Approaches to the Evaluation of Conformity Taking into Account the Uncertainty of the Value of the Monitored Parameter

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Abstract — **The article discusses approaches to the formation of rules for deciding on the compliance of an object with established standards, taking into account the measurement uncertainty of the tested parameter, recommended by international documents. The features of accounting for uncertainty in each of them are shown. An approach is proposed using an adaptive iterative decision-making algorithm, which is a compromise between risk, effort and cost of the conformity assessment procedure. The algorithm uses the introduction of a guard interval within the limits of the tolerance field, as well as the relationship between the tolerance interval on the parameter, measurement uncertainty and the likelihood of a decision on compliance. The possibility of implementing the proposed algorithm on a specific example is show.**

Keywords — measurement uncertainty, decision role, guard interval, conformity assessment, acceptance interval, tolerance, probability of compliance.

I. Introduction

Conformity assessment is any activity undertaken to directly or indirectly determine whether a product, process, system, person, or body with relevant standards and fulfills specified requirements [1]. In the measurement inspection of the characteristic properties / parameters of products, the assessment of compliance with specified requirements is made on the basis of measurements of the monitored quantity. Since the measurement result is presented in the form of an uncertainty interval, and the requirements are specified in the form of a tolerance interval for a parameter, then, with certain ratios of these quantities, there are risks of making an incorrect decision on compliance.

Before commencing the comparison of measurement results with specified requirements and making a decision on conformity / non-conformity, it is advisable to determine in advance the method of comparison of measurement results and established standards. There are several approaches to the development of a decisive rule on compliance (non-compliance) with the established requirements, which do not provide unambiguous recommendations.

II. Formulation of the Problem

In current regulations there is no unified approach to deciding on compliance and non-compliance. Therefore, the proposal of the decision rule, which would take into account both the interests of the consumer and the interests of the producer, is relevant, being a compromise solution.

III. Approaches to the Formation of the Decision Rule Taking into Account Uncertainty

The approach outlined in the international standard [2] involves a two-step procedure, the use of which reduces the risk of making incorrect decisions. A two-step procedure involves repeated measurements when the limits of the uncertainty interval, calculated after the first stage, are outside the tolerance interval. In this case, the measurement result is in the area where it is the inconclusive result of the conformity assessment. The value of the measured quantity and its uncertainty are established as a combination of the results of measurements of the two stages.

Reducing the area of inconclusive results is possible by reducing the measurement error. This can be achieved by using measurement tools with less instrumental uncertainty, additional measurements. However, such measures significantly increase the cost of measurements incompatible with their goals [3].

The decision on compliance (non-compliance) with the requirements is made depending on whether the entire uncertainty interval is in the zone of permissible (unacceptable) values. However, uncertainty can be estimated for different confidence values. This approach allows the producer of products to make decisions related to conformity assessment, as when comparing the measured value with the limits of the tolerance interval does not give unambiguous recommendations. It is impossible to unambiguously answer whether the result is acceptable when with a certain probability the measured characteristic is in the zone of unacceptable values.

The standard does not deal with situations where a non-final conformity assessment result has been obtained, although standards for testing methods of specific technical objects may contain other rules for conformity assessment.

So, standards [4, 5] state that the decision on compliance is made only if the corresponding limit of the one-sided interval of uncertainty does not exceed the standard value. In this case, the measured value increased by the expanded uncertainty is compared with the tolerance limit. However, it is difficult to put it into practice due to the fact that the principles of assigning the maximum permissible uncertainty values for normalized indicators are not clear [6].

The approach outlined in [7] is technique an iterative of management uncertainty management technique based on GUM principles. It is based on the sequential Wald algorithm [8].

Its feature is a differentiated approach to the evaluation of each object to be monitored, in contrast to the traditional integral approach with multiple measurements, followed by averaging the results obtained. This iterative technique is based on the “upper limit value” strategy, i.e. estimation of uncertainty at all levels, but with repeated inspection, as shown in Fig.1.

