IEEE Ukraine Section SP/AES
Societies Joint Chapter

National Aviation University

IEEE Central Ukraine
Joint Chapter 2007

2018 IEEE 5th International Conference

Methods and Systems of Navigation and Motion Control (MSNMC)

Proceedings

October 16-18, 2018
Kyiv, Ukraine

IEEE Catalog Number: CFP1852Y-PRT
ISBN: 978-1-5386-5869-7

CONFERENCE ORGANIZERS:

IEEE Ukraine Section SP/AES Societies Joint Chapter (Kyiv, Ukraine)

National Aviation University (Kyiv, Ukraine)

International Research and Educational Center of Information Technologies and Sy National Academy of Science and Ministry of Education and Science of Ukraine (Ukraine)

National Technical University of Ukraine "Igor Sykorsky Kyiv Polytechnic Institute" (Kyiv, Ukraine)

National Aerospace University named after N. Zhukovsky "KhAI" (Kharkov, Ukrain

Central Ukrainian Technical University (Kropivnickiy, Ukraine)

PROGRAMME COMMITTEE:

Final papers presented in author's edition

IEEE Catalog Number: CFP1852Y-PRT

ISBN: 978-1-5386-5869-7

SESSION B. DESIGN OF OPTIMAL CONTROL SYSTEMS AND AVIATION INSTRUMENTS

Setting the PID Controller for Controlling Quadrotor Flight: a Gradient Approach

D. Kucherov
Department of Computerized Control Systems
National Aviation University
Kyiv, Ukraine
d_kucherov@ukr.net

A. Kozub
Usage of Space Systems and GIS Software Department
National University of Defence of Ukraine
Kyiv, Ukraine
kozubzks404@gmail.com

A. Rasstrygin
Navy Department of Central Rese and Development Institute of Arm. and Military Equipment of Arm Forces of Ukraine
Kyiv, Ukraine
0679450438r@ukr.net

Abstract— **The paper deals with the design of an autopilot for a quadrotor based on a PID controller. A mathematical description of the control object in form of the system of second-order differential equations is also presented. Setting the parameters of the PID controller is an iterative procedure. The paper investigates three optimal tuning algorithms based on minimizing the quality integral: coordinate descent, gradient descent, and steepest descent. During the research, a number of advantages of the gradient search algorithm are established. Gradient search is used as the main design method; on its basis, the autopilot model is synthesizing. Some simulation results are presented, which are confirmed by the Simulink autopilot model.**

Keywords—PID controller, unmanned aerial vehicle, controller setting, Quadrotor, optimization, gradient

I. Introduction

Currently, there has been an extraordinary activity in research into the governing of unmanned aerial vehicles (UAVs) with a multi-rotor design. Interest in the study of these means is explained by the increased capabilities of microelectronics and their exceptional characteristics of maneuverability. UAVs of this type have the properties of a helicopter design, namely, vertical takeoff and landing, as well as hovering in the air for the required time. In addition, unlike helicopters, they are able to dramatically change course without additional time spent on turning the aircraft.

These capabilities proved to be useful in military matters for solving reconnaissance and guidance problems, are widely used for carrying out high-quality photography and filming under the necessary angle, and are also useful for the delivery of small loads, monitoring of road traffic, a state of supply electricity lines, etc.

II. Related Works

To control an unmanned flying object with four rotors with a vertical take-off and landing (VTOL) with four rotors, known as a quadrotor, the most commonly used proportional-integral-derivative (PID) regulator [1]–[8]. The development of a PID regulator for stabilizing motion is described by *Salih et al* [1]. The authors propose quadrotor stabilization with the help of four PID controllers, one pair is independent regulators used to stabilize the flight and roll heights, another pair of regulators combined (on one chassis) to stabilize the yaw and pitch of the aircraft. The simulation was carried out using the Simulink tools of the Matlab package.

The design of a reliable PID controller for high-order systems, using the example of an unmanned aerial vehicle, is presented by *Kada et al* [2]. The developed methodology combines an insensitivity reaction, reliable control, and methods for reducing models to increase the perforn and reliability of the PID controller under conditior parametric uncertainty. The approach is based on the u the settings to a single cascade gain, which allows authors to obtain a practical method for tuning the controller.

The SISO method for controlling, stabilizing suppressing the interference of the quadrotor ratio subsy was developed by *Bolandi et al* [3]. The PID controll synthesized analytically for the desired response of closed loop. The performance of the constructed co structure is estimated by factors in the time domain, su overshoot, settling time and an integrated error index reliability. A comparison is made between the devel controller and the controller used in the basic quad model.

