MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
National Aviation University

VIVERE!
VINCERE!
CREARE!

CONTINUING AIRCRAFT AIRWORTHINESS (ICAO Doc 9760)

SELF-STUDY METHOD GUIDE

Part I

Reliability Measures to Assess the Aircraft Maintenance Process Improvements for the Students of the Field of Study 27 "Transport", Specialty 272 "Aviation Transport", Specialization 01 "Aircraft and Aero-Engines Maintenance and Repair"

Kyiv 2018

Ministry of Education and Science of Ukraine
National Aviation University

CONTINUING AIRCRAFT AIRWORTHINESS (ICAO Doc 9760)

SELF-STUDY METHOD GUIDE

Part I

Reliability Measures to Assess the Aircraft Maintenance Process Improvements for the Students of the Field of Study 27 "Transport", Specialty 272 "Aviation Transport", Specialization 01 "Aircraft and Aero-Engines Maintenance and Repair"

Kyiv 2018

УДК 629.73:656.017:629.7.017(076.5)
С76

Compiler *A. V. Goncharenko*

Reviewers:
V. V. Panin, Doctor of engineering sciences, professor (State University of Infrastructure and Technology);

S. R. Ignatovich, Doctor of engineering sciences, professor (National Aviation University);

English language adviser *I. S. Kozeletska*, senior lecturer (NAU)

Approved by the Methodical and Editorial Board of the National Aviation University (Minutes № 2/18 of 19.06.2018).

Містять декілька рекомендацій для самостійної роботи щодо показників надійності для оцінки удосконалення процесу технічного обслуговування повітряного судна в рамках елементів наукового дослідження.

Для студентів 1-го курсу галузі знань 27 «Транспорт», спеціальності 272 «Авіаційний транспорт», спеціалізації 01 «Технічне обслуговування та ремонт повітряних суден і авіадвигунів».

С76 **Continuing Aircraft Airworthiness (ICAO Doc 9760)** : Self-Study Method Guide . Part I . Reliability Measures to Assess the Aircraft Maintenance Process Improvements / compiler A. V. Goncharenko. – K. : NAU, 2018. – 48 p.

The **Method Guide** contains a few recommendations on the Self-Study in regards with the reliability measures to assess the aircraft maintenance process improvements in the framework of the scientific research elements.

Designed for the 1st year students of the Field of Study 27 "Transport", Specialty 272 "Aviation Transport", Specialization 01 "Maintenance and Repair of Aircraft and Aircraft Engines".

Contents

INTRODUCTION

This **METHOD GUIDE ON THE SELF-STUDY** (SS) is contemplated in response to the needs of our students in more detailed elaborations concerning problems stated, set or given for the students' independent work, possibly used in their course projecting, further graduation papers or even Ph.D. studies. The whole material is split into portions. Each portion is intended to cover a fraction of probable applications aimed at **CONTINUING AIRCRAFT AIRWORTHINESS** or its retaining measures.

The presented in this, first, **PART I** of the **METHOD GUIDE ON THE** SS problems are dedicated, and a special attention is drawn here, to the scientific component of the SS work. Specifically, the objectives of the **PART I** are to help students cope with the challenging problems relating to the **AIRCRAFT** (A/C) technical operation in regards with the aeronautical engineering **MAINTENANCE** (M/T) optimal periodicities.

The set of the considered problems is based upon the **RECOMMENDED LITERATURE SOURCES** (the list is presented, but not limited to it). The **LIST OF LITERATURE** at the end of the **METHOD GUIDE** is basic (major) and compiled in the alphabetic order with respect to the matter of supposed (assumed) importance.

The **REFERENCES LIST** is selected, set in the order [1-144], does not pretend for completeness, but instead it is aimed at developing the students' abilities of thinking and to analyze, contemplate in the specified directory rather than their abilities to know and memorize. However, these are very significant too. Actually, in the contemporary informative boom world, the needed or required data can easily be retrieved from the internet, found in multiple references, guidance materials [1, 69, 71, 74, 111, 112, 134, 135, 137], studies [2, 66-68, 75-88, 113-116, 119-133, 136, 139-144], dictionaries [70, 110], comprehensive books [3, 72] or monographs [64, 65, 86-88, 113, 114, 117, 118, 123] etc. The **METHOD GUIDE** is designed for the 1st year students of the Field of Study: 27 "Transport", Specialty: 272 "Aviation Transport", Specialization: 01 "Maintenance and Repair of Aircraft and Aircraft Engines". It includes detailed solutions for obtaining reliability objective measures allowing assessing the improvements of the A/C functional system M/T process considered in reference [112].

THE SIMPLEST CASE OF THE MAINTENANCE OPTIMAL PERIODICITY DETERMINATION

The principal theoretical provisions can be found out in references [141-143, 3, 72, 76, 82, 84, 87, 110, 114, 116, 124-133].

1. Determination of the periodicity with the use (help) of the normative for the specified (predetermined) reliability level

It is supposed, [143, pp. 162-174, Chapter 15, especially Sub-Chapter 15.4, pp. 169, 170], that on the basis of some directive documents or researches (investigations) the necessary or required reliability level of a certain engineering unit (product) is predetermined (given probability of the non-failure running, acceptable number of damages or failures per 1,000 flight hours or in-between scheduled maintenance period etc.), which must be ensured (guaranteed) in the process of operation. To that end, from the established (determined) normative reliability and with the help of the accepted model, the maintenance maximal periodicity is found that would ensure the specified reliability, [143, pp. 169, 170].

The process of the damages emergence on the units of aeronautical engineering in some cases can be considered as the simplest flow of uniform events, characterized with stationary, absence of after-action, and ordinary. Such a flow has a property that the probability of the damages emergence on the unit is predetermined by the *Poisson* law, [143, p. 170].

In the considered case, the probability that there will arise k number of damages in the given period between maintenances t_p is [143, Sub-Chapter 15.4, p. 170]:

$$P_k(t_p)=\frac{(\omega t_p)^k}{k!}e^{-\omega t_p}, \qquad (1)$$

where ω – parameter of the damages flow (damage intensity) in the between-scheduled maintenance period.

From the calculation results of the specified probabilities (Eq. (1)) depending upon the maintenance periodicity, for example, for the fuel-governing aggregate of the powerplant (Fig. 1) [143, p. 170, Fig. 15.1] it is visible that if the emergence of two damages on the aggregate is a danger (threat), then at the accepted probability level of their appearance $P_2(t_p) = 0.05$ the maintenance periodicity of the aggregate should not exceed (go beyond) 383 h. The damage intensity (parameter of the damages flow) has been got by the statistical data, $\omega = 1 \cdot 10^{-3}$ h–1, [143, p. 170].

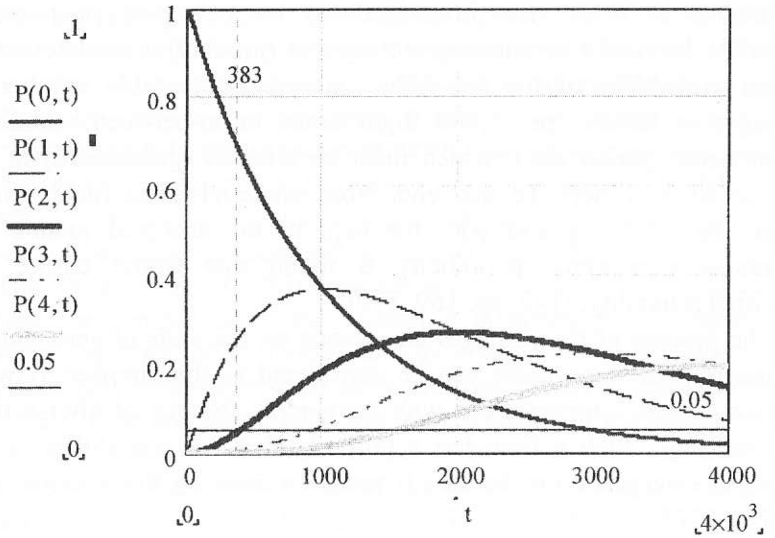

Fig. 1. Dependence of the fuel-governing aggregate malfunctioning coming out probability distribution upon running hours [143, p. 170, Fig. 15.1]

The given method can be used for complex products consisting of a great number of elements, failure of one or two of which does not influence upon the working state of the product as a whole, [143, p. 170].

2. Determination of the maintenance optimal periodicity with taking into account (consideration) the rate (speed) of the failure development

The essence of the method is that at the optimal timing intervals of the maintenance execution the probability of the conjoint event – emergence of a damage and the failure not coming out is maximized. It is supposed (assumed) that elimination (removal) of the damage in the predetermined time intervals prevents (prevails) the failures occurrences [143, pp. 170-172].