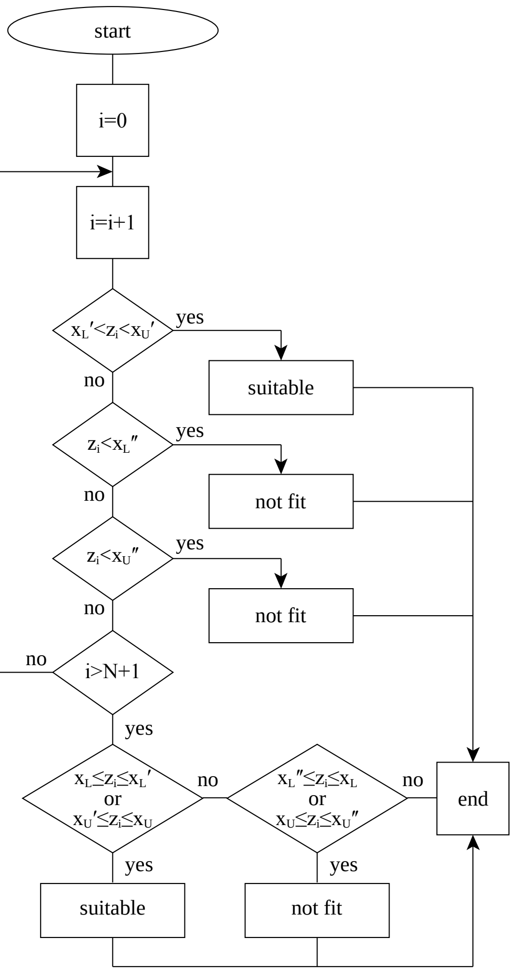

Fig.1 Sequential decision algorithm

Re-evaluation must be monitored, given the economic feasibility. The iterative technique makes it possible to compromise between risk, complexity and cost of the conformity assessment procedure.

An acceptable approach to assessing uncertainty in the field of conformity assessment activities in the electrical sector, according to the international organizations ILAC and IAF, is the Guide IEC 115 [9]. The approach is based on:

- minimization of significant sources of variability by monitoring them;
- exclusion from the analysis of those sources that have little effect on the result;
- using known test equipment accuracy as a representation of the measurement uncertainty of the test result.

Approach [9] is to limit the impact of significant sources of uncertainty by limiting the permissible limits of their variability. Figure 2 shows a recommendation that should be used in cases where the essential components of uncertainty cannot be limited in accordance with the so-called “accuracy method” (when the maximum measurement error relates to the instrument). In this case, the calculated measurement uncertainty is compared with the interval of uncertainty and with the established limit of the monitored quantity. A measurement result is considered acceptable if its best estimate is below the established limit and the probability of it being within at least 50%.

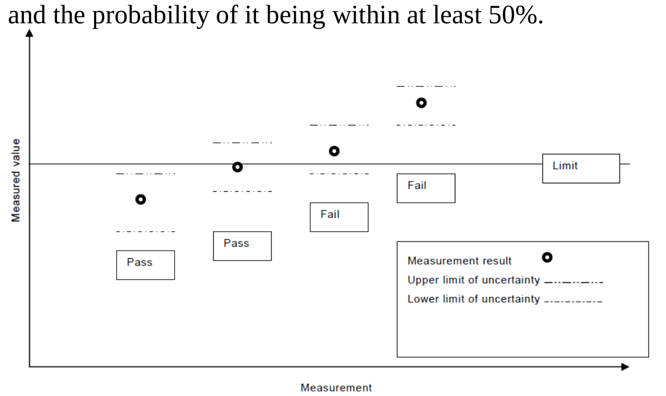

Fig.2. Decision rules with estimated uncertainty

Figure 3 presents a recommendation for the case when sources of uncertainty are monitored and there is no need to estimate uncertainty. At the same time, in accordance with the “accuracy method”, the variability of the test parameters and the accuracy of the instruments are within acceptable limits. A measurement result is deemed acceptable if it is within specified limits.

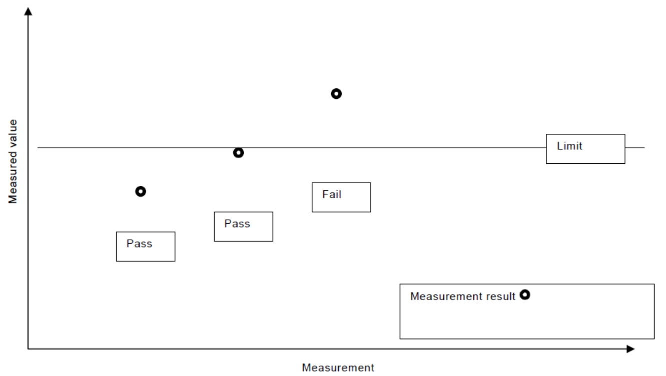

Fig.3. Decision rules for controlled sources of uncertainty

The “accuracy method” is a traditional approach to estimating uncertainty in the electrical sector. It requires less time and cost to implement than detailed calculation of measurement uncertainty and allows you to compare the result with the established limit of the monitored value.