A comparison of the different types of controllers can be used to control the quadrotor is presented by *Arge et al* [4]. In this paper, the PID ITAE, LQR regulator, PID are compared with the LQR loop, tuned to the performance of the dynamic platform. The results prese by the authors concern only the control in the vertical pla

A simulation study of a PID controller and a li quadrature regulator (LQR) under noise conditions to s the stability of a hexacopter relative to the terrestrial fr using the LabVIEW software package is presented *Ibrahim et al* in [5]. The authors have established advantages of optimization techniques for constructir controller based on solving a linear quadratic problen similar conclusion was obtained in [6] for the synthesi the effective ship's autopilot when comparing the controller and the time-optimal regulator.

The construction of an aircraft with H-optimal digital type autopilot is presented *Zhiteckii et al* in [7]. The autoj consists of two regulators P and PI-type, which are cov by feedback on position and speed. The parameters of regulators are found in the condition of ensuring the stab of the autopilot. The approach allows providing ro control under conditions of the uncertainty of parametric nonparametric type.

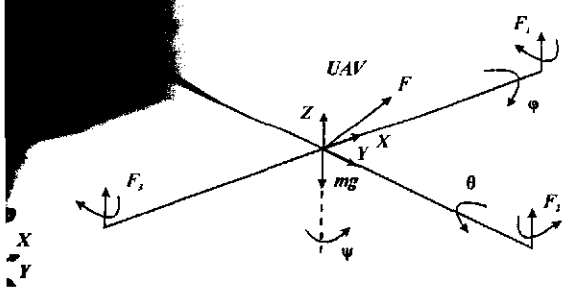
A complete mathematical model of the quadroto presented in [8]. The quadrotor model is described b system of second-order differential equations in Carte and Eulerian coordinates. The model is approved longitudinal and lateral movement.

In this paper discusses the problem of synthesizing I controller to govern the UAV to supply the desired respo of object control on position and velocity in the parame and nonparametric gust.

978-1-5386-5869-7 18/$31.00 © 2018 IE

LEM STATEMENT

V in the Earth's inertial coordinate considered. We also introduce a V-frame) associated with a flying nnected to the center of gravity of coordinate system is given by the X, Y, Z)^T, Fig. 1.



ertial and angular coordinate systems associated with UAVs.

e angular position of the aircraft in relation to the l coordinate system is determined through the angular $\zeta = (\varphi, \theta, \psi)^T$, where the angles φ, θ, ψ denote roll, yaw, respectively, the angles φ, θ are in the interval π/2[, and ψ∈]-π; π[.

also introduce the vector of linear velocities of the l object $V = (V_x, V_y, V_z)^T = (\dot{x}, \dot{y}, \dot{z})^T$, and the angular y vector $\Omega = (\Omega_\varphi, \Omega_\theta, \Omega_\psi)^T = (\dot{\varphi}, \dot{\theta}, \dot{\psi})^T$. Then, using sults of [8], the generalized system of differential ns describing the linear and rotational dynamics of trol object can be represented in the form

$$\begin{vmatrix} mI & 0 \\ 0 & J \end{vmatrix} \begin{vmatrix} \dot{V} \\ \dot{\Omega} \end{vmatrix} + \begin{vmatrix} \Omega \times mV \\ \Omega \times I\Omega \end{vmatrix} = \begin{vmatrix} F \\ M \end{vmatrix}. \quad (1)$$

equation (1), m is the mass, $I \in \Re^{3\times3}$ is the inertia . $J \in \Re^{3\times3}$ is the inertia of the rotor, F is the total force, is the total moment acting on the center of mass of ng object.

lecting the small values of the components of the forces operating at the aircraft, we write the total hen moving in the vertical plane in the form

$$F = \begin{vmatrix} 0 \\ 0 \\ -g \end{vmatrix}. \quad (2)$$

us also write down the forces of the total forces u_1, which ensure the movement of the aircraft, providing al movements

$$M_1 + M_3 - M_4 - M_2 = k_1\Omega_1^2 + k_3\Omega_3^2 - k_4\Omega_4^2 - k_2\Omega_2^2, \quad (3)$$

$$u_2 = M_4 - M_2 = k_4\Omega_4^2 - k_2\Omega_2^2, \quad (4)$$

$$u_3 = M_1 - M_3 = k_1\Omega_1^2 - k_3\Omega_3^2. \quad (5)$$

In equations (3) – (5) k_i is some proportionality coefficient that relates the moment to the angular velocity Ω_i. In more detail, the equations of dynamics describing the translational motion of a UAV are of the form