Let us say, [143, p. 170], that at the work of a product at a random moment of time τ there happens a damage, the further development of which leads to the failure at the moment of time t. Maintenance has to be performed in the interval (τ, t) at the moment t_p, Fig. 2.

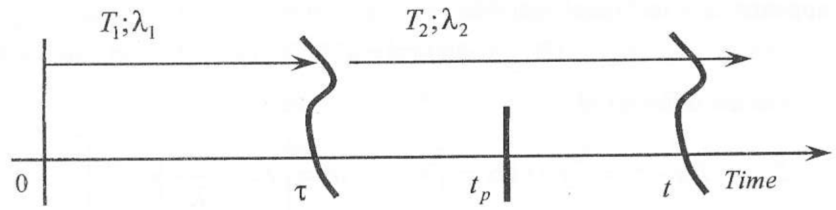

Fig. 2. Illustration of the failure after a damage development

Then, for the considered product, the probability that the damage will happen at the moment τ but there will not occur the failure during the time interval of $(t-\tau)$, [143, p. 171],

$$dP(t,\tau) = f(\tau)d\tau P(t-\tau), \qquad (2)$$

where $f(\tau)$ – distribution density of running hours before the damage emergence; $P(t-\tau)$ – probability of not materialization of the failure in the interval $(t-\tau)$ (on condition that the damage appeared at the moment of time τ), [143, p. 171].

Integrating with respect to τ from 0 up to t we will get, [143, p. 171],

$$P(t,\tau) = \int_0^t f(\tau) P(t-\tau) d\tau. \qquad (3)$$

Accepting, for instance, the law of the damages appearance times distribution, as well as of their development times to the failure happening, as the exponential ones with the corresponding rates of λ_1 and λ_2, [143, p. 171], that is

$$f_1(t)=\lambda_1 e^{-\lambda_1 t}, \qquad f_2(t)=\lambda_2 e^{-\lambda_2 t}, \tag{4}$$

$$P_{\bar{F}|D} = P(t-\tau)=1-Q(t-\tau)=1-\int_0^{t-\tau} f_2(x)dx = 1-\int_0^{t-\tau} \lambda_2 e^{-\lambda_2 x}dx =$$

$$=1-\left[\lambda_2\left(-\frac{1}{\lambda_2}\right)e^{-\lambda_2 x}\right]_0^{t-\tau} = 1+e^{-\lambda_2(t-\tau)}-e^{-\lambda_2\cdot 0} = 1+e^{-\lambda_2(t-\tau)}-1,$$

where $P_{\bar{F}|D}$ – conditional probability of the failure not happening, there is the “negation mark” over F, i.e. $\bar{F}$, on the condition that the damage appears; x – technical variable,

$$P_{\bar{F}|D} = P(t-\tau)=e^{-\lambda_2(t-\tau)}. \tag{5}$$

On the other hand

$$P_{\bar{F}|D} = P(t-\tau)=\int_{t-\tau}^{\infty} f_2(x)dx = \int_{t-\tau}^{\infty}\lambda_2 e^{-\lambda_2 x}dx = \left[\lambda_2\left(-\frac{1}{\lambda_2}\right)e^{-\lambda_2 x}\right]_{t-\tau}^{\infty} =$$

$$=-e^{-\lambda_2\cdot\infty}+e^{-\lambda_2(t-\tau)} = e^{-\lambda_2(t-\tau)}-0.$$

Thus, the result of Eq. (5) is also obtained.

Then equation (3) yields, [143, p. 171],

$$P_{D\bar{F}} = P(t,\tau)=P(t)=\int_0^t e^{-\lambda_2(t-\tau)}\lambda_1 e^{-\lambda_1\tau}d\tau = \lambda_1 e^{-\lambda_2 t}\int_0^t e^{(\lambda_2-\lambda_1)\tau}d\tau =$$

$$=\frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2 t}\left[e^{(\lambda_2-\lambda_1)\tau}\right]_0^t = \frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2 t}\left[e^{(\lambda_2-\lambda_1)t}-1\right]. \tag{6}$$

or, [143, p. 171, (15.1)],

$$P_{D\bar{F}} = P(t,\tau)=P(t)=\frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t}-e^{-\lambda_2 t}\right). \tag{7}$$

The optimal time interval for the scheduled maintenance works performance will be found at

$$\frac{dP_{D\bar{F}}}{dt}=0. \tag{8}$$

Accordingly to the condition of Eq. (8), it yields, [143, p. 171, (15.2)],

$$\frac{dP_{D\bar{F}}}{dt}=\frac{\lambda_1}{\lambda_2-\lambda_1}\left(\lambda_2 e^{-\lambda_2 t}-\lambda_1 e^{-\lambda_1 t}\right)=0. \tag{9}$$

Solving equation (9), we find the sought optimal periodicity of the scheduled maintenance works performance, [143, p. 171]:

$$\lambda_2 e^{-\lambda_2 t_p}-\lambda_1 e^{-\lambda_1 t_p}=0, \qquad \lambda_2 e^{-\lambda_2 t_p}=\lambda_1 e^{-\lambda_1 t_p}, \qquad e^{(\lambda_1-\lambda_2)t_p}=\frac{\lambda_1}{\lambda_2},$$

$$(\lambda_1-\lambda_2)t_p=\ln\frac{\lambda_1}{\lambda_2}, \qquad t_p=\frac{\ln\lambda_1-\ln\lambda_2}{\lambda_1-\lambda_2}. \tag{10}$$

Practically, the optimal value of the maintenance caring out timing (10) can be determined from a diagram in the way of plotting the dependencies of the: probability of the damages, in the between-scheduled period, absence $P_{\bar{D}}(t)$; probability of the damages happening, but not coming forward the failure $P_{D\bar{F}}(t)$; probability of the failure, in the between-scheduled period, coming out $P_F(t)$ [143, p. 171].

3. Determination of the maintenance optimal periodicity via conditional probabilities

In the considered above case (2)-(10) all those three situations together constitute (comprise, make up, form, compose) the complete (total/whole/full/entire) group of non-conjunctive events [143, p. 171]:

$$P_{\bar{D}}(t)+P_{D\bar{F}}(t)+P_F(t)=1. \tag{11}$$

This is the so-called normalizing condition.

Let us remind that if there is a set of two random events A and B possibly happening together, then the total probability for the whole group of non-conjunctive events, [82],

$$P_{AB}+P_{A\bar{B}}+P_{\bar{A}B}+P_{\bar{A}\bar{B}}=1. \tag{12}$$

For example, the probability of event A is $P_A=0.1$; for not A, i.e. $\bar{A}$, it makes $P_{\bar{A}}=1-P_A=1-0.1=0.9$; conditional probabilities, for instance: event B happens on condition of event A has already happened, $P_{B|A}=0.4$; then $P_{\overline{B}|A}=1-P_{B|A}=1-0.4=0.6$; $P_{B|\overline{A}}=0.2$; $P_{\overline{B}|\overline{A}}=1-P_{B|\overline{A}}=1-0.2=0.8$.

Probabilities for conjunctive (conjoint) events, for Eq. (12):

$$P_{AB}=P_A P_{B|A}=0.1\cdot 0.4=0.04;$$

$$P_{A\overline{B}}=P_A P_{\overline{B}|A}=0.1\cdot 0.6=0.06;$$

$$P_{\overline{A}B}=P_{\overline{A}} P_{B|\overline{A}}=0.9\cdot 0.2=0.18;$$

$$P_{\overline{A}\overline{B}}=P_{\overline{A}} P_{\overline{B}|\overline{A}}=0.9\cdot 0.8=0.72. \qquad (13)$$

Substituting results of Eq. (13) for corresponding values of Eq. (12) it yields that result. An illustration of the concept expressed with Eq. (11)-(13) is represented in Fig. 3.

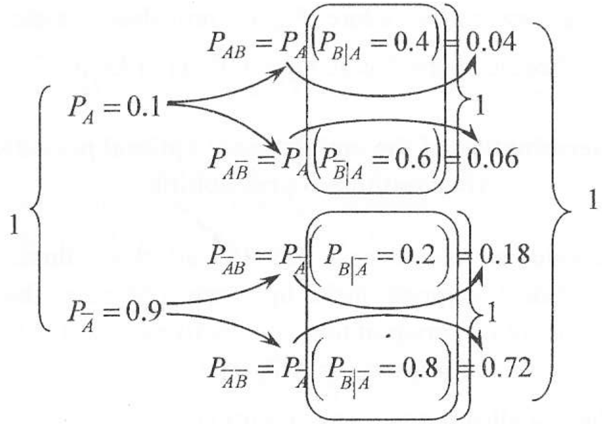

Fig. 3. Illustration of the concept for
a set of two conjoint events

In the considered problem setting, Eq. (2)-(11), the failure can occur (arise) only if a damage has already taken place. For probabilities, it may be interpreted in such, for instance, way (Fig. 4).