International documents [10, 11] propose two approaches to the development of a decisive rule on compliance (non-compliance) with established requirements: based on the so-called shared risk and on the basis of establishing restrictions on errors of the 1st and 2nd kind when testing the compliance hypothesis.

With regard to decision-making procedures based on the analysis of errors of the 1st and 2nd kind, it should be noted that in the framework of the GUM [12] these procedures are incorrect. Error analysis of the 1st and 2nd kind is an integral part of the statistical hypothesis testing procedure based on the frequency interpretation of the probability, in which the concept of the true value of the measured quantity is used and the result of the measurement is represented as a realization of a random variable. The concept of GUM is based on a Bayesian interpretation of probabilities, in which randomness is expressed not as a result of measurement, but in a subjective view of the measured quantity, characterized by the corresponding distribution.

The rule of shared risk is that the requirement is deemed to be met if the resulting value of the measured value lies within the established tolerance, provided that the measurement uncertainty is within certain limits. In this case, if the decision made turns out to be wrong, then the damage from this, given the restriction on the measurement uncertainty, will be insignificant.

The rule of shared risk has a limited scope. It cannot be used in cases where exceeding the established limit is unacceptable. A way out of this situation is to establish a guard interval (guard band [9]) near the limit value, as shown in Figure 4, where x_L is the lower limit of the tolerance interval; x_L' – lower limit of the acceptance interval; guard interval w separates the tolerance interval from the acceptance area. It is considered that the object does not meet the requirement, not only in the case when the value of the monitored parameter is out of tolerance, but also when it fell into the guard interval. This decision rule will allow you to set a guard interval taking into account the uncertainty of the measurement result in the form:

$$w = ku,$$

where $k = 2$ is the coverage factor for a confidence level of 0.95

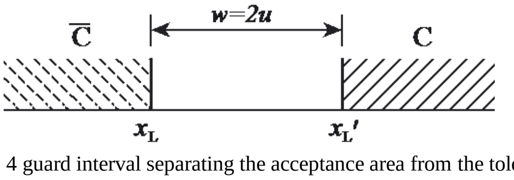

Fig. 4 guard interval separating the acceptance area from the tolerance interval

The guard interval is determined in such a way that the probability of an erroneous decision on the acceptability / inadmissibility of a measurement result lying in the range of acceptable values is less than or equal to the specified value of the significance level. Based on the technical requirements and decision rules, the decision limit is calculated, as well as the range of acceptable / invalid values.

In general, it is necessary to talk about two different uncertainties – the current measurement and the specified limits. Uncertainty of limits depends on the principle of their formation. The procedure for comparing a measurement result with given limits should take into account, in addition to the uncertainties of the current measurement and limits, the rule for the formation of these limits.

In order to apply the Bayes decision procedure, it is necessary that all possible alternative hypotheses about conformity be assigned some probability of their fulfillment. With regard to the problem of conformity assessment, this means that it is necessary to have a priori information about the probability with which a parameter can take one or another value. This information can be formalized in the form of a priori probability density. A priori, we can assume the normal distribution of a monitored indicator on the basis of the law of large numbers, for which, for random variables, which are sums of a large number of independent random variables, under some general conditions it turns out that this amount has a distribution, near to normal although each of the terms may not obey the normal probability distribution [13].

Consequently, under the normal distribution of possible deviations of measurement results z from the measured value of x, the probability of conformity is calculated [13] as:

$$p_c = \Phi\left(\frac{x_U - x}{u_c}\right) - \Phi\left(\frac{x_L - x}{u_c}\right), \tag{1}$$

where $\Phi(\cdot) = \int_{-\infty}^{a} e^{-t^2}\,dt$ is the Laplace function;
u_c - is the combined standard uncertainty of the result z.

Since the relevant objects (parameters) are included, the value of the measured value is in the tolerance interval $T = x_U - x_L$, the relative value in the tolerance zone is introduced:

$$\tilde{x} = (x - x_L)/T, \tag{2}$$

which takes value

$$\tilde{x} = \begin{cases} 0 \text{ при } x = x_L \\ 1 \text{ при } x = x_U \end{cases}.$$

Substituting the expression (2) in expression (1), we obtain the dependence:

$$p_c = \Phi[4C_m(1-\tilde{x})] - \Phi(-4C_m\tilde{x}) = p_c(\tilde{x}, C_m). \tag{3}$$

where C_m – measurement capability index of the measuring system, taking into account the ratio of its uncertainty and tolerance interval.

