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Investigation of estimation of the mixed second-order moment in bilinear unbiased estimates class

The best estimate (with minimum variance) of the mixed second-order moment in a class of bilinear unbiased estimates is proposed.

The solution of problems related to the reliability of products and choice of optimal technological options for the production is a very important issue in modern terms. It is connected with both a permanent complication of the manufacturing technology and the need improving the economic performance of production. To solve technological problems the probabilistic and statistical methods are used. The application of these techniques allows for the statistical analysis of accumulated data with the aim of generalization of the information in formulas. One of the main tasks that arise is the definition of distribution law of random variables and corresponding numerical characteristics on experimental data.

The main characteristics of random variables include the expectation, variance, covariance.

Let $K(x,y)=m[x-m(x)][y-m(y)]$ be the covariance of random variables x,y where $m(x)$ and $m(y)$ are expectations of x and y correspondingly. Covariance is known to be one of the most important characteristics of measure of random variables dependence.

The covariance is equal to zero for independent random variables. But the inverse statement is not correct because there are examples of dependent random variables with zero covariance [1]. But for many random variables x, y having the given common distribution $F(u,v)=p(x<u,y<v)$ which belongs to a certain class (for instance to the class of compatible normal distributions) of independent random variables the following equality is hold $K(x,y)=0$ [2]. That is why in practice statistical analysis of the results of empirical observations the small covariance is considered a sign of low dependence of variables *x* and *y*.

Let x, y be the pair of random variables and $K(x,y)=m[x-m(x)][y-m(y)]$ is covariance. Let us transform

$$K(x,y)=m(xy-xm(y)-m(x)y+m(x)m(y))=m(xy)-m(x)m(y)-m(x)m(y)+$$

$$+m(x)m(y)=m(xy)-m(x)m(y)\,.$$

It follows that for the evaluation of covariance $K(x,y)$ it’s necessary to assess the mixed second-order moment $\gamma=m(xy)$ and expectations $\alpha=m(x),\beta=m(y)$.

Let $(x_1, y_1),\dots,(x_n y_n)$ be the set of pairs of sample values obtained from the general population using simple random selection. It means that all random values (x_i, y_i) have compatible and two-dimensional distribution $F(u,v)$ and these pairs are independent each other. Hence for any two Borel's sets M_1, M_2 in the plane
$p((x_i, y_i)\in M_1,(x_k, y_k)\in M_2)= p((x_i, y_i)\in M_1)\cdot p((x_k, y_k)\in M_2)$
if $i \neq k$.

We consider the class of linear unbiased estimates of the mixed second-order moment $\gamma = m(xy)$

$$T = \left\{ u = \sum_{i,k=1}^{n} a_{ik} x_i y_k = (Ax, y), A = \|a_{ik}\|_{i,k=\overline{1,n}}, x = (x_1,\dots,x_n), y = (y_1,\dots,y_n), \right.$$

$$\left. m(u) = \gamma \right\}.$$

From the condition of unbiasedness of assessment $m(u) = m(xy)$ follows

$$m(u) = m\left(\sum_{i,k=1}^{n} a_{ik} x_i y_k\right) = m\left(\sum_{\substack{i\neq k \\ i,k=1}}^{n} a_{ik} x_i y_k + \sum_{i=1}^{n} a_{ii} x_i y_i\right) = m(x)m(y)\sum_{\substack{i\neq k \\ i,k=1}}^{n} a_{ik} +$$

$$+ m(xy)\sum_{i=1}^{n} a_{ii} = \alpha\beta \sum_{\substack{i\neq k \\ i,k=1}}^{n} a_{ik} + m(xy)\sum_{i=1}^{n} a_{ii} = m(xy) .$$

Because this equality should be carried out at random $m(xy)$ then

$$\sum_{\substack{i,k=1 \\ i\neq k}}^{n} a_{ik} = 0, \qquad \sum_{i=1}^{n} a_{ii} = 1 .$$

Thus, the class of bilinear unbiased estimates of the mixed second-order moment has the following view

$$T = \left\{ u = \sum_{i,k=1}^{n} a_{ik} x_i y_k = (Ax, y), A = \|a_{ik}\|, x = (x_1,\dots,x_n), y = (y_1,\dots,y_n), \right.$$

$$\left.\sum_{\substack{i,k=1\\ i\neq k}}^{n} a_{ik} = 0, \sum_{i=1}^{n} a_{ii} = 1 \right\}.$$

Let $(x_1, y_n),...,(x_1, y_n)$ be a set of pairs of sample values obtained from the general population using simple random the distribution of which has the final mixed moment of the fourth order. We consider the estimation

$$\bar{\gamma} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i \quad .$$

This estimation $\bar{\gamma}$ belongs to the class T .

Really, $\bar{\gamma}$ is the estimation of the view $u = \sum_{i,k=1}^{n} a_{ik} x_i y_k$ with coefficients

$a_{ii} = \frac{1}{n}, \forall i = \overline{1,n}$, $a_{ik} = 0$, $i \neq k$ satisfying unbiasedness:

$$\sum_{\substack{i,k=1\\ i\neq k}}^{n} a_{ik} = 0 \ , \ \sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{n} \frac{1}{n} = 1 \, .$$

Namely $\bar{\gamma} \in T$.

Now prove that $D(\bar{\gamma}) \leq D(u), \forall u \in T$. According to unbiasedness condition $\forall u \in T$: $D(u) = m(u^2) - m^2(u) = m(u^2) - \gamma^2$.

A minimum $D(u)$ reachs where the function $m(u^2)$ acquires the smallest value.

If the value $m(u^2)$ is find and the unbiasedness conditions $a_{nn} = 1 - \sum_{i=1}^{n-1} a_{ii}$, $a_{nn-1} = - \sum_{\substack{i\neq k\\ (i,k)\neq(n,n-1)}}^{n} a_{ik}$ are taken into account then the problem of searching of a conditional extremum will be reduced to the problem of absolute extremum research.

As a result of transformations we obtain a positive quadratic function with respect to variables a_{ik} which reaches a minimum at the point where the partial derivatives are equal to zero.

Finding the partial derivatives and sabstituting coefficients $a^*_{ik} = 0, \ i \neq k; \quad a^*_{ii} = \frac{1}{n}$ (from the estimation $\bar{\gamma} = \frac{1}{n}\sum_{k=1}^{n} x_k y_k$), we see in the conditions.

Resume

The estimation $\bar{\gamma} = \frac{1}{n}\sum_{k=1}^{n} x_k y_k$ is the best (another words it has minimum variance) in class T of unbiased bilinear estimates of the mixed second-order moment. незміщених білінійних оцінок змішаного моменту другого порядку.

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