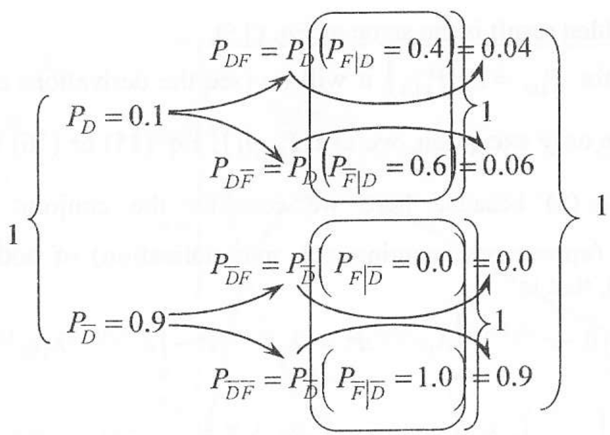

Fig. 4. Illustration of the concept for a set of damage
and failure conjoint events

That is why in Eq. (11) $P_{\overline{D}}(t)$ represents both $P_{\overline{D}F}=P_{\overline{D}}\left(P_{F|\overline{D}}=0.0\right)=0.0$ and $P_{\overline{D}\overline{F}}=P_{\overline{D}}\left(P_{\overline{F}|\overline{D}}=1.0\right)=P_{\overline{D}}$; and $P_F(t)$ is for $P_{DF}=P_D\left(P_{F|D}\right)$. Therefore all of them, together with $P_{D\overline{F}}=P_D\left(P_{\overline{F}|D}\right)$, see Eq. (7), make the entire group of non-conjoint events. For the probability of $P_{\overline{D}\overline{F}}=P_{\overline{D}}\left(P_{\overline{F}|\overline{D}}=1.0\right)=P_{\overline{D}}(t)$ we have

$$P_{\overline{D}}(t)=\int_t^{\infty}\lambda_1 e^{-\lambda_1 x}dx=-e^{-\lambda_1\cdot\infty}+e^{-\lambda_1 t}=e^{-\lambda_1 t}-0=e^{-\lambda_1 t}. \qquad (14)$$

For $P_{DF}=P_D\left(P_{F|D}\right)$, see the derivation of Eq. (5) also,

$$P_{F|D}(t)=1-P_{\overline{F}|D}(t)=Q(t-\tau)=1-e^{-\lambda_2(t-\tau)}. \qquad (15)$$

Or on the other hand

$$P_{F|D}(t)=Q(t-\tau)=\int_0^{t-\tau}f_2(x)dx=\int_0^{t-\tau}\lambda_2 e^{-\lambda_2 x}dx=\left[\lambda_2\left(-\frac{1}{\lambda_2}\right)e^{-\lambda_2 x}\right]_0^{t-\tau}=$$

$$=-e^{-\lambda_2(t-\tau)}+e^{-\lambda_2\cdot 0}=-e^{-\lambda_2(t-\tau)}+1. \qquad (16)$$

The yielded result is the same as Eq. (15).

At last for $P_{DF}=P_D\left(P_{F|D}\right)$ it will be (see the derivations of Eq. (6), (7) with the only exception, we use $P_{F|D}(t)$, Eq. (15) or (16) instead of $P_{\overline{F}|D}(t)$, Eq. (5) because here we consider the conjoint event of emergence (appearance, coming out, materialization) of both damage and failure), that is

$$P_{DF}(t)=\int_0^t\left[1-e^{-\lambda_2(t-\tau)}\right]\lambda_1 e^{-\lambda_1\tau}d\tau=\int_0^t\lambda_1 e^{-\lambda_1\tau}d\tau-\int_0^t e^{-\lambda_2(t-\tau)}\lambda_1 e^{-\lambda_1\tau}d\tau=$$

$$=\left[\lambda_1\left(-\frac{1}{\lambda_1}\right)e^{-\lambda_1\tau}\right]_0^t-\lambda_1 e^{-\lambda_2 t}\int_0^t e^{(\lambda_2-\lambda_1)\tau}d\tau=-e^{-\lambda_1 t}+e^{-\lambda_1\cdot 0}-$$

$$-\frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2 t}\left[e^{(\lambda_2-\lambda_1)\tau}\right]_0^t=1-e^{-\lambda_1 t}-\frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2 t}\left[e^{(\lambda_2-\lambda_1)t}-e^{(\lambda_2-\lambda_1)0}\right]=$$

$$=1-e^{-\lambda_1 t}-\frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2 t}\left[e^{(\lambda_2-\lambda_1)t}-1\right]. \qquad (17)$$

Finally

$$P_{DF}(t)=1-e^{-\lambda_1 t}-\frac{\lambda_1}{\lambda_2-\lambda_1}\left[e^{-\lambda_1 t}-e^{-\lambda_2 t}\right]. \qquad (18)$$

Thus, see Eq. (7), (11), (14), (18), the normalizing condition (11) will be satisfied:

$$P_{DF}(t)+P_{D\overline{F}}(t)+P_{\overline{D}F}(t)+P_{\overline{D}\overline{F}}(t)=$$

$$=1-e^{-\lambda_1 t}-\frac{\lambda_1}{\lambda_2-\lambda_1}\left[e^{-\lambda_1 t}-e^{-\lambda_2 t}\right]+\frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t}-e^{-\lambda_2 t}\right)+0+e^{-\lambda_1 t}=1. \qquad (19)$$

The diagrams plotted by the numerical simulation with the formulae Eq. (2)-(19) are shown in Fig. 5. Mathematical modeling has been realized for such data: $\lambda_1=5\cdot10^{-3}$ h–1; $\lambda_2=1\cdot10^{-3}$ h–1; $t=0\ldots1.5\cdot10^3$ h. $t_{opt}\approx402$ h. $P_{DnF}(t)$ is for $P_{D\overline{F}}(t)$; $P_{nD}(t)$ – $P_{\overline{D}}(t)$; $P_{DF}(t)$ – $P_{DF}(t)$.

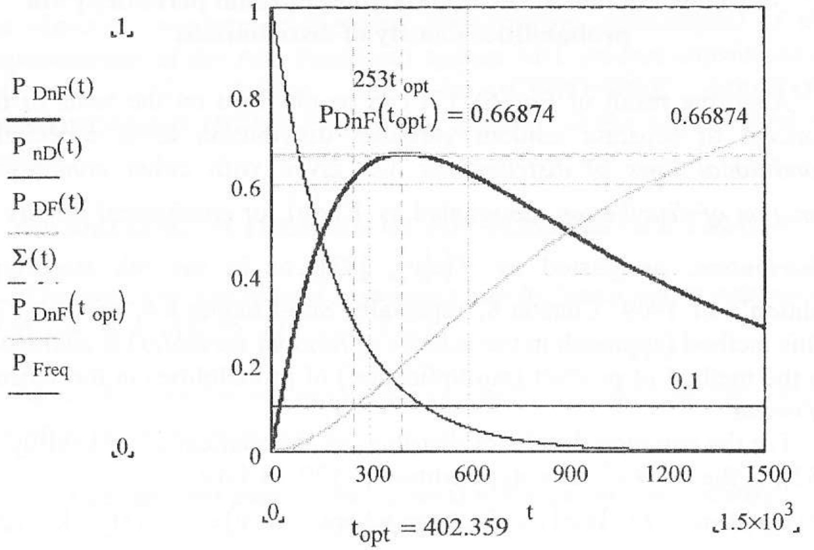

Fig. 5. Results of computer calculation experiments on the MathCad platform

It is obvious that the function of $P_{\overline{D}}(t)$ is a monotonously decreasing one, and the function of $P_{DF}(t)$ – non-decreasing [143, p. 171]. The character of the all three situations probability changes is shown in the Fig. 5, [143, p. 172, Fig. 15.2]. at the optimal periodicity t_{opt} determination one should follow the requirement that the probability of failure $P_{DF}(t)$ is not higher than the predetermined (prescribed) one $P_{DF}(t)_{req}$, [143, p. 171].

At the required level of the non-failure state probability, for example, $P_{DF}(t)_{req}\le0.1$, designated as P_{Freq} in Fig. 5, the maintenance periodicity must not exceed 253 hours (also see Fig. 5).

4. Determination of the maintenance optimal periodicity via probabilities density of distributions

Also, the result of Eq. (6), (7) can be obtained on the basis of the concept of separate random variables distribution laws, especially, *conditional laws of distributions*, i.e. given with either *conditional function of distribution*, designated as $F(x|y)$, or *conditional density of distribution*, designated as $f(x|y)$, [82], or in the 4th stereotype edition's of 1969: Chapter 8, especially Sub-Chapter 8.4, pp. 168-171. This method (approach in the *scheme of random variables*) is analogous to the method of product (multiplication) of probabilities in the *scheme of events*.

