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Research paper

Synthesis of the switching control law for a quadrotor autopilot

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Abstract

The paper deals with the problem of synthesizing the time-optimal control law by the angular coordinates of an unmanned aerial vehicle with stabilization in the pitch and roll directions. The full mathematical model of the unmanned aerial vehicle is reduced to a system of the first-order differential equations, based on which the optimal control law is constructed. Control action in each plane depends only on the measured coordinates and is calculated in real time. It is believed that the dynamic model, described by a system of differential equations, contains complex roots, which indicate the oscillatory response of the controlled object to the control action. Some properties of the switching line and switching control are also considered in the paper. Some results of simulating the dynamics of the object under examination with a synthesized control law are presented.

Keywords: *Dynamic System; Quadrotor; Switching Control Law; Time-Optimal Control; Unmanned Aerial Vehicle.*

1. Introduction

Nowadays the quadrotor is the most widespread unmanned aerial vehicle (UAV). This has been proved by numerous publications. Such interest in quadrocopters is caused by their maneuverability characteristics, which exceed the similar characteristics of manned aircraft. Unmanned aerial vehicles of this type are capable of hovering and dramatically changing the flight direction, which has huge advantages when flying in conditions of stationary and moving obstacles.

Obtaining adequate mathematical models and effective characteristics of maneuverability are the main goals of most studies. The basis for obtaining a mathematical model of quadrotor is the helicopter model. The assessment of dynamic properties of the quadrotor was initially grounded on the choice of transfer function parameters of the servo drives for controlling the speed of quadrotor screws, for example, as authors did in [1]. Despite the complexity of the procedure, it is possible to obtain an adequate mathematical model using classical methods. The mathematical model and control law on angular coordinates, which provide UAV stabilization of the roll and pitch movement using the Lyapunov criterion, was proposed in [2]. The method requires a careful choice of the model parameters to ensure its stability. In works [3, 4], the control is implemented using the technique of backstepping, which allows stabilizing the angular coordinates of the unmanned aircraft in a finite time. In [5] there have been proposed the problem solution for optimal control of angular coordinates based on the synthesis of a linearly quadratic regulator without noise by means of a quadratic criterion. The development of a robust PID-controller for quadrature control was proposed in [6], [7]. The further improvement of system robustness can be achieved using the results given in [8]. Control for a finite time by means of application of a multivariable super-twisting-like algorithm was presented in [9]. A feature of this control law is using of the discontinuous integral component that allowed avoiding an influence of disturbances on the device.

Realization of potential possibilities for control of angular coordinates lies in the use of a time-optimal control law. The following problem can be solved by implementing the switching control law as it has been presented in the book of well-known American scientists Athans and Falb [10]. The algorithm and adaptive control law of the aircraft operating in the roll plane in the presence of noise in the coordinate measurement channels are given in [11]. In this case, the mathematical model of a dynamic object is described by a system of differential equations. The scheme for implementing the above-mentioned law in the manual control mode in two perpendicular planes, such as pitch and roll ones, is presented by Athans and Falb. It should be noted about a difficulty in presenting the switching line in a closed form in [10].

The main purpose of this paper is to synthesize the time-optimal control law using the UAV dynamic model represented by a system of differential equations with completely known parameters without noise in the channels of measuring the controlled coordinates in the pitch and roll planes.

2. Problem statement

We will assume that a UAV's spatial orientation can be represented by means of the Lagrangian function of generalized coordinates $(x, y, z, \theta, \varphi, \psi) \in R^6$. Let $q = (x, y, z) \in R^3$ denote the position of a centre of UAV mass in a Cartesian coordinate system and in the inertial reference frame. $\xi = (\psi, \theta, \varphi)^T \in R^6$ are Euler angles called yaw, pitch, and roll, respectively. The unmanned aerial vehicle is a four-motor rotorcraft, ith motor creates thrust f_i, ($i=\overline{1,4}$), P is a constrained value, i.e. $f_i \le P$. The quadrotor scheme is shown in Figure 1.

