^\alpha$, instead of β, because of that in subjective analysis it is the coefficient at the cognitive function in the functional of the view of (1) which is indicated as β, t – time.

Extremizing the functional of the view of (19) the firm can optimize its own payouts for the official state taxation through the determination of an extremal of $\alpha(t)_{opt}^{(T)}$ from the Euler-Lagrange equation.

The optimization of the payouts by the firm the shadow taxation and total expenses by the both components in time, following the same method (variational principle) is correspondingly with the functionals of:

$$I_C[\alpha]=\int_{t_0}^{t_1} V_C\cdot dt=\int_{t_0}^{t_1} T\cdot V_p\cdot\frac{\xi}{1+\xi^\alpha}\cdot dt,\ \Rightarrow\ \alpha(t)_{opt}^{(C)}; \tag{20}$$

$$I_F^\Sigma[\alpha]=\int_{t_0}^{t_1} V_F^\Sigma\cdot dt=\int_{t_0}^{t_1} T\cdot V_p\cdot\frac{\xi+\xi^\alpha}{1+\xi^\alpha}\cdot dt,\ \Rightarrow\ \alpha(t)_{opt}^{(F_\Sigma)}. \tag{21}$$

Numerical experimenting for a static model by the formulae of (20, 21) at the values of: $T=0.3$, $V_p=10{,}000$, $t_0=0$, $t_1=100$ is illustrated with the plots in Fig. 2.

In Fig. 2, there can be traced a presence of a saddle point in a game for an optimization of the shadow taxation.

For a static model of the type of (19-21), at the condition of $C=k\cdot\alpha$, where k – coefficient of proportionality, the necessary conditions for an extremum existence in the view of the Euler-Lagrange equations satisfaction:

$$\frac{\partial V_T}{\partial\alpha}-\frac{d}{dt}\left(\frac{\partial V_T}{\partial\dot\alpha}\right)=0,\qquad \frac{\partial V_C}{\partial\alpha}-\frac{d}{dt}\left(\frac{\partial V_C}{\partial\dot\alpha}\right)=0,\qquad \frac{\partial V_F^\Sigma}{\partial\alpha}-\frac{d}{dt}\left(\frac{\partial V_F^\Sigma}{\partial\dot\alpha}\right)=0, \tag{22}$$