For the common density of distribution, the element of probability is, [82], in the 1969 4th stereotype edition p. 170, (8.4.4):

$$f(x,y)dxdy = f_1(x)dxf(y|x)dy, \text{ from where } f(x,y) = f_1(x)f(y|x), \qquad (20)$$

where x and y – specific values of random variables of X and Y; $f_1(x)$ – distribution law of random variable $T_1 = \tau - 0$ given with the density of its distribution, the first of the equations of (4); $f(y|x)$ – conditional density of distribution of random variable $T_2 = t - \tau$ given with the density of its distribution on condition that the first random variable T_1 has got the specified value of τ, the second Eq. (4).

Thus, we have the system of the two random variables T_1 and T_2. Therefore, integrating the element of probability, it yields

$$P_{D\bar{F}} = P(t,\tau) = P(t) = \int_0^t \lambda_1 e^{-\lambda_1 x} dx \left(\int_{t-\tau}^{\infty} \lambda_2 e^{-\lambda_2 y} dy \right) =$$

$$= \int_0^t \lambda_1 e^{-\lambda_1 x} \left(\left[-e^{-\lambda_2 y} \right]_{t-\tau}^{\infty} \right) dx = \int_0^t \lambda_1 e^{-\lambda_1 x} \left[-e^{-\lambda_2 \cdot \infty} + e^{-\lambda_2 (t-\tau)} \right] dx =$$

$$= \int_0^t \lambda_1 e^{-\lambda_1 x} \left[-0 + e^{-\lambda_2 (t-\tau)} \right] dx. \qquad (21)$$

And we have come to Eq. (6), (7) again.

At the next section, it is going to be considered a few aspects of the modeling, described above with the formulae of (1)-(21), with taking into account provisions of the ***MASS SERVICE THEORY.***

This will allow solving the reliability problems [143], relating with the objective measures evaluations (assessments, estimations) of the improvements of the A/C functional system M/T process considered in reference [112], dealing with the **CONTINUING AIRCRAFT AIRWORTHINESS (ICAO DOC 9760)** [1-144], at the higher level of study.

MODELLING ON THE BASIS OF THE *MASS SERVICE THEORY*

The principal theoretical provisions can be found out in references [136, 76-79, 81-84, 86, 113, 139, 140].

More elite (exclusive) modelling can be performed with the use of *Theory of Mass Service*, [136].

1. The simplest case of the optimal maintenance periodicity determination

First, let us consider a graph which corresponds to the previous problem. It is represented in Fig. 6.

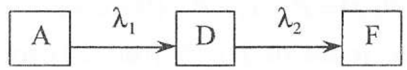

Fig. 6. Graph of three states of an aircraft functional system

Here, in Fig. 6, "A" designates the up state of the system; "D" – damage; "F" – failure. The corresponding system of differential equations by Erlang will have the view of

$$\left.\begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - \lambda_2 P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D. \end{aligned}\right\} \qquad (22)$$

Here, P_A, P_D, and P_F – probabilities of the corresponding states (see Fig. 6); t – time.

From the first equation of the system (22) we find

$$\frac{dP_A}{dt}=-\lambda_1 P_A; \qquad \frac{dP_A}{P_A}=-\lambda_1 dt; \qquad \int\frac{dP_A}{P_A}=-\int\lambda_1 dt+C, \tag{23}$$

where C – constant of integration, it will be found from the initial conditions:

$$t_0=0; \qquad P_A\big|_{t=t_0}=1; \qquad P_D\big|_{t=t_0}=0; \qquad P_F\big|_{t=t_0}=0, \tag{24}$$

Eq. (23) yields

$$\ln P_A=-\lambda_1 t+C, \tag{25}$$

from where

$$\ln 1=-\lambda_1\cdot 0+C, \qquad 0=-0+C, \qquad C=0. \tag{26}$$

Thus,

$$P_A(t)=e^{-\lambda_1 t}. \tag{27}$$

The same result as Eq. (14), which is correct because state "A" means not transition into the state of damage "D" and transition from "A" strait to the state of failure "F" is impossible as well as any of the backward transitions. Therefore

$$P_A(t)=P_{\overline{D}}(t)=P_{D\overline{F}}(t)+P_{\overline{DF}}(t)=0+e^{-\lambda_1 t}=e^{-\lambda_1 t}. \tag{28}$$

Substituting the result of Eq. (27) or (28) for the corresponding value of $P_A(t)$ in the second equation of the system (22) we get

$$\frac{dP_D}{dt}=\lambda_1 e^{-\lambda_1 t}-\lambda_2 P_D. \tag{29}$$

Thus, we have got a linear not uniform (homogeneous) differential equation of the first order. It can easily be solved with the method represented in [139, 140]. Exactly, accordingly to [140, Chapter XIII, especially § 7, pp. 30-33] the Eq. (29) has the view of [140, Chapter XIII, § 7, p. 30, (1)]

$$\frac{dy}{dx}+P(x)y=Q(x). \tag{30}$$

where $P(x)$ and $Q(x)$ – given continuous functions of x (or constants).

T h e s o l u t i o n o f t h e l i n e a r e q u a t i o n (3 0) . We will be finding the solution of the equation (30) in the view of a product of two functions of x [140, Chapter XIII, § 7, p. 30, (2)]:

$$y=u(x)v(x). \tag{31}$$

One of these functions can be taken arbitrary, the other one will be determined on the basis of Eq. (30).

Differentiating both parts of Eq. (31), we find, [140, Chapter XIII, § 7, p. 31],

$$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}. \tag{32}$$

Substituting the obtained expression of the derivative of $\frac{dy}{dx}$ into Eq. (30), we will have, [140, Chapter XIII, § 7, p. 31],

$$u\frac{dv}{dx}+v\frac{du}{dx}+Puv=Q, \tag{33}$$

or, [140, Chapter XIII, § 7, p. 31, (3)],

$$u\left(\frac{dv}{dx}+Pv\right)+v\frac{du}{dx}=Q. \tag{34}$$

Let us choose function v as such as, [140, Chapter XIII, § 7, p. 31, (4)],

$$\frac{dv}{dx}+Pv=0. \tag{35}$$

Dividing variables in this differential equation with respect to function v, we find, [140, Chapter XIII, § 7, p. 31],

$$\frac{dv}{v}=-Pdx. \tag{36}$$

Integrating, we get, [140, Chapter XIII, § 7, p. 31],

$$-\ln|C_1|+\ln|v|=-\int Pdx, \tag{37}$$

where C_1 – constant of integration; or, [140, Chapter XIII, § 7, p. 31],

$$v=C_1 e^{-\int Pdx}. \tag{38}$$

Since it is enough to have some different from zero solution of Eq. (35), then we will take [140, Chapter XIII, § 7, p. 31, (5)] as the function of $v(x)$, assuming and accepting $C_1=1$:

$$v(x) = e^{-\int P dx}, \tag{39}$$

where $\int P dx$ – some antiderivative (counterderivative). It is obviously that $v(x) \neq 0$.

Substituting the found value of $v(x)$ into Eq. (34), [140, Chapter XIII, § 7, p. 31, (3)], we will get (taking into account that $\left.\frac{dv}{dx} + Pv = 0\right)$, [140, Chapter XIII, § 7, p. 31],

$$v(x)\frac{du}{dx} = Q(x), \tag{40}$$

or, [140, Chapter XIII, § 7, p. 31],

$$\frac{du}{dx} = \frac{Q(x)}{v(x)}, \tag{41}$$

from where, [140, Chapter XIII, § 7, p. 31],

$$u = \int \frac{Q(x)}{v(x)} dx + C, \tag{42}$$

where C – constant of integration.