$$\ddot{\varphi}(t) = \frac{(I - I_3)}{I}\dot{\theta}(t)\dot{\psi}(t) - \frac{Jr}{I}\dot{\theta}(t)\Omega + \frac{c}{I}u_2(t), \quad (6)$$

$$\ddot{\theta}(t) = -\frac{(I - I_3)}{I}\dot{\psi}(t)\dot{\varphi}(t) + \frac{Jr}{I}\dot{\varphi}(t)\Omega + \frac{c}{I}u_3(t), \quad (7)$$

$$\ddot{\psi}(t) = \frac{1}{I}u_1(t), \quad (8)$$

$$\ddot{x} = \frac{\bar{u}_1\left(\cos(\psi)\sin(\theta)\cos(\varphi) + \sin(\psi)\sin(\varphi)\right)}{m}, \quad (9)$$

$$\ddot{y} = \frac{\bar{u}_1\left(\cos(\psi)\sin(\theta)\cos(\varphi) - \cos(\psi)\sin(\varphi)\right)}{m}, \quad (10)$$

$$\ddot{z} = \frac{\bar{u}_1\cos(\theta)\cos(\varphi)}{m} - g. \quad (11)$$

In equations (9)–(11) $\bar{u}_1$ is sum of M_i. Then the dynamics of the controlled object model described by equations (6)–(11) can be represented by such a scheme, Fig. 2.

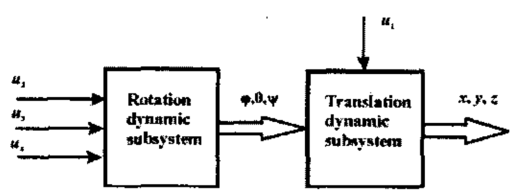


Fig. 2. Scheme of quadrotor dynamics.

As an autopilot, consider, typically used to control moving objects, a PID controller whose output signal is written as

$$u(t) = k_p e(t) + \frac{1}{T_i}\int e(t)dt + T_d\frac{de(t)}{dt}, \quad (12)$$

where k_p, T_i, T_d are the adjustable parameters of the PID controller, denote the transmission coefficient of the proportional link, and the time constants of the integral and differentiating link. In the interest of further synthesis, we introduce the notation for the parameters of the PID controller $q_1 = k_p$, $q_2 = 1/T_i$, $q_3 = T_d$, then (12) can be rewritten as

$$u(t) = q_1 e(t) + q_2\int e(t)dt + q_3\frac{de(t)}{dt}. \quad (13)$$

The diagram of the quadrotor dynamics taking into account the PID controller has the form shown in Fig. 3.

The quality of the control system will be estimated by an integral index of productivity in the form

$$I = \int_0^\infty (e^2 + \mu_1\dot{e}^2 + \mu_2\ddot{e}^2)dt, \quad (14)$$

where $e, \dot{e}, \ddot{e}$ is the vector of the deviation of the controlled quantity and its time derivatives, parameters μ_1, μ_2 are the weight coefficients specified by the system designer. The minimum value of the quality index (14) provides the minimum values of the components of the integrand on the control interval, which ensures smoothness and time of the transient process.

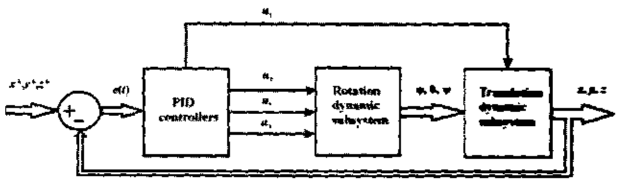


Fig. 3. Structure of the control system with autopilot based on the PID controller

The problem of optimal adjustment of parameters of PID controllers satisfying the minimum value (14) for the control object, whose dynamics is described by equations (6)–(11), is set and solved in the paper.

IV. Optimization PID Controller

Setting the parameters of the PID controller is a long iterative procedure, which does not always lead to optimal tuning in practice. In most cases, the use of regulators of this type is associated with a priori uncertainty about the parameters of the controlled objects. Therefore, in practice, methods of experimental adjustment of the PID regulator are used.

