Section A. Nomenclature Topics

Part I. Thermodynamic Processes, Real and Ideal Gases

Chapter 3. Theoretical Dependencies for an Ideal Gas

LECTURE 4. PARAMETERS OF AN IDEAL GAS IN INTERRELATIONSHIPS

§ 1.5. The equation of state for an ideal gas

An equation that establishes (sets up) the connection between pressure, temperature, and specific volume of a medium of a constant content (composition, compound) is called the **thermodynamic equation of state** [113, pp. 14-15], [79, p. 381]. The general view of this equation is [113, p. 14, (2.6)]

$$f(p, v, T) = 0. \tag{2.6}$$

The equation of (2.6) in space depicts (portrays, shows, represents, describes, illustrates, gives a picture of) a **surface**, which characterises all possible equilibrium states of a regular (uniform) thermodynamic system. This surface is called the **thermodynamic surface** or the **surface of state**. At the thermodynamic surface a certain point corresponds to each state of the system.

In the equation of (2.6) the number of the independent variables will be equal to two of the three of (p, v, T), because any of the three parameters (state variables) is the **one-valued function** of the other two given. For example (instance), if one accepts v and T as the independent variables, then p can be expressed as a function of v and T, i.e. (that is) $p = f_p(v, T)$ (the lower index (subscript) has a sense of the analytical view of the function of f_p to get the p as the dependent variable

(function); not to be confused with the constant state variables given below for iso-processes and partial derivatives); if one accepts p and T as the independent variables, then specific volume $v = f_v(p, T)$.

If one of the parameters of state (state variables) is the constant value, then there will be two variable values, and points, representing the state of the system, will lay (lie) upon one plane, which intersects the thermodynamic surface perpendicularly to the coordinate axis, upon which the constant value is taken. Such coordinate systems on plane are called **diagrams of state of substance**. The diagrams with coordinates pv, pT, vT, Ts, is are applied (used) most often.

$$* \qquad * \qquad * \qquad * \qquad *$$

The equation of state for the ideal gas first was obtained (found, got, deduced) by **Benoît Paul Émile Clapeyron** in 1834 by the way of joining (unifying, combining) the equations of the **Robert's Boyle's-Edme's Mariotte's** and **Joseph's Louis's Gay-Lussac's laws**

$$\frac{pv}{T} = \text{const}.$$

Denoting (indicate, designating, specifying) the constant as R, we will get [113, p. 14, (2.7)]

$$pv = RT, \qquad\qquad (2.7)$$

where R – specific gas constant, relatively to the mass of the gas is equaled to 1 kg; J/(kg·K).

Benoît Paul Émile Clapeyron (26 February 1799 – 28 January 1864) was a French engineer and physicist, one of the founders of thermodynamics.

Robert Boyle, Fellow of the Royal Society (FRS), (25 January 1627 – 31 December 1691) was a 17th century natural philosopher, chemist, physicist, and inventor, also noted for his writings in theology. He has been variously described as Irish, English and Anglo-Irish, his father having come to Ireland from England during the time of the Plantations.

Edme Mariotte ([Til-Châtel](#) c. 1620 – [Paris](#) 12 May 1684) was a [French](#) [physicist](#) and priest.

Joseph Louis Gay-Lussac (pronounced: [[ʒɔsɛf lwi gɛlysak]](#); also **Louis Joseph Gay-Lussac**, 6 December 1778 – 9 May 1850) was a [French](#) [chemist](#) and [physicist](#). He is known mostly for [two laws](#) related to [gases](#), and for his work on alcohol-water mixtures, which led to the [degrees Gay-Lussac](#) used to measure alcoholic beverages in many countries.