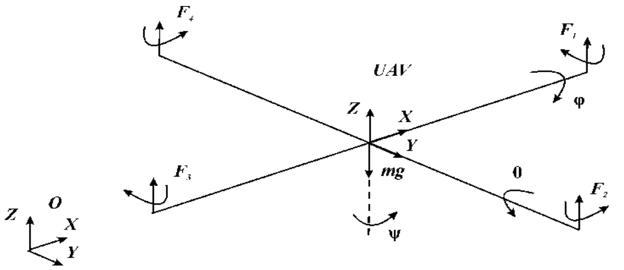

Fig. 1: The Configuration of Four-Motor UAV with Fixed Body Frame and Inertial Frame *XYZ*.

To obtain the equations of a quadrotor's motion, the Lagrange function can be represented as a sum of the kinetic and potential energy of a dynamical system

$$L(\dot{q},\dot{\xi},t) = K(\dot{q},\dot{\xi},t) - U(q,t) \tag{1}$$

Here $K(\dot{q},\dot{\xi},t)$ is the kinetic energy of rotorcraft, and $\dot{q}_i \equiv \frac{dq_i}{dt}$, $\dot{\xi}_i \equiv \frac{d\xi_i}{dt}$, value t means time. This type of energy is formed as a sum of translational and rotational constituents,

$$K(\dot{q},\dot{\xi},t) = \frac{m\dot{q}^2(t)}{2} + \frac{m\dot{\xi}^2(t)}{2} \tag{2}$$

and

$$U(q,t) = mgz(t). \tag{3}$$

The dynamics equations can be obtained by differentiating the Lagrange function (1) with respect to the derivatives of coordinates and in time, namely,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = F_i, \tag{4}$$

where F_i is the external force that acts on ith coordinate q_i. In common case,

$$F = m\ddot{q}, \tag{5}$$

where

$$\ddot{q} \equiv \frac{d^2 q}{dt^2}.$$

We will assume that these forces are able to move a quadrotor to the given point of the trajectory. The force applied to the input can be treated as a control signal of the object. Considering the main type of displacement in accordance with Figure 1, one can write

$$F = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}, \tag{6}$$

where

$$u = \sum_{i=1}^{N} G_i, \tag{7}$$

and

$$G_i = k_i \Omega_i^2. \tag{8}$$

In (7) $N = 4$, factor k_i is some constant, and Ω_i is the angular rate of the ith motor.
In the Eulerian coordinate system, the UAV orientation relative to the inertial coordinate system *XYZ* can be represented by rotation matrix R

$$F_q = R(\psi,\theta,\varphi)F, \tag{9}$$

where

$$R = \begin{pmatrix} C_\psi C_\theta & C_\psi S_\theta S_\varphi - S_\psi C_\varphi & C_\psi S_\theta C_\varphi + S_\psi S_\varphi \\ S_\psi C_\theta & S_\psi S_\theta S_\varphi + C_\psi C_\varphi & S_\psi S_\theta C_\varphi - C_\psi S_\varphi \\ -S_\theta & C_\theta S_\varphi & C_\theta C_\varphi \end{pmatrix}. \tag{10}$$

The matrix (10) uses notations $C_\alpha = \cos(\alpha)$, $S_\alpha = \sin(\alpha)$. Taken into consideration (10) and (6) we can rewrite formula (9) in the following form

$$m\ddot{x} = u(C_\psi S_\theta C_\varphi + S_\psi S_\varphi), \tag{11}$$

$$m\ddot{y} = u(S_\psi S_\theta C_\varphi - C_\psi S_\varphi), \tag{12}$$

$$m\ddot{z} = uC_\theta C_\varphi - mg. \tag{13}$$

Let I_1, I_2, I_3 are moments of quadrotor inertia with respect to axes X, Y, and Z passing through its centre of mass, respectively. $\dot{\varphi}, \dot{\theta}, \dot{\psi}$ are angular rates regarding axes X, Y, and Z. As it is known [10, 12], that if external forces are absent, the differential equations of angular rates can be represented in the form