In the problem under consideration, we will assume that the mathematical model of the control object is given, it is a third-order dynamic object. As in [3], the mathematical model of the control object is represented by a transfer function of the form

$$W(s) = \frac{ke^{-\tau_d p}}{s^2(Ts+1)}, \qquad (15)$$

assuming that the small delay τ_d is a presence. In equation (15), k, T, ξ are the parameters of the control object characterizing the gain, and inertia, respectively, $s \equiv d/dt$ is the differentiation operator.

The search values are the parameters of the PID controller, so the optimized function is transformed into a

$$I(\mathbf{q}) = \int_0^\infty \left(e^2(\mathbf{q}) + \mu_1 \dot{e}^2(\mathbf{q}) + \mu_2 \ddot{e}(\mathbf{q})\right)dt, \qquad (16)$$

where $\mathbf{q} = (q_1, q_2, q_3)^T$ is the vector of controller parameters.

A. Coordinate descent method

This method allows you to find the direction along which the optimization criterion decreases most strongly. The algorithm for changing the values of the adjustable parameters in the selected direction can be written in expression

$$q_i[l] = q_i[l-1] - h[l]\,\mathrm{sign}\frac{\partial I(q_i[l-1])}{\partial q_i}, \qquad (17)$$

where $q_i[l]$ is the value of the parameter being ac the lth step of the algorithm, $h(l)$ is the magnitude · step, sign(·) is the sign function.

The direction of the greatest decrease in the opti criterion corresponds to the largest values of the con of the partial derivatives. The algorithm is executed condition

$$\sum_{i=1}^{3}\left(\frac{\partial I}{\partial q_i}\right)^2 \le \varepsilon$$

is true.

In expression (18), ε is a sufficiently small quant idea of the algorithm for determining the paramete illustrated as path A in Fig. 4. Letter O is the optimun

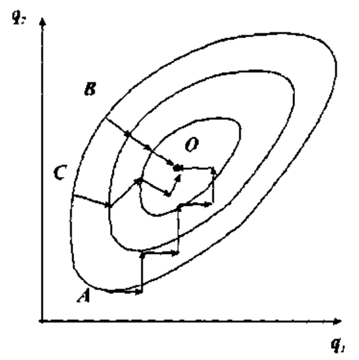


Fig. 4. Search paths for optimization algorithms.

B. Gradient descent method

The algorithm for setting parameters, in this ca defined as follows

$$q_i[l] = q_i[l-1] - h_i[l]\frac{\partial I(q_i[l-1])}{\partial q_i}.$$

In accordance with this algorithm, the step of measu each parameter is made proportional to the correspon component of the gradient. The stopping criterion is (17). For the convergence of the algorithm to a l minimum, the step size should be chosen from condition

$$h_i[l] = (\mathbf{q}[l] - \mathbf{q}[l-1])\frac{\frac{\partial I(\mathbf{q}[l])}{\partial \mathbf{q}} - \frac{\partial I(\mathbf{q}[l-1])}{\partial \mathbf{q}}}{\left\|\frac{\partial I(\mathbf{q}[l])}{\partial \mathbf{q}} - \frac{\partial I(\ldots}{\ldots}\right.}$$

When approaching the endpoint, the reducing, and therefore the determination parameters is more accurate. It is the ad approach. In Figure 4 this search path in indicated by the letter B.

C. Method of steepest descent

The main idea of this method is to direction of the fastest decrease of the func which a step is taken in the same direction is determined, and further, a new direction of

The following mathematical
algorithm

$$\ldots - h[l]\frac{\partial I(q_i)}{\partial q_i}, \quad (21)$$

$$\ldots I\left(\mathbf{q} - h\frac{\partial I(\mathbf{q})}{\partial \mathbf{q}}\right). \quad (22)$$

(22) serves as the Lagrange multiplier. h-step in determining the direction makes the ...m too slow, which is its essential disadvantage. For ...orithm, the search path of the optimal parameters q_i ...n by the letter C in Fig. 4.

V. Design PID Controllers

...t the model of the dynamics of UAV in the plane of ...d pitch also, as in [11], be described by a transfer ...n of the form

$$W_{R,P}(s) = \frac{27.5e^{-0.022s}}{s^2(s+4.1)}, \quad (23)$$

...r yaw

$$W_Y(s) = \frac{17.42e^{-0.022s}}{s^2(s+4.1)}. \quad (24)$$

...dex of productivity is given in the form

$$I = \int_0^{\infty}\left(e^2 + \dot{e}^2 + \ddot{e}^2\right)dt. \quad (25)$$

...itial parameters for the gradient adjustment algorithm ...e value $q_1 = q_2 = q_3 = 1$; the application of the gradient ...nent algorithm allows to obtain the values of the PID ...llers parameters satisfying (25). The optimum values ...ven in Table I. The results are obtained using the ...ink Matlab software.