One of the possible ways to deduce (derive, get, gain, obtain, receive, take, develop, originate) the equation of state (2.7) is the next (following). The **Boyle's-Mariotte's law** (R. Boyle (1662), E. Mariotte (1676)) is

$$pv = \text{const}, \qquad T = \text{const};$$

and the **Gay-Lussac's law** (1802) is

$$\frac{v}{T} = \text{const}, \qquad p = \text{const}.$$

I. Let us consider a chain of two of iso-processes that transfer the system from the state 1 with the coordinates of (v_1, p_1), through the intermediate state Tp with the coordinates of (v_{N_21}, p_2), to the state 2 with the coordinates of (v_2, p_2) (fig. 2.a3). The iso-curves are plotted for nitrogen with the following (next) parameters (the specific gas constant see below, at the end of this paragraph).
[XXX, pp. XXX-XXXX]

Pa		$\frac{m^3}{kg}$		K	
$p1 := 1.01332 \cdot 10^5$	$v1 := \dfrac{22.4146 \cdot 10^{-3}}{\mu N2}$	$v1 = 0.79995$	$T1 := \dfrac{p1 \cdot v1}{RN2}$	$T1 = 273.182$
$p2 := 8 \cdot p1 \quad p2 = 8.107 \times 10^5$	$v2 := \dfrac{v1}{2}$	$v2 = 0.39998$	$T2 := \dfrac{p2 \cdot v2}{RN2}$	$T2 = 1.093 \times 10^3$

First, we compress the gas in the cylinder at the constant temperature T_1; carry out the compression and at the same time cooling the gas in order to keep the temperature of the gas constant (or doing the compression so slowly that the

temperature of the system relaxes). Then, at the constant pressure p_2 we expand the gas by heating it.
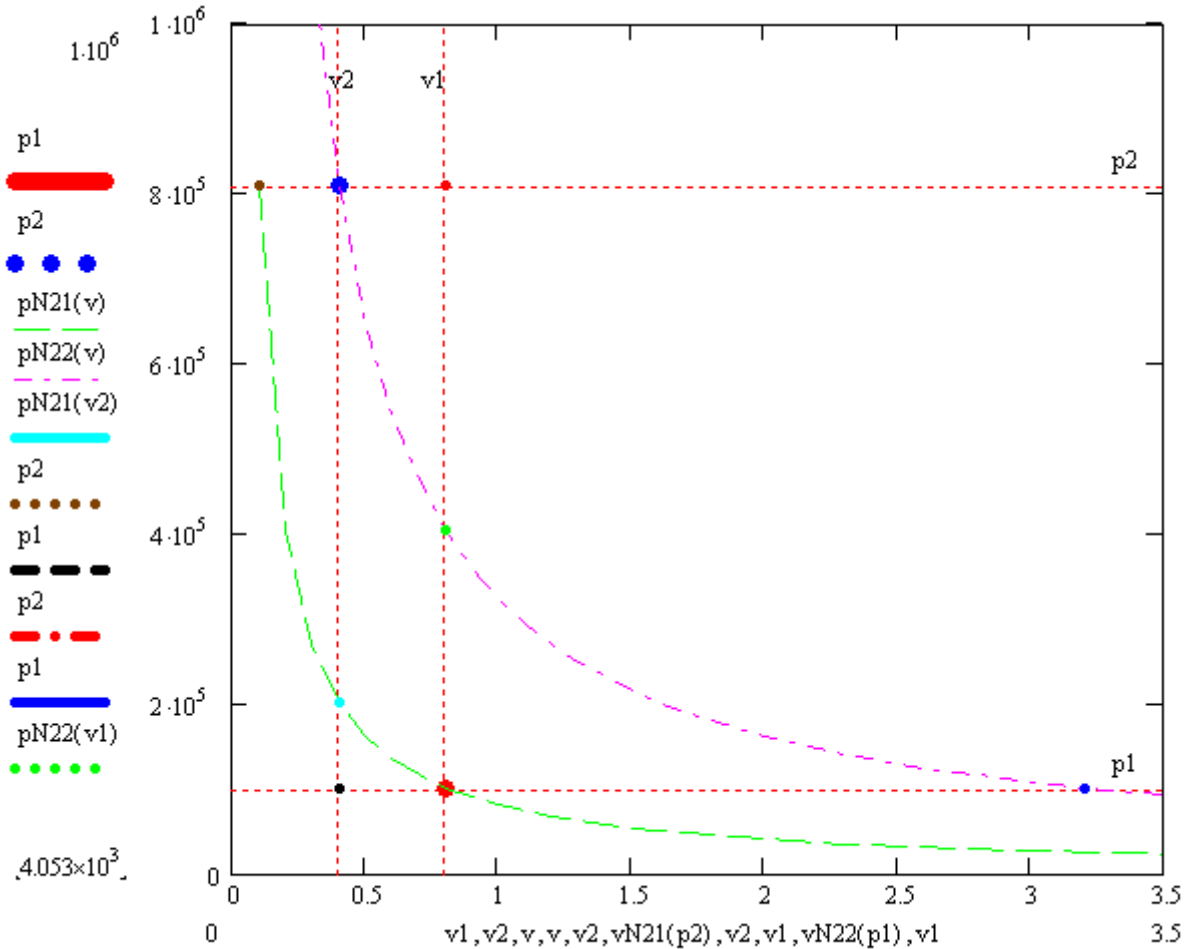