Substituting u and v into the formula of Eq. (31), [140, Chapter XIII, § 7, p. 30, (2)], we finally get

$$y = v(x)\left[\int \frac{Q(x)}{v(x)} dx + C\right], \tag{43}$$

or, [140, Chapter XIII, § 7, p. 32, (6)],

$$y = v(x)\int \frac{Q(x)}{v(x)} dx + Cv(x). \tag{44}$$

In the considered problem setting (see, compare, and substitute the corresponding functions, expressions, variables, and values of Eq. (29), (30), (44))

$$\frac{dP_D}{dt} + \lambda_2 P_D = \lambda_1 e^{-\lambda_1 t}, \qquad \frac{dy}{dx} + P(x)y = Q(x),$$

$$y = P_D, \quad x = t, \quad P(x) = \lambda_2, \quad Q(x) = \lambda_1 e^{-\lambda_1 t}, \quad v(x) = e^{-\int \lambda_2 dt} = e^{-\lambda_2 t}. \tag{45}$$

Thus

$$P_D = e^{-\lambda_2 t}\int \frac{\lambda_1 e^{-\lambda_1 t}}{e^{-\lambda_2 t}} dt + Ce^{-\lambda_2 t}. \tag{46}$$

Then

$$P_D = e^{-\lambda_2 t}\lambda_1 \int e^{(\lambda_2 - \lambda_1)t} dt + Ce^{-\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t}\left[e^{(\lambda_2 - \lambda_1)t}\right] + Ce^{-\lambda_2 t}. \tag{47}$$

From the initial conditions, expressions (24) $t_0 = 0$; $P_D|_{t=t_0} = 0$. It yields

$$0 = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 \cdot 0}\left[e^{(\lambda_2 - \lambda_1)\cdot 0}\right] + Ce^{-\lambda_2 \cdot 0} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \cdot 1 \cdot 1 + C \cdot 1. \tag{48}$$

Therefore

$$C = -\frac{\lambda_1}{\lambda_2 - \lambda_1}. \tag{49}$$

Then

$$P_D = \frac{\lambda_1}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t}\right] - \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right). \tag{50}$$

Final result Eq. (50) is the same as (6), (7). It is the probability of the damage state "D" of the system (see Fig. 6).

The probability of the failure state "F" will be obtained as the solution from the third equation of the system (22), i.e.

$$\frac{dP_F}{dt} = \lambda_2 P_D = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right). \tag{51}$$

Integration yields

$$P_F = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1}\left[\int e^{-\lambda_1 t} dt - \int e^{-\lambda_2 t} dt\right] + C, \tag{52}$$

where C – constant of integration.

Then

$$P_F = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1}\left[\left(-\frac{1}{\lambda_1}\right)e^{-\lambda_1 t} + \frac{1}{\lambda_2} e^{-\lambda_2 t}\right] + C. \tag{53}$$

From the initial conditions, expressions (24) $t_0 = 0$; $P_F|_{t=t_0} = 0$. It yields

$$0=\frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left[\left(-\frac{1}{\lambda_1}\right)e^{-\lambda_1\cdot 0}+\frac{1}{\lambda_2}e^{-\lambda_2\cdot 0}\right]+C=\frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)+C=$$

$$=\frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left(\frac{\lambda_1-\lambda_2}{\lambda_2\lambda_1}\right)+C=-1+C \quad\Rightarrow\quad C=1. \qquad (54)$$

Therefore

$$P_F=\frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left[\frac{e^{-\lambda_2 t}}{\lambda_2}-\frac{e^{-\lambda_1 t}}{\lambda_1}\right]+1=\frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left[\frac{\lambda_1 e^{-\lambda_2 t}-\lambda_2 e^{-\lambda_1 t}}{\lambda_2\lambda_1}\right]+1=$$

$$=\frac{\lambda_1 e^{-\lambda_2 t}-\lambda_2 e^{-\lambda_1 t}}{\lambda_2-\lambda_1}+1=\frac{\lambda_1 e^{-\lambda_2 t}-\lambda_2 e^{-\lambda_1 t}}{\lambda_2-\lambda_1}+1+e^{-\lambda_1 t}-e^{-\lambda_1 t}=$$

$$=1-e^{-\lambda_1 t}+\frac{\lambda_1 e^{-\lambda_2 t}-\lambda_2 e^{-\lambda_1 t}}{\lambda_2-\lambda_1}+e^{-\lambda_1 t}=$$

$$=1-e^{-\lambda_1 t}+\frac{\lambda_1 e^{-\lambda_2 t}-\lambda_2 e^{-\lambda_1 t}+\lambda_2 e^{-\lambda_1 t}-\lambda_1 e^{-\lambda_1 t}}{\lambda_2-\lambda_1}.$$

Finally

$$P_F=1-e^{-\lambda_1 t}+\frac{\lambda_1 e^{-\lambda_2 t}-\lambda_1 e^{-\lambda_1 t}}{\lambda_2-\lambda_1}=1-e^{-\lambda_1 t}+\frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_2 t}-e^{-\lambda_1 t}\right). \qquad (55)$$

The result (55) is the same as (17), (18).

Numerical simulation results as a conducted calculation experiment are shown in Fig. 7. The data for the performed calculations are the same as the mentioned above at fulfilling modeling portrayed in Fig. 5.

In Fig. 7 y0, y1, y2 represent numerical integration of the differential equations system (22) at the initial conditions (24). Symbols "0", "1", "2" depict the system states of "A", "D", "F" respectively. The curve y1 plotted for the damage state "D" coincides with $P_{DnF}(t)$ for $P_{D\bar{F}}(t)$; these are the solutions Eq. (6), (7), and P_D, Eq. (50) of the second equation of the system (22) correspondingly.

Both analytical research results described with the sequences of the Eq. (1)-(55) and computer simulation numerical solutions for the differential equations system (22) at the initial conditions (24) give the identical results.

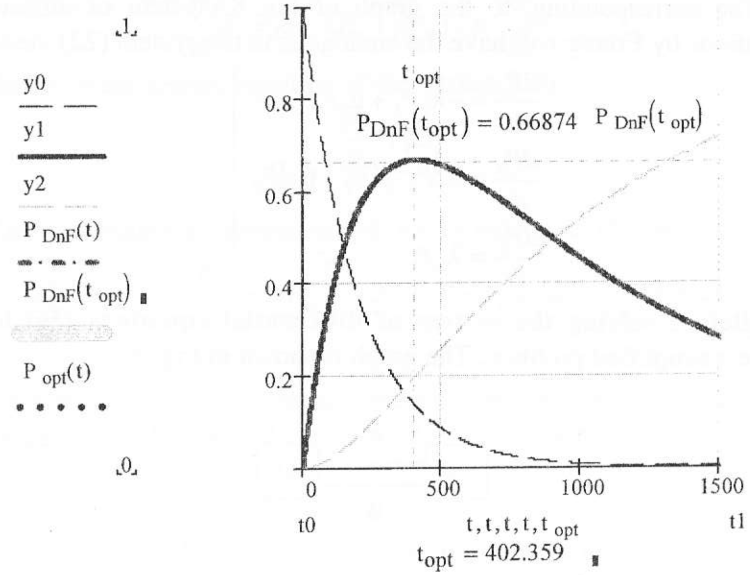

Fig. 7. Results of computer calculation experiments on the MathCad platform

Thus, the detailed solutions for obtaining the reliability objective measures allow successfully assessing the improvements of the A/C functional system M/T process considered in reference [112] on the basis of [136, 140].

2. More general modelling implying a possibility of restoration from the damaged into initial state

More general modelling will be done with a step that takes into account a possible return of the system from the damage state "D" into the normal state of "A". This transition is carried out with the parameter of μ_1 illustrated on the graph, see Fig. 8.

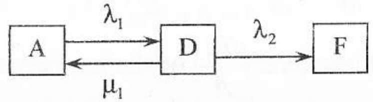

Fig. 8. Graph of three states of an aircraft functional system with a possible transition from the damaged into normal state

The corresponding, to the graph of Fig. 8, system of differential equations by Erlang will have the analogous to the system (22) view of

$$\left.\begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A + \mu_1 P_D; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - (\lambda_2 + \mu_1) P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D. \end{aligned}\right\} \quad (56)$$

Before solving the system of differential equations (56) let us solve a simplified problem. The graph is shown in Fig. 9.