The document [10] shows a diagram showing which relations C_m and $\tilde{x}$ the probability value of p_c remain constant and equal to 95% for the $0 \le \tilde{x} \le 1$. Curve in the diagram divides the areas of correspondence and discrepancies. As follows from the diagram for $C_m = 1$ ($u =$ = T / 4), the values corresponding to the probability $p_c \ge$ 95% will only be for relative monitored values in the range $0{,}45 \le \tilde{x} \le 0{,}55$. To expand the range of possible monitored values, it is necessary to increase the value of C_m. The direct way to achieve this is to reduce the uncertainty of the measurement u_c. It is essential in the application of a decisive rule based on a guard acceptance, since the length of the guard interval is proportional to the uncertainty.

According to [14], with a guard acceptance, the consumer's risk of getting inappropriate products is reduced, since objects whose monitored parameter values fall into the guard band are considered inappropriate, although in reality this is not the case, hence the producer suffers a loss. If you gradually reduce uncertainty and the guard interval associated with it, then you can reduce the loss of the producer.

IV. Iterative Adaptive Procedure for Making a Decision on Conformity

Often there is a case when the standard uncertainty u_c corresponding to the estimate of the monitored parameter has a fixed value, which depends on the design of the measuring system, but does not depend on the parameter itself. In this case, the measurement capability index C_m is fixed, and the question of whether the value of the monitored parameter meets the technical conditions with an acceptable probability can be solved on the basis of an estimate of this parameter using expressions (2) and (3) with a fixed C_m.

This situation can be observed in the practice of industries that assess the conformity of their products, when the range of products can vary two or more times a day. In this case, components – components of products may remain unchanged, but the tolerance intervals on them are changing. Since the measuring instruments on the technological line remain the same, the instrumental component of measurement uncertainty remains the same [15]. This leads to a change in C_m, which in turn affects the likelihood of compliance and, with the rest, the probability of making the correct decision by the results of inspection.

As a result of the analysis of the approaches in the formation of decision rules on compliance, a decision rule is proposed based on an iterative multicyclic inspection organization procedure using a guard interval and control of the measurement capability index.

A gradual reduction of the guard band can be achieved by applying a sequential procedure. The essence of the sequential procedure is as follows. At the first main stage, all objects are monitored without exception. The transition to the next additional step is carried out in the case when the result of measuring the parameter of the object has fallen into the zone of making doubtful decisions - the zone of uncertainty (x_L, x_L') or / and (x_U', x_U). The size of the uncertainty zone is determined depending on the metrological characteristics of the measuring system and corresponds to the expanded uncertainty of measurement U. Output is the combine standard uncertainty of measurement u_c, which defines the limit x_L' or / and x_U' and the limiting number of successive stages when the result z hits an additional interval (x_L, x_L') or / and (x_U', x_U).

At the beginning of the decision-making process, calculate the initial value:

$$C_m = T/4u_c. \tag{4}$$

Based on the actual correlation between the length of the tolerance interval and the uncertainty of measurement, the initial relative lower and upper limit values are found which correspond to the probability of making a decision on the correspondence of 95%:

$$\tilde{x}_{L1} = \frac{x_{L1} - x_L}{T}; \tag{5}$$

They are represented in absolute values, with which the primary result of the measurement is compared:

$$x_{L1} = \tilde{x}_{L1}T + x_L;$$
$$x_{U1} = \tilde{x}_{U1}T + x_L \tag{6}$$

If it is in the acceptance interval, then with a probability of 95% it is decided to match the object to the given rules, and this procedure of inspection is over. In the opposite case, they proceed to a consistent procedure for reducing uncertainty of measurement and finding additional acceptance limits that reduce the guard band. The average and uncertainty of the result is determined on (i + 1)-th stage:

$$\bar{x}_{i+1} = \frac{i\bar{x}_i}{i+1} + \frac{x_{i+1}}{i+1}, \left(i = \overline{0,n}\right); \tag{7}$$

$$u_{c(i+1)} = u_c / \sqrt{i+1}, \tag{8}$$

Then calculate a new value of the measurement capability index:

$$C'_m = \frac{T\sqrt{i+1}}{2U}. \tag{9}$$

The number of stages is selected provided the compliance is at 95%. If at nth stage the probability of compliance within tolerances is less than 95%, then a decision is made about the discrepancy of the object. The procedure for reducing the guard band is shown in Fig. 5.