$$I_1\ddot{\varphi}(t) = (I_2 - I_3)\dot{\theta}(t)\dot{\psi}(t), \tag{14}$$

$$I_2\ddot{\theta}(t) = (I_3 - I_1)\dot{\psi}(t)\dot{\varphi}(t), \tag{15}$$

$$I_3\ddot{\psi}(t) = (I_1 - I_2)\dot{\varphi}(t)\dot{\theta}(t). \tag{16}$$

The equations (14) – (16) are Euler's dynamic equations. When taking into account the action of motors $M_1 - M_4$, equations (14) – (16) look like

$$I_1\ddot{\varphi}(t) = (I_2 - I_3)\dot{\theta}(t)\dot{\psi}(t) + cu_1(t), \tag{17}$$

$$I_2\ddot{\theta}(t) = (I_3 - I_1)\dot{\psi}(t)\dot{\varphi}(t) + cu_2(t), \tag{18}$$

$$I_3\ddot{\psi}(t) = (I_1 - I_2)\dot{\varphi}(t)\dot{\theta}(t) + u_3(t). \tag{19}$$

In formulas (17) – (19) c is the distance between the centre of mass of UAV and motor M_i; u_1, u_2, u_3 are mechanical forces produced by motors M_i. These forces can be described in the following way

$$u_1(t) = G_4(t) - G_2(t) = k_4\Omega_4^2(t) - k_2\Omega_2^2(t), \tag{20}$$

$$u_2(t) = G_1(t) - G_3(t) = k_1\Omega_1^2(t) - k_3\Omega_3^2(t), \tag{21}$$

$$\begin{aligned} u_3(t) &= G_1(t) + G_3(t) - G_4(t) - G_2(t) \\ &= k_1\Omega_1^2(t) + k_3\Omega_3^2(t) - k_4\Omega_4^2(t) - k_2\Omega_2^2(t), \end{aligned} \tag{22}$$

The rotation of the motors relative to the body-axis reference frame leads to the Coriolis Effect, namely, to the change in the centre-of-mass accelerations in the inertial reference frame. Taking into consideration this effect, we can write the following system of equations

$$I_1\ddot{\varphi}(t)=(I_2-I_3)\dot{\theta}(t)\dot{\psi}(t)-I_r\dot{\theta}(t)+cu_1(t), \quad (23)$$

$$I_2\ddot{\theta}(t)=(I_3-I_1)\dot{\psi}(t)\dot{\varphi}(t)+I_r\dot{\varphi}(t)\Omega+cu_2(t), \quad (24)$$

$$I_3\ddot{\psi}(t)=(I_1-I_2)\dot{\varphi}(t)\dot{\theta}(t)+u_3(t). \quad (25)$$

Thus, our purpose is to synthesize the control law that provides the movement of the quadrotor from one point to another in 3D space.

3. Synthesizing the control law

To synthesize the control law it is necessary to analyze the equations (23) – (25). Since the axis Z is the axis of symmetry and the motors M_i are the same and rigidly fixed to the apparatus, the moments of inertia I_1 and I_2 are equal to each other, i.e.

$$I_1=I_2=I. \quad (26)$$

Then the motion equations (23) – (25) can be reduced to the form

$$\ddot{\varphi}(t)=\frac{1}{I}\left[(I-I_3)\dot{\theta}(t)\dot{\psi}(t)-I_r\dot{\theta}(t)+cu_1(t)\right], \quad (27)$$

$$\ddot{\theta}(t)=\frac{1}{I}\left[-(I-I_3)\dot{\psi}(t)\dot{\varphi}(t)+I_r\dot{\varphi}(t)\Omega+cu_2(t)\right], \quad (28)$$

$$\ddot{\psi}(t)=\frac{1}{I}u_3(t). \quad (29)$$

The research of these equations allows us to conclude that if thrust u_3 and initial states $\varphi(0)$, $\theta(0)$, $\psi(0)$ are known at the instant of time $t = 0$, we can always find a coordinate $\psi(t)$ by means of double integration

$$\psi(t)=\psi(0)+\frac{1}{I}\iint u_3 dt^2 \quad (30)$$

Which is independent on the angular coordinates $\varphi(t)$, $\theta(t)$. Stabilization of coordinate ψ is possible if the angular rates $\dot{\varphi}(t),\dot{\theta}(t)$ take zero values for the minimum possible time, and equality $u_3(t)$ = const is provided, whence $\psi(t) = \Psi$ = const. We assume that the thrusts $u_1(t)$, $u_2(t)$ are bounded in amplitude