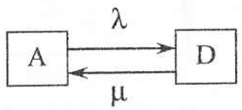

Fig. 9. Graph of two states of an aircraft functional system with a possible transition from the damage into normal state

The corresponding, to the graph of Fig. 9, system of differential equations by Erlang will have the analogous to the system (56) view of

$$\left.\begin{aligned} \frac{dP_A}{dt} &= -\lambda P_A + \mu P_D; \\ \frac{dP_D}{dt} &= \lambda P_A - \mu P_D. \end{aligned}\right\} \quad (57)$$

Let us convert (transform) the system of two ordinary linear differential equations of the first order (57) into one second order ordinary linear differential equation. Differentiate the second equation of the system (57) once more with respect to time.

$$\frac{d^2 P_D}{dt^2} = \frac{d}{dt}\frac{dP_D}{dt} = \frac{d}{dt}(\lambda P_A) - \frac{d}{dt}(\mu P_D) = \lambda \frac{dP_A}{dt} - \mu \frac{dP_D}{dt}. \quad (58)$$

Substitute the derivative $\frac{dP_A}{dt}$ of the expression (58) with its value from the first equation of the system (57).

$$\frac{d^2 P_D}{dt^2} = \lambda(-\lambda P_A + \mu P_D) - \mu \frac{dP_D}{dt}. \quad (59)$$

But from the second equation of the system (57)

$$P_A = \frac{\frac{dP_D}{dt} + \mu P_D}{\lambda}. \quad (60)$$

Thus, substituting expression (60) for its value into (59) we get

$$\ddot{P}_D = \lambda\left[-\lambda\left(\frac{\dot{P}_D + \mu P_D}{\lambda}\right) + \mu P_D\right] - \mu \dot{P}_D = \lambda(-\dot{P}_D) - \mu \dot{P}_D = -(\lambda + \mu)\dot{P}_D. \quad (61)$$

Here we have obtained the one second order ordinary linear uniform (homogeneous) differential equation, [140, Chapter XIII, § 21, p. 74, (1)]:

$$\ddot{P}_D + (\lambda + \mu)\dot{P}_D = 0. \quad (62)$$

In order to find the general integral of this equation, it will be sufficient (enough), as that was proven, for instance, in [140, Chapter XIII, § 20, pp. 68-74, Theorem 6, (8)], to find two linearly independent partial solutions, [140, Chapter XIII, § 21, pp. 74-79].

Let the sought partial solutions will be (let us be finding the partial solutions) in the view of [140, Chapter XIII, § 21, p. 74, (2)]:

$$P_D = e^{kt}, \quad \text{where} \quad k = \text{const}; \quad (63)$$

then [140, Chapter XIII, § 21, p. 74]:

$$\dot{P}_D = ke^{kt}, \qquad \ddot{P}_D = k^2 e^{kt}. \quad (64)$$

Substituting the obtained expressions of the derivatives into Eq. (62), we are finding [140, Chapter XIII, § 21, p. 74]:

$$k^2 e^{kt} + (\lambda + \mu)ke^{kt} = e^{kt}\left[k^2 + (\lambda + \mu)k\right] = 0. \quad (65)$$

Since (in view of the fact that) $e^{kt} \neq 0$, hence (for this reason), [140, Chapter XIII, § 21, p. 74, (3)]:

$$k^2 + (\lambda + \mu)k = 0. \quad (66)$$

Therefore, if k satisfies Eq. (66), then e^{kt} will be the solution of Eq. (62). Eq. (66) is called the *characteristic equation* in relation with Eq. (62).

The characteristic equation is a quadratic equation, having two roots; let us denote them as k_1 and k_2. At this, [140, Chapter XIII, § 21, p. 75]:

$$k_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}, \qquad k_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}, \tag{67}$$

where a, b, c – coefficients of a corresponding quadratic equation likewise

$$ak^2 + bk + c = 0. \tag{68}$$

There can be the following cases, [140, Chapter XIII, § 21, p. 75]:

I. k_1 and k_2 – real, being unequal one to the other numbers $(k_1 \neq k_2)$;

II. k_1 and k_2 – complex numbers;

III. k_1 and k_2 – real, being equal one to the other numbers $(k_1 = k_2)$.

Each case is considered separately at many reference books and literature sources. For example, [140, Chapter XIII, § 21, p. 75-79].

In the stated problem

$$a = 1, \qquad b = \lambda + \mu, \qquad c = 0. \tag{69}$$

The roots k_1 and k_2 from the equation (66)

$$k(k+\lambda+\mu) = 0, \qquad k_1 = 0, \qquad k_2 + \lambda + \mu = 0, \qquad k_2 = -(\lambda+\mu). \tag{70}$$

This is the condition # I, [140, Chapter XIII, § 21, p. 75]: $(k_1 \neq k_2)$. In such case, the partial solutions will be the functions [140, Chapter XIII, § 21, p. 75]:

$$P_{D_1} = e^{k_1 t}, \qquad P_{D_2} = e^{k_2 t}. \tag{71}$$

These solutions are linearly independent, since [140, Chapter XIII, § 21, p. 75]:

$$P_{D_1} = e^{k_1 t}, \qquad \frac{P_{D_2}}{P_{D_1}} = \frac{e^{k_2 t}}{e^{k_1 t}} = e^{(k_2-k_1)t} \neq \text{const}. \tag{72}$$

Hence (for this reason), the general integral has the view of [140, Chapter XIII, § 21, p. 75]:

$$P_D = C_1 e^{k_1 t} + C_2 e^{k_2 t} = C_1 e^{0\cdot t} + C_2 e^{-(\lambda+\mu)t} = C_1 + C_2 e^{-(\lambda+\mu)t}. \tag{73}$$

From the initial conditions similar to (24)

$$t_0 = 0; \qquad P_A\big|_{t=t_0} = 1; \qquad P_D\big|_{t=t_0} = 0. \tag{74}$$

It yields from Eq. (73)

$$0 = C_1 + C_2 e^{-(\lambda+\mu)0}; \qquad 0 = C_1 + C_2; \qquad C_1 = -C_2. \tag{75}$$

From where

$$P_D = -C_2 + C_2 e^{-(\lambda+\mu)t} = C_2\left[e^{-(\lambda+\mu)t} - 1\right] = C_1\left[1 - e^{-(\lambda+\mu)t}\right]. \tag{76}$$

Solving Eq. (60) for P_A with the use of Eq. (76)

$$P_A = \frac{\dot{P}_D + \mu P_D}{\lambda} = \frac{C_1\left[(\lambda+\mu)e^{-(\lambda+\mu)t}\right] + \mu C_1\left[1 - e^{-(\lambda+\mu)t}\right]}{\lambda}. \tag{77}$$

Again, from the initial conditions (74)

$$1 = \frac{C_1\left[(\lambda+\mu)e^{-(\lambda+\mu)\cdot 0}\right] + \mu C_1\left[1 - e^{-(\lambda+\mu)\cdot 0}\right]}{\lambda} = \frac{C_1\left[(\lambda+\mu)\right] + \mu C_1[1-1]}{\lambda}.$$

$$1 = \frac{C_1(\lambda+\mu)}{\lambda}; \qquad C_1 = \frac{\lambda}{\lambda+\mu}; \qquad C_2 = -\frac{\lambda}{\lambda+\mu}. \tag{78}$$

Then, from Eq. (76) finally we are having

$$P_D = \frac{\lambda}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right]. \tag{79}$$

On the other hand, let

$$\dot{P}_D = y. \tag{80}$$

Then, for the system (57) converted (transformed) into Eq. (62)

$$\dot{y} + (\lambda+\mu)y = 0. \tag{81}$$

Now we have a differential equation of the first order (81) with splitting variables. Let us split the variables:

$$\frac{dy}{dt} = -(\lambda+\mu)y; \qquad \frac{dy}{y} = -(\lambda+\mu)dt. \tag{82}$$

Integration yields

$$\ln|y| = -(\lambda+\mu)t + \ln|C_1|; \qquad \ln\left|\frac{y}{C_1}\right| = -(\lambda+\mu)t; \qquad \frac{y}{C_1} = e^{-(\lambda+\mu)t}; \tag{83}$$

where C_1 – constant of integration; or

$$y = C_1 e^{-(\lambda+\mu)t}. \tag{84}$$

Next, see Eq. (80)

$$\frac{dP_D}{dt} = C_1 e^{-(\lambda+\mu)t}. \tag{85}$$

Integrating one more time (once more, again), i.e. from Eq. (85) it is obtained

$$P_D = -\frac{C_1}{\lambda+\mu} e^{-(\lambda+\mu)t} + C_2, \tag{86}$$

where C_2 – constant of integration.

From the initial conditions (74)

$$0 = -\frac{C_1}{\lambda+\mu} e^{-(\lambda+\mu)\cdot 0} + C_2; \qquad C_2 = \frac{C_1}{\lambda+\mu}; \qquad \frac{C_1}{C_2} = \lambda+\mu. \tag{87}$$

Then Eq. (86) will take the view of

$$-C_2 e^{-(\lambda+\mu)t} + C_2 = C_2\left[1 - e^{-(\lambda+\mu)t}\right]. \tag{88}$$

For the probability of the normal state "A" from the second equation of the system (57), analogously to Eq. (77) with the help of Eq. (86)

$$P_A = \frac{\dot{P}_D + \mu P_D}{\lambda} = \frac{-\frac{C_1}{\lambda+\mu}\left[-(\lambda+\mu)\right]e^{-(\lambda+\mu)t} + \mu\left(-\frac{C_1}{\lambda+\mu} e^{-(\lambda+\mu)t} + C_2\right)}{\lambda} =$$

$$= \frac{C_1 e^{-(\lambda+\mu)t} + \mu\left[-C_2 e^{-(\lambda+\mu)t} + C_2\right]}{\lambda} = \frac{C_1 e^{-(\lambda+\mu)t} + \mu C_2\left[1 - e^{-(\lambda+\mu)t}\right]}{\lambda}. \tag{89}$$

Once more from the initial conditions (74) the Eq. (89) yields

$$1 = \frac{C_1 e^{-(\lambda+\mu)0} + \mu C_2\left[1 - e^{-(\lambda+\mu)0}\right]}{\lambda}; \qquad \lambda = C_1 + \mu C_2[1-1]; \qquad C_1 = \lambda. \tag{90}$$

In accordance to relations (87)

$$C_2 = \frac{\lambda}{\lambda+\mu}. \tag{91}$$

Thus, substituting expression (91) for its value into Eq. (88) we obtain

$$P_D = \frac{\lambda}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right]. \tag{92}$$

Or for Eq. (86), applying expressions (90) and (91), it yields,

$$P_D = -\frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} + \frac{\lambda}{\lambda+\mu} = \frac{\lambda}{\lambda+\mu}\left(1 - e^{-(\lambda+\mu)t}\right), \tag{93}$$

the same result.