Fig. 2.a3

The first process 1-Tp is the isothermal. Accordingly to the Boyle's-Mariotte's law

$$pv = \text{const} = p_1 v_1 = p_2 v_{Tp}, \qquad T = \text{const} = T_1. \qquad (2.7.1)$$

The second process Tp-2 is the isobaric. Accordingly to the Gay-Lussac's law

$$\frac{v}{T} = \text{const} = \frac{v_{Tp}}{T_1} = \frac{v_2}{T_2}, \qquad p = \text{const} = p_2. \qquad (2.7.2)$$

Expressing v_{Tp} from the condition of (2.7.2) we get

$$v_{Tp} = \frac{T_1}{T_2} v_2. \qquad (2.7.3)$$

Substituting v_{Tp} of the equation of (2.7.3) into the relation of (2.7.1) we get

17

$$p_1 v_1 = p_2 \frac{T_1}{T_2} v_2, \qquad \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \text{const}. \qquad (2.7.4)$$

Thus, we come to the expression of (2.7), i.e. (that is) the equation of state by Clapeyron for the ideal gas.

Actually, the equation of (2.7) can be obtained by leading the process from the state 1 to the state 2 through the five other points.

II. For 1-*Tv* and *Tv*-2

$$pv = \text{const} = p_1 v_1 = p_{Tv} v_2, \qquad T = \text{const} = T_1. \qquad (2.7.5)$$

The **Charles's law** (also the **second Gay-Lussac's law** (1808) is

$$\frac{p}{T} = \text{const}, \qquad v = \text{const}. \qquad (2.7.6)$$

Then,

$$\frac{p}{T} = \text{const} = \frac{p_{Tv}}{T_1} = \frac{p_2}{T_2}, \qquad v = \text{const} = v_2. \qquad (2.7.7)$$

$$p_{Tv} = p_2 \frac{T_1}{T_2}. \qquad (2.7.8)$$

$$p_1 v_1 = p_2 \frac{T_1}{T_2} v_2, \qquad \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \text{const}. \qquad (2.7.9)$$

Jacques Alexandre César Charles (November 12, 1746 – April 7, 1823) was a French inventor, scientist, mathematician, and balloonist.

III. For 1-*vp* and *vp*-2

$$\frac{p}{T} = \text{const} = \frac{p_1}{T_1} = \frac{p_2}{T_{vp}}, \qquad v = \text{const} = v_1. \qquad (2.7.10)$$

$$\frac{v}{T} = \text{const} = \frac{v_1}{T_{vp}} = \frac{v_2}{T_2}, \qquad p = \text{const} = p_2. \qquad (2.7.11)$$

$$\frac{1}{T_{vp}} = \frac{v_2}{T_2 v_1}. \qquad (2.7.12)$$

$$\frac{p_1}{T_1} = \frac{p_2 v_2}{T_2 v_1}, \qquad \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \text{const}. \qquad (2.7.13)$$

IV. For 1-*vT* and *vT*-2

$$\frac{p}{T} = \text{const} = \frac{p_1}{T_1} = \frac{p_{vT}}{T_2}, \qquad v = \text{const} = v_1. \qquad (2.7.14)$$

$$pv = \text{const} = p_{vT} v_1 = p_2 v_2, \qquad T = \text{const} = T_2. \qquad (2.7.15)$$

$$p_{vT} = \frac{p_2 v_2}{v_1}. \qquad (2.7.16)$$

$$\frac{p_1}{T_1} = \frac{p_2 v_2}{T_2 v_1}, \qquad \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \text{const}. \qquad (2.7.17)$$