$$|u_1(t)|\le\bar{u}_1=U, |u_2(t)|\le\bar{u}_2=U.$$

Introducing the new notations

$$\omega=\frac{I-I_3}{I}\dot{\psi},\ \ \Phi=\dot{\varphi},\ \ \Theta=\dot{\theta},\ \ l=\frac{c}{I},$$

and substituting these values into equations (27) – (29), we will obtain

$$\dot{\Phi}(t)=\omega\Theta(t)-\frac{I_r}{I}\Theta(t)\Omega+l\bar{u}_1(t)=\omega'\theta(t)+l\bar{u}_1(t), \quad (31)$$

$$\dot{\Theta}(t)=-\omega\Phi(t)-\frac{I_r}{I}\Phi(t)\Omega+l\bar{u}_2(t)=\omega'\Phi(t)+l\bar{u}_2(t). \quad (32)$$

In the equations (11) – (13) we also use $|u_3(t)| \le \bar{u}_3$, and $x_1=\dot{x},\ \ x_2=\dot{y},\ \ x_3=\dot{z}$. Then equations (11) – (13) take the form

$$\dot{x}_1=\bar{u}_3(C_\psi S_\theta C_\varphi+S_\psi S_\varphi)/m, \quad (33)$$

$$\dot{x}_2=\bar{u}_3(S_\psi S_\theta C_\varphi-C_\psi S_\varphi)/m, \quad (34)$$

$$\dot{x}_3=\bar{u}_3 C_\theta C_\varphi/m-g. \quad (35)$$

As a result of the transformations, we will obtain a system of equations in the state space with the state vector $\{v\}\in\Re^{12}$, $v=(x_1,x_2,x_3,\dot{x}_1,\dot{x}_2,\dot{x}_3,\varphi,\theta,\psi,\dot{\varphi},\dot{\theta},\dot{\psi})^T$. The dynamic object implements a motion in this space from the initial state

$$v(0)=(x_1(0),x_2(0),x_3(0),0,0,0,\varphi(0),\theta(0),\psi(0),0,0,\psi)^T$$

to the finite state

$$v(0)=(0,0,0,0,0,0,0,0,0,0,0,\psi)^T$$

With control actions $|\bar{u}_1(t)|\le U,\ |\bar{u}_2(t)|\le U,\ |\bar{u}_3(t)|\le U'$ for any instant of time t.

Control action for the dynamic system (31), (32) can be obtained by synthesizing the time-optimal control algorithms. In this case, control problem lies in minimizing states $\Phi(t)$, $\Theta(t)$ for the minimum time and states x_1, x_2, x_3 fully defined by angles φ, θ, ψ, and control input U'. Then, space angular rates are divided into four subspaces

$$S_1=\{u_1(t)=U,\ \ u_2(t)=U\}, \quad (36)$$

$$S_2=\{u_1(t)=U,\ \ u_2(t)=-U\}, \quad (37)$$

$$S_3=\{u_1(t)=-U,\ \ u_2(t)=U\}, \quad (38)$$

$$S_4=\{u_1(t)=-U,\ \ u_2(t)=-U\}. \quad (39)$$

Switching the sign of control is implemented in accordance with the curve

$$\Im\equiv\left[\bigcup_{j=0}^{\infty}\Im_{S_1}^j(\cdot)\right]\cup\left[\bigcup_{j=0}^{\infty}\Im_{S_2}^j(\cdot)\right]\cup\left[\bigcup_{j=0}^{\infty}\Im_{S_3}^j(\cdot)\right]\cup\left[\bigcup_{j=0}^{\infty}\Im_{S_4}^j(\cdot)\right]. \quad (40)$$