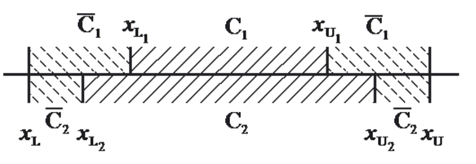

Fig. 5. Adaptive variation of the guard interval

If, when using a guard band, 5% of the corresponding objects is discarded, but when using an adaptive sequential algorithm, it is not. Thus, those objects whose measurement result did not fall into the acceptance interval are discarded.

In the method of adaptive control of limits, the actual value of the parameter is refined, the measurement result of which fell into the interval ($x_L \ldots x_{L1}$) или ($x_{U1} \ldots x_U$). For objects where the measurement result is less than x_L or more than x_U, a decision is made about the nonconformity. It should be noted that the initial acceptance limits x_{L1} and x_{U1} coincide with the acceptance limits using the guard band only for p_c = 0,95. So, in the case of p_c > 0.95, the compliance curve [10] for this case will be higher, and, therefore, the value of the acceptance limits will be closer to x_L and x_U, respectively.

V. Numerical Example

An object with a nominal value of a monitored parameter of 100 units has a maximum deviation of ± 3%. Limit values:

x_L = 97 units, x_U = 103 units; tolerance T = 6 units

In the passport of the measuring instrument the maximum difference between two consecutive measurements is indicated r = 0,42 units.

Sources of uncertainty taken:

- permitting instrument capacity d = 0,005;

- influence of random variables.

Standard uncertainty by type B:

$$u\,(d) = 0.003 \text{ units.}$$

Based on the ratio $r = 2{,}8\cdot\sigma$ [16] we define

$$\sigma = r/2{,}8 = 0{,}15 \text{ units.}$$

With information on the standard deviation, one can determine the combine standard measurement uncertainty:

$$u_c = \sqrt{\sigma^2 + u^2(d)} \approx \sigma .$$

Determine the value of the guard band:

$$w = 2\sigma = 0{,}3 \text{ units.}$$

Thus, the limits of the acceptance interval:

$$x_L = 97{,}3 \text{ units and } x_U = 102{,}7 \text{ units}$$

However, the use of a guard band leads to unnecessary losses of the producer, which in material terms can be very significant.

The same result can be achieved in the first step by applying the method of adaptive limits. Indeed, calculating the measurement capability index:

$$C_{m1} = T/\,4u_c = 6/4\cdot 0{,}15 = 10,$$

according to the nomogram [9] we define the relative acceptance limits:

$$\tilde{x}_{L1} = 0{,}05 \text{ units and } \tilde{x}_{U1} = 0{,}95 \text{ units}.$$

Then we determine the acceptance limits in absolute units:

$$x_{L1} = \tilde{x}_{L1}T + x_L = 0{,}05\cdot 6 + 97 = 97{,}3 \text{ units;}$$

$$x_{U1} = \tilde{x}_{U1}T + x_L = 0{,}95\cdot 6 + 97 = 102{,}7 \text{ units.}$$

In the case of failure of the measurement result in this interval, carry out a re-measurement, estimate the uncertainty of type A associated with the inaccuracy of the implementation of the measurement procedure and the influence of environmental factors, and calculate a new measurement capability index for two successive results:

$$C_{m2} = \frac{6*\sqrt{2}}{4*0{,}15} = 14{,}1 .$$

For this value, by the nomogram [10], find new relative limits:

$$\tilde{x}_{L1} = 0{,}03 \text{ units and } \tilde{x}_{U1} = 0{,}97 \text{ units}.$$

According to these values we find absolute acceptance limits:

$$x_{L1} = \tilde{x}_{L1}T + x_L = 0{,}03\cdot 6 + 97 = 97{,}18 \text{ units}$$

$$x_{U1} = \tilde{x}_{U1}T + x_L = 0{,}97\cdot 6 + 97 = 102{,}82 \text{ units}$$

For objects whose result the measurement does not fall within the "new" acceptance limits, carry out the third measurement, etc. The number of successive stages is determined by the permissible "residual" probability of making an incorrect decision about the matching of the object.

Thus, the use of an adaptive sequential algorithm reduces the size of the guard band and ensures the correct decision-making about compliance and thus prevents loss of the producer.

Conclusion

Some approaches to the formation of the decision rule when assessing compliance with the uncertainty of measurements are analyzed. The analysis showed that there are no single decisive rules for conformity assessment, and standards for test methods for specific technical objects may contain different rules for conformity assessment.

An approach to the formation of a decision rule is proposed, the basis for which is the principle of guard acceptance, followed by the use of consistent adaptive expansion of acceptance limits to maximize the satisfaction of the interests of both the consumer and the producer.

The approach is applicable to any objects of conformity assessment.

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