TABLE I PID CONTROLLER PARAMETERS

...l	q_1	q_2	q_3	t_r, s	σ, %
...'	0.027	0.00036	0.3036	0.672	10.5
	0.89	0.00187	0.073	0.38	15.4

...alysis of Table I shows that on all channels the rise ...atisfies the requirement $t_r < 1$ s the algorithm setting is ...ted in ideal conditions when measurement noise is not ...nto account. Future research is planned to focus on the ...of the effect of noise in measurement channels.

VI. Conclusions

This paper presented the problem of PID controller design to control the quadrotor system. The parameters of the PID controller are set from the condition of the minimum of the quality functional, the components of which are the deviation of the output value from the job and its derivatives. The representation of the quality functional by a dependence on unknown parameters made it possible to apply iterative optimization methods.

During the design, such optimization methods were studied: the method of coordinate-wise descent, gradient, and *steepest descent. Of greatest interest are the methods of* gradient search, where the direction of the step at the next *iteration turns out to be strictly perpendicular to the lines of* equal values of the quality function.

The resulting controller was compared to a similar model built using Simulink. The results of the comparison make it possible to establish the possibility of applying the tuning algorithm developed in the problems of constructing a quadrotor control system for constructing an effective flying vehicle.

References

[1] A. L. Salih, M. Moghavvemi, H. A. F. Mohamed, and K. S. Gaeid, "Flight PID controller design for a UAV quadrotor," Academic Journals, Scientific Research and Essays, vol. 5(23), pp. 3660–3667, 2010.

[2] B. Kada and Y. Ghazzawi, "Robust PID Controller Design for a UAV Flight Control System," Proceedings of the World Congress on Engineering and Computer Science, 19-21 October 2011, San Francisco, USA: proceedings: WCECS, vol. II, 2011, pp. 945–950.

[3] H. Bolandi, M. Rezaei, R. Mohsenipour, H. Nemati, and S. M. Smailzadeh, "Attitude Control of a Quadrotor with Optimized PID Controller," Intelligent Control and Automation, 4, pp. 335–342, 2013.

[4] L. M. Argentim, W. C. Rezende, P. E. Santos, and R. A. Aguiar, "PID, LQR and LQR-PID on a Quadcopter Platform," 2nd International Conference on Informatics, Electronics and Vision (ICIEV 2013), Dhaka, Bangladesh, May 17-18, 2013: proceedings: ICIEV, 2013, pp. 1–6.

[5] I. N. Ibrahim, M. A. A. Akkad and I. V. Abramov, "UAV Efficient PID and LQR Controllers Design based on its Stochastic State Space Dynamics Model including Disturbances," Moscow Workshop on Electronic and Networking Technologies (MWENT), 14-16 March 2018, Moscow, Russia, pp. 1–9.

[6] O. V. Vasilenko, D. P. Kucherov, and B. P. Ivanov, "On two approaches to the synthesis control system of the ship: optimal on a time and proportional – differentia controller," Academic Journal. Control, Navigation and Communication System, no. 3(11), pp. 80–85, 2009.

[7] V. N. Azarskov, A. Yu. Pilchevsky, L. S. Zhiteckii, and K. Yu. Solovchuk, "Is the l1-optimal Lateral Autopilot of PI Type Robust?," Proceedings 2016 IEEE 4th International Conference of Navigation and Motion Control (MSNMC), October 18-20, 2016, Kyiv, Ukraine: proceedings: IEEE, 2016, pp. 170–174.

[8] D. P. Kucherov and A. M. Kozub, "Mathematical model of UAV as agent of multi-agent system," Dependable Systems, Services, and Technologies (DESSERT-2018), May 24-27, 2018, Kyiv, Ukraine: proceedings: IEEE, 2018, pp. 358–362.

[9] J. A. Snyman, "Practical mathematical optimization. An Introduction to Basic Optimization Theory and Classical and New Gradient-Based Algorithms," Springer, NY, USA, 2005, p. 247.

[10] J. Nocedal and S. J. Wright, "Numerical Optimization," Springer, Second Edition, 2006, p. 664.

[11] P. Pounds, R. Mahony, P. Hynes, and J. Roberts, "Design of a Four-Rotor Aerial Robot," Proc. 2002 Australasian Conference on Robotics and Automation, Auckland, 27-29 November 2002, pp. 145–150.