Also, using instead of Eq. (86) yielding (89), the result (92) or (93) can be obtained with the help of Eq. (88), which gives

$$P_A = \frac{\dot{P}_D + \mu P_D}{\lambda} = \frac{C_2(\lambda+\mu)\left[e^{-(\lambda+\mu)t}\right] + \mu C_2\left[1 - e^{-(\lambda+\mu)t}\right]}{\lambda}. \tag{94}$$

Applying the initial conditions (74) once again the Eq. (94) yields

$$1 = \frac{C_2(\lambda+\mu)e^{-(\lambda+\mu)0} + \mu C_2\left[1 - e^{-(\lambda+\mu)\cdot 0}\right]}{\lambda} = \frac{C_2(\lambda+\mu) + \mu C_2[1-1]}{\lambda},$$

$$1 = \frac{C_2(\lambda+\mu)}{\lambda}. \tag{95}$$

And finally we come to expression (91):

$$C_2 = \frac{\lambda}{\lambda+\mu}. \tag{96}$$

In such case, for probability of the normal state denoted as "A", P_A, from Eq. (89) or (94), substituting constants of integration (90) and (91) or (96) for their values into Eq. (89) or (94), we will get

$$P_A = \frac{\lambda\left[e^{-(\lambda+\mu)t}\right] + \mu\frac{\lambda}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right]}{\lambda} = e^{-(\lambda+\mu)t} + \frac{\mu}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right] =$$

$$= \frac{(\lambda+\mu)e^{-(\lambda+\mu)t} + \mu\left[1 - e^{-(\lambda+\mu)t}\right]}{\lambda+\mu} = \frac{\left(\lambda e^{-(\lambda+\mu)t} + \mu e^{-(\lambda+\mu)t}\right) + \left[\mu - \mu e^{-(\lambda+\mu)t}\right]}{\lambda+\mu}. \tag{97}$$

Finally, cancelling members $+\mu e^{-(\lambda+\mu)t}$ and $-\mu e^{-(\lambda+\mu)t}$ in the obtained expression (97), it yields

$$P_A = \frac{\lambda e^{-(\lambda+\mu)t} + \mu}{\lambda+\mu}. \tag{98}$$

It is obvious that normalizing condition for the probabilities of the normal "A" and damaged "D" states, P_A and P_D respectively, satisfies:

$$P_A + P_D = \frac{\lambda e^{-(\lambda+\mu)t} + \mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu}\left[1 - e^{-(\lambda+\mu)t}\right],$$

$$P_A + P_D = \frac{\lambda e^{-(\lambda+\mu)t} + \mu + \lambda - \lambda e^{-(\lambda+\mu)t}}{\lambda + \mu}. \qquad (99)$$

After cancelling the similar members $+\lambda e^{-(\lambda+\mu)t}$ and $-\lambda e^{-(\lambda+\mu)t}$ and reducing for the member of the denominator $\lambda + \mu$ in the obtained expression (99), it yields

$$P_A + P_D = 1. \qquad (100)$$

Now we may return to the system for the three states, namely: normal – "A", damaged – "D", failure – "F", corresponding system of equations is (56).

Here, in accordance with [140, Chapter XIII, § 30, pp. 108-113], the characteristic equation for system (56) will be similarly (likewise) [140, Chapter XIII, § 30, p. 109, (5)]:

$$\begin{vmatrix} -\lambda_1 - k & \mu_1 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) - k & 0 \\ 0 & \lambda_2 & 0 - k \end{vmatrix} = 0. \qquad (101)$$

Determinant (101) yields

$$(-\lambda_1 - k)[-(\lambda_2 + \mu_1) - k](0 - k) + \lambda_1\lambda_2 \cdot 0 + \mu_1 \cdot 0 \cdot 0 -$$
$$-[-(\lambda_2 + \mu_1) - k] \cdot 0 \cdot 0 - \lambda_1\mu_1(0 - k) - (-\lambda_1 - k)\lambda_2 \cdot 0 = 0. \qquad (102)$$

Which means

$$(-\lambda_1 - k)[-(\lambda_2 + \mu_1) - k](-k) - \lambda_1\mu_1(-k) = 0,$$
$$-(\lambda_1 + k)[\lambda_2 + \mu_1 + k]k + \lambda_1\mu_1 k = 0. \qquad (103)$$

From where

$$k[\lambda_1\mu_1 - (\lambda_1 + k)(\lambda_2 + \mu_1 + k)] = 0. \qquad (104)$$

Thus, we have already known at least one root:

$$k_1 = 0. \qquad (105)$$

Then, for finding two other roots from Eq. (104)

$$\lambda_1\mu_1 - (\lambda_1 + k)(\lambda_2 + \mu_1 + k) = 0,$$

$$\lambda_1\mu_1 - \lambda_1\lambda_2 - \lambda_1\mu_1 - \lambda_1 k - k\lambda_2 - k\mu_1 - k^2 = 0. \qquad (106)$$

Reducing Eq. (106) and cancelling the similar members $\lambda_1\mu_1$ and $-\lambda_1\mu_1$, we get the quadratic equation

$$-k^2 - k(\lambda_1 + \lambda_2 + \mu_1) - \lambda_1\lambda_2 = 0. \qquad (107)$$

The sought roots are

$$k_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}; \qquad k_3 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}; \qquad (108)$$

where $a = -1$, $b = -(\lambda_1 + \lambda_2 + \mu_1)$, $c = -\lambda_1\lambda_2$ – corresponding coefficients of Eq. (107).

For each root k_i of Eq. (101)-(104), (106), (107), namely k_1, k_2, k_3 Eq. (105) and (108) we will write down the system of linear uniform (homogenous) algebraic equations with respect to their coefficients $\alpha_1^{(i)}$, $\alpha_2^{(i)}$, $\alpha_3^{(i)}$, [140, Chapter XIII, § 30, p. 108, (3)]:

$$\left.\begin{array}{lll} (-\lambda_1 - k)\alpha_1 & +\mu_1\alpha_2 & +0 \cdot \alpha_3 = 0; \\ \lambda_1\alpha_1 & +[-(\lambda_2 + \mu_1) - k]\alpha_2 & +0 \cdot \alpha_3 = 0; \\ 0 \cdot \alpha_1 & +\lambda_2\alpha_2 & +(0 - k)\alpha_3 = 0. \end{array}\right\} \qquad (109)$$

The system of Eq. (109) derives from an assumption of a partial solution existence in the view of [140, Chapter XIII, § 30, p. 108, (2)]:

$$P_A = \alpha_1 e^{kt}; \qquad P_D = \alpha_2 e^{kt}; \qquad P_F = \alpha_3 e^{kt}; \qquad (110)$$

for the system of Eq. (56).