V. For 1-*pv* and *pv*-2

$$\frac{v}{T} = \text{const} = \frac{v_1}{T_1} = \frac{v_2}{T_{pv}}, \qquad p = \text{const} = p_1. \qquad (2.7.18)$$

$$\frac{p}{T} = \text{const} = \frac{p_1}{T_{pv}} = \frac{p_2}{T_2}, \qquad v = \text{const} = v_2. \qquad (2.7.19)$$

$$\frac{1}{T_{pv}} = \frac{p_2}{T_2 p_1}. \qquad (2.7.20)$$

$$\frac{v_1}{T_1} = \frac{v_2 p_2}{T_2 p_1}, \qquad \frac{v_1 p_1}{T_1} = \frac{v_2 p_2}{T_2} = \text{const}. \qquad (2.7.21)$$

VI. For 1-*pT* and *pT*-2

$$\frac{v}{T} = \text{const} = \frac{v_1}{T_1} = \frac{v_{pT}}{T_2}, \qquad p = \text{const} = p_1. \qquad (2.7.22)$$

$$pv = \text{const} = p_1 v_{pT} = p_2 v_2, \qquad T = \text{const} = T_2. \qquad (2.7.23)$$

$$v_{pT} = \frac{p_2 v_2}{p_1}. \qquad (2.7.24)$$

$$\frac{v_1}{T_1} = \frac{v_2 p_2}{T_2 p_1}, \qquad \frac{v_1 p_1}{T_1} = \frac{v_2 p_2}{T_2} = \text{const}. \qquad (2.7.25)$$

The equation of (2.7) is put (written) down for 1 kg of an ideal gas. For *m* kg, the equation of state will have the view of [113, p. 15, (2.8)]

$$pV = mRT, \qquad (2.8)$$

where V – the volume of the gas, m³.

The gas which state is accurately described by the equation of (2.7) is called the **ideal**.

Multiplying the both parts of the equation of (2.7) by the molecular mass (weight) μ, we will get [113, p. 15, (2.9)]

$$pV_\mu = \mu RT ,\qquad\qquad(2.9)$$

where $V_\mu = v\mu$ – the volume occupied by one mole of the gas.

Molecular mass (weight) represents

$$\ast\qquad\qquad\ast\qquad\qquad\ast\qquad\qquad\ast\qquad\qquad\ast$$

The volume of one mole of gas at the normal physical conditions $(T_0 = 273.15 \text{ K}, \ p_0 = 101{,}332 \text{ Pa})$ $V_\mu = \mu v = 22.4146 \cdot 10^{-3}$ m^3/mole. Substituting this value into the formula of (2.9) we will get [113, p. 15, (2.9a)]

$$\mu R = \frac{p_0 V_\mu}{T_0} = 8.3143 \text{ J/(mole·K)},\qquad\qquad(2.9a)$$

where μR – **universal gas constant**, the same for any of gases [79, p. 381].

From here the equation of state for one mole of the idea gas will be [113, p. 15, (2.10)]

$$pV_\mu = 8.3143T .\qquad\qquad(2.10)$$

The equation of (2.10) was deduced by **D.I. Mendeleyev** in 1874 and is called the equation of state by **Clapeyron-Mendeleyev** [113, p. 14-15].

The molecular masses (weights) of hydrogen, nitrogen, and oxygen [XXX, pp. XXX-XXXX]

$$\mu H2 := 2.016 \cdot 10^{-3} \quad \mu N2 := 28.02 \cdot 10^{-3} \quad \mu O2 := 32 \cdot 10^{-3} \qquad \frac{kg}{mol}$$

The specific gas constants for hydrogen, nitrogen, and oxygen

$$RH2 := \frac{8.3143}{\mu H2} \qquad RN2 := \frac{8.3143}{\mu N2} \qquad RO2 := \frac{8.3143}{\mu O2} \qquad \frac{\dfrac{\text{Joules}}{\text{mol·K}}}{\dfrac{kg}{mol}}$$

$$RH2 = 4124.157 \qquad RN2 = 296.727 \qquad RO2 = 259.822 \qquad \frac{\text{Joules}}{kg \cdot K}$$