In formula (40) $\Im_{S_i}^j(\cdot)=\Im_{S_i}^j(\Phi(t),\Theta(t))$, j is the number of the ring sector that eliminates optimal trajectory for a pair of signs of control, j = 0, 1, , N; N is the quantity of switching. Hence

$$\Im_{S_1}^j(\cdot)\equiv\left\{\begin{array}{l}(\Phi,\Theta)\in F_{++}:\\ \Theta=c_1\sqrt{1-(-c_2\Phi+(2j+1))^2},\\ \Phi=c_1\sqrt{1-(c_2\Phi+(2j+1))^2},\end{array}\right\} \quad (41)$$

$$\Im_{S_2}^j(\cdot)\equiv\left\{\begin{array}{l}(\Phi,\Theta)\in F_{+-}:\\ \Theta=c_1\sqrt{1-(-c_2\Phi+(2j+1))^2},\\ \Phi=-c_1\sqrt{1-(c_2\Phi-(2j+1))^2},\end{array}\right\} \quad (42)$$

$$\Im_{S_3}^j(\cdot)\equiv\left\{\begin{array}{l}(\Phi,\Theta)\in F_{-+}:\\ \Theta=-c_1\sqrt{1-(-c_2\Phi-(2j+1))^2},\\ \Phi=c_1\sqrt{1-(c_2\Phi+(2j+1))^2},\end{array}\right\} \quad (43)$$

$$\Im_{S_4}^j(\cdot)\equiv\left\{\begin{array}{l}(\Phi,\Theta)\in F_{--}:\\ \Theta=-c_1\sqrt{1-(-c_2\Phi-(2j+1))^2},\\ \Phi=-c_1\sqrt{1-(c_2\Phi-(2j+1))^2}.\end{array}\right\} \quad (44)$$

The appearance of the switching line is shown in Figure 2.

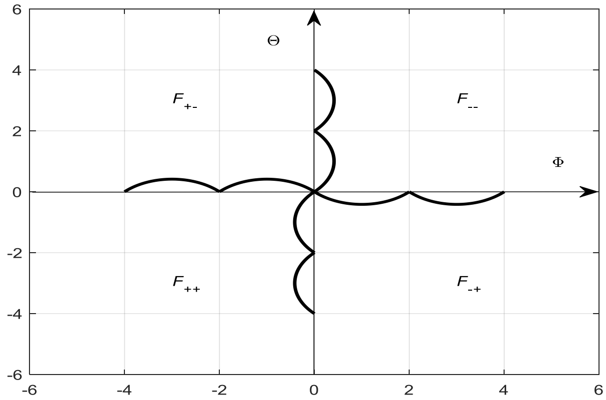

Fig. 2: The Form of the Switching Curve.

Proposition. *Optimal control law can be designed in the form*

$$\bar{u}_1(t) = \begin{cases} +U, \ if \ \Im^j_{S_1}(\cdot) > 0, \ \Im^j_{S_2}(\cdot) > 0, \\ -U, \ if \ \Im^j_{S_3}(\cdot) < 0, \ \Im^j_{S_4}(\cdot) < 0, \\ \bar{u}_1(t-0), \ if \ \Im^j_{S_1}(\cdot) = 0, \\ \Im^j_{S_2}(\cdot) = 0, \ \Im^j_{S_3}(\cdot) = 0, \Im^j_{S_4}(\cdot) = 0, \end{cases} \tag{45}$$

$$\bar{u}_2(t) = \begin{cases} +U, \ if \ \Im^j_{S_1}(\cdot) > 0, \ \Im^j_{S_3}(\cdot) > 0, \\ -U, \ if \ \Im^j_{S_2}(\cdot) < 0, \Im^j_{S_4}(\cdot) < 0, \\ \bar{u}_1(t-0), \ if \ \Im^j_{S_1}(\cdot) = 0, \\ \Im^j_{S_2}(\cdot) = 0, \ \Im^j_{S_3}(\cdot) = 0, \ \Im^j_{S_4}(\cdot) = 0. \end{cases} \tag{46}$$

Corollary 1. *If* $\Theta(t) = 0$, $\Phi(t) = 0$, $t \neq 0$, *it is the finite point of control, and* $U(T) = 0$.
Corollary 2. *If* $\Theta(t) \neq 0$, $\Phi(t) \neq 0$, $t = 0$, *the optimal control is a switch type, and the number of switching is not more than N.*
Corollary 3. *The number of switching control of the dynamic system* (31), (32) *with the control law* (45), (46) *is minimal, and it is defined* $\Phi(0)$, $\Theta(0)$ *according to*

$$N_{\min} = \left[\frac{\omega}{2l} \sqrt{(\Phi(0))^2 + (\Theta(0))^2} \right] \tag{47}$$

where [·] means integer part of a number.
Proof. Analogically to [10, section 7.9] and therefore omitted.

4. Case study and discussion

An example of simulating the motion of a four-motor UAV is considered for the initial angular rate of 0.8 *rad* / *s* and 0.4 *rad* / *s* for roll and pitch, respectively. The control action was assumed to be equal to the normalized values, i.e. $U = \pm 1$, $\omega = 5\ s^{-1}$, $l = 1\ m$, and according to (47) the minimal number of switching the sign of control signal for these initial conditions is equal to 2. The results of testing the initial mismatch by the algorithm (45), (46) with the separating function (40) – (44) in different planes are shown in Figure 3 – 6.
The change of the initial mismatch with respect to the angular rates $\Phi(t)$, $\Theta(t)$ in the phase plane $\Phi O \Theta$ is shown in Figure 3. The variation of the same variables shown in the upper graph in Figure 4 in the time plane, as well as the control actions $u_1(t)$, $u_2(t)$ providing angular stabilization of the quadrotor, are shown in the lower graph in Figure 4. The test results confirm an initial guess concerning the number of switching control signal.

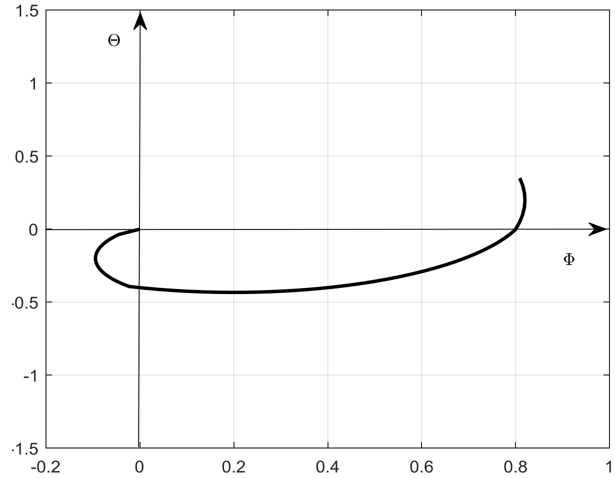

Fig. 3: Changes of Variables Φ, Θ in the Phase Plane.

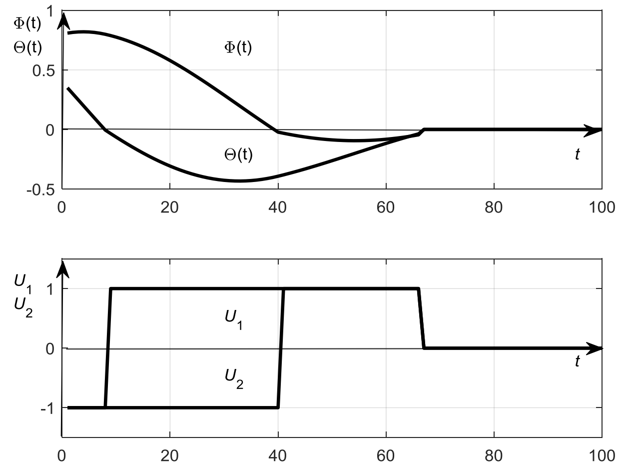

Fig. 4: Changes of Variables Φ, Θ, U_1, U_2 in the Time Plane.

Figures 5, 6 show the changes in the coordinates $x_1(t)$, $x_2(t)$, $x_3(t)$ in the time plane and in the phase plane respectively.

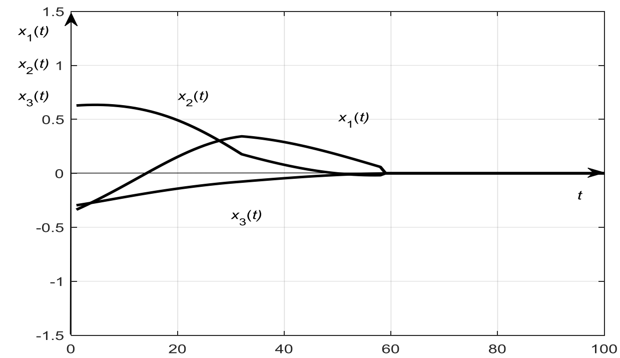

Fig. 5: Changes of Variables $x_1(t)$, $x_2(t)$, $x_3(t)$ in the Time Plane.

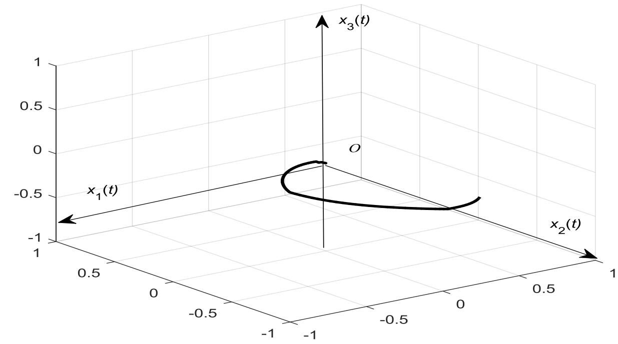

Fig. 6: Changes of Variables $x_1(t)$, $x_2(t)$, $x_3(t)$ in the Phase Plane.

The control law was researched by means of the MatLab software package for various initial mismatches for $\psi = 30°$. These results are presented in Table 1.

Table 1: Some Study Results

θ, *rad*	φ, *rad*	*N*	Θ, *rad/s*	Φ, *rad/s*	$x_1(t)$, *m*	$x_2(t)$, *m*	$x_3(t)$, *m*
0	0.08	1	-0.0198	0.0495	0.0076	-0.0529	-0.0014
0.05	0.1	1	0.039	-0.0397	0.014	0.054	-0.0015
0.1	0.12	1	0.0475	-0.0489	0.0168	0.0663	-0.0023
0.15	0.14	1	0.0469	-0.0441	0.0187	0.0618	-0.0021
0.2	0.16	1	0.0465	-0.0389	0.0055	0.0594	-0.0018
0.25	0.18	1	-0.0237	0.0482	0.0141	0.0644	-0.0022
0.35	0.2	1	0.041	0.0464	0.0602	-0.014	-0.0019
0.4	0.3	1	-0.0354	0.0465	-0.0156	-0.0585	-0.0018
0.45	0.4	2	-0.0428	-0.022	-0.0457	-0.0184	-0.0012
0.5	0.6	2	0.0476	0.0493	-0.046	-0.0185	-0.0012
0.55	0.8	2	0.0058	-0.0486	-0.0192	0.0452	-0.0012

The achievable accuracy of setting the angular rate is $\sigma_\Phi = 0.0466$ rad/s, $\sigma_\Theta = 0.0422$ *rad/s*. To achieve the effect of optimal control in time, the initial angular mismatches should exceed 0.047 *rad/s*. In all cases of the fulfillment of the last condition, the simulation yielded satisfactory results, i.e. the stabilization error does not exceed $\sigma_{x1} = 0.03$ *m*, $\sigma_{x2} = 0.049$ *m*, $\sigma_{x3} = 0.000413$ *m*.

5. Conclusions

The control algorithm for the UAV of the quadrotor type with stabilizing angular coordinates by the time-optimal control is proposed in the paper. This algorithm assumes that the main parameters of quadrotor are known and angular coordinates of quadrotor's motion are measured. In contrast to other control laws, we propose the control law of switching type. The main possibilities of control law are formulated in corollaries. Here we analytically set time-optimal control law and the dependence of the minimal number of switching control signal and initial conditions of a dynamic system. Modelling process of UAV motion proves the correctness of the control law that is proposed in the paper.
The proposed control law is effective at large mismatches. For small discrepancies, a linear control law should be applied. The task of the future research is to study the control law in the absence of precise values of the control object parameters.

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