Since having three roots in the stated problem setting, we obtain, [140, Chapter XIII, § 30, p. 109], the solution of the system of Eq. (56):

for the root of k_1

$$P_A^{(1)} = \alpha_1^{(1)} e^{k_1 t}; \qquad P_D^{(1)} = \alpha_2^{(1)} e^{k_1 t}; \qquad P_F^{(1)} = \alpha_3^{(1)} e^{k_1 t}; \qquad (111)$$

for the root of k_2

$$P_A^{(2)} = \alpha_1^{(2)} e^{k_2 t}; \qquad P_D^{(2)} = \alpha_2^{(2)} e^{k_2 t}; \qquad P_F^{(2)} = \alpha_3^{(2)} e^{k_2 t}; \qquad (112)$$

for the root of k_3

$$P_A^{(3)} = \alpha_1^{(3)} e^{k_3 t}; \qquad P_D^{(3)} = \alpha_2^{(3)} e^{k_3 t}; \qquad P_F^{(3)} = \alpha_3^{(3)} e^{k_3 t}. \qquad (113)$$

In the way (by the method) of direct substitution of partial solutions (111)-(113) into equations, one can be convinced that the system of functions, similarly to [140, Chapter XIII, § 30, p. 109, (6)]:

$$\left.\begin{aligned} P_A &= C_1P_A^{(1)} + C_2P_A^{(2)} + C_3P_A^{(3)} = C_1\alpha_1^{(1)}e^{k_1t} + C_2\alpha_1^{(2)}e^{k_2t} + C_3\alpha_1^{(3)}e^{k_3t};\\ P_D &= C_1P_D^{(1)} + C_2P_D^{(2)} + C_3P_D^{(3)} = C_1\alpha_2^{(1)}e^{k_1t} + C_2\alpha_2^{(2)}e^{k_2t} + C_3\alpha_2^{(3)}e^{k_3t};\\ P_F &= C_1P_F^{(1)} + C_2P_F^{(2)} + C_3P_F^{(3)} = C_1\alpha_3^{(1)}e^{k_1t} + C_2\alpha_3^{(2)}e^{k_2t} + C_3\alpha_3^{(3)}e^{k_3t};\end{aligned}\right\} \tag{114}$$

where C_1; C_2; C_3; – arbitrary constants; also is the solution of the differential equations system (56). This is t h e g e n e r a l s o l u t i o n o f t h e d i f f e r e n t i a l e q u a t i o n s s y s t e m (5 6), [140, Chapter XIII, § 30, p. 110].

Satisfying the condition of Eq. (105) for root $k_1 = 0$ from the system of Eq. (109) we have

$$\left.\begin{aligned} &(-\lambda_1 - k_1)\alpha_1^{(1)} && +\mu_1\alpha_2^{(1)} && +0\cdot\alpha_3^{(1)} = 0;\\ &\lambda_1\alpha_1^{(1)} && +[-(\lambda_2+\mu_1) - k_1]\alpha_2^{(1)} && +0\cdot\alpha_3^{(1)} = 0;\\ &0\cdot\alpha_1^{(1)} && +\lambda_2\alpha_2^{(1)} && +(0-k_1)\alpha_3^{(1)} = 0.\end{aligned}\right\} \tag{115}$$

Thus

$$\left.\begin{aligned} &-\lambda_1\alpha_1^{(1)} && +\mu_1\alpha_2^{(1)} && +0\cdot\alpha_3^{(1)} = 0;\\ &\lambda_1\alpha_1^{(1)} && -(\lambda_2+\mu_1)\alpha_2^{(1)} && +0\cdot\alpha_3^{(1)} = 0;\\ &0 && +\lambda_2\alpha_2^{(1)} && -0\cdot\alpha_3^{(1)} = 0.\end{aligned}\right\} \tag{116}$$

From where, immediately the coefficients are

$$\alpha_2^{(1)} = 0; \qquad \alpha_1^{(1)} = 0; \qquad \alpha_3^{(1)} = 1; \tag{117}$$

since $\alpha_3^{(1)}$ is an arbitrary number, supposedly (by an assumption) $\alpha_3^{(1)} = 1$, [140, Chapter XIII, § 30, p. 109].

For the Eq. (107) roots of k_2 and k_3, Eq. (108), the system of Eq. (109) analogous to the system of Eq. (115) it yields

$$\left.\begin{aligned} &(-\lambda_1 - k_{2,3})\alpha_1^{(2,3)} && +\mu_1\alpha_2^{(2,3)} && +0\cdot\alpha_3^{(2,3)} = 0;\\ &\lambda_1\alpha_1^{(2,3)} && +[-(\lambda_2+\mu_1) - k_{2,3}]\alpha_2^{(2,3)} && +0\cdot\alpha_3^{(2,3)} = 0;\\ &0\cdot\alpha_1^{(2,3)} && +\lambda_2\alpha_2^{(2,3)} && +(0-k_{2,3})\alpha_3^{(2,3)} = 0.\end{aligned}\right\} \tag{118}$$

The system of Eq. (118) may be solved for unknown sought (wanted/needed) coefficients in a few different ways. One of them is the next.

Since one of the alpha coefficients can be chosen arbitrary, accordingly with [140, Chapter XIII, § 30, p. 109], let us assume

$$\alpha_2^{(2,3)} = 1. \tag{119}$$

Then, from the first equation of system (118)

$$(-\lambda_1 - k_{2,3})\alpha_1^{(2,3)} + \mu_1 = 0; \qquad \alpha_1^{(2,3)} = \frac{\mu_1}{\lambda_1 + k_{2,3}}. \tag{120}$$

Or from the second equation

$$\lambda_1\alpha_1^{(2,3)} - (\lambda_2 + \mu_1) - k_{2,3} = 0; \qquad \alpha_1^{(2,3)} = \frac{\lambda_2 + \mu_1 + k_{2,3}}{\lambda_1}. \tag{121}$$

Or summing the first and second equations, reducing, and cancelling the similar members

$$(-\lambda_1 - k_{2,3})\alpha_1^{(2,3)} + \lambda_1\alpha_1^{(2,3)} + \mu_1\alpha_2^{(2,3)} + [-(\lambda_2+\mu_1) - k_{2,3}]\alpha_2^{(2,3)} = 0;$$

$$-k_{2,3}\alpha_1^{(2,3)} - [\lambda_2 + k_{2,3}]\alpha_2^{(2,3)} = 0; \qquad \alpha_1^{(2,3)} = -\frac{\lambda_2 + k_{2,3}}{k_{2,3}}. \tag{122}$$

All three expressions for $\alpha_1^{(2,3)}$, i.e. Eq. (120)-(122) are equivalent because all of them use the roots k_2 and k_3, Eq. (108) of the initial quadratic equation Eq. (107).

Indeed.

Equalizing Eq. (120) and (121) we get

$$\frac{\mu_1}{\lambda_1 + k_{2,3}} = \frac{\lambda_2 + \mu_1 + k_{2,3}}{\lambda_1}; \qquad \lambda_1\mu_1 = (\lambda_2 + \mu_1 + k_{2,3})(\lambda_1 + k_{2,3});$$

$$\lambda_1\mu_1 = \lambda_1\lambda_2 + \lambda_1\mu_1 + \lambda_1 k_{2,3} + \lambda_2 k_{2,3} + \mu_1 k_{2,3} + k_{2,3}^2. \tag{123}$$

And cancelling for $\lambda_1\mu_1$ in both parts of Eq. (123) it yields Eq. (107):

$$\lambda_1\lambda_2+(\lambda_1+\lambda_2+\mu_1)k_{2,3}+k_{2,3}^2=0. \qquad (124)$$

The same result is obtained if make equal Eq. (120) and (122):

$$\frac{\mu_1}{\lambda_1+k_{2,3}}=-\frac{\lambda_2+k_{2,3}}{k_{2,3}}; \qquad \mu_1 k_{2,3}=-(\lambda_2+k_{2,3})(\lambda_1+k_{2,3});$$

$$\mu_1 k_{2,3}=-\lambda_1\lambda_2-\lambda_1 k_{2,3}-\lambda_2 k_{2,3}-k_{2,3}^2;$$

$$k_{2,3}^2+(\lambda_1+\lambda_2+\mu_1)k_{2,3}+\lambda_1\lambda_2=0. \qquad (125)$$

When equalling Eq. (121) and (122) it gives the same. Indeed:

$$\frac{\lambda_2+\mu_1+k_{2,3}}{\lambda_1}=-\frac{\lambda_2+k_{2,3}}{k_{2,3}}; \qquad (\lambda_2+\mu_1+k_{2,3})k_{2,3}=-(\lambda_2+k_{2,3})\lambda_1;$$

$$(\lambda_2+\mu_1)k_{2,3}+k_{2,3}^2=-\lambda_1\lambda_2-\lambda_1 k_{2,3};$$

$$k_{2,3}^2+(\lambda_1+\lambda_2+\mu_1)k_{2,3}+\lambda_1\lambda_2=0. \qquad (126)$$

For the values of coefficient $\alpha_3^{(2,3)}$, we have, from the third equation of the system of Eq. (118) and condition (119),

$$\lambda_2\alpha_2^{(2,3)}-k_{2,3}\alpha_3^{(2,3)}=0; \qquad \alpha_3^{(2,3)}=\frac{\lambda_2}{k_{2,3}}. \qquad (127)$$

Thus, turning back to the system of Eq. (114), we determine the unknown coefficients of the general solution of the differential equations system (56), [140, Chapter XIII, § 30, p. 110], satisfying the initial conditions (24): $t_0=0$; $P_A|_{t=t_0}=1$; $P_D|_{t=t_0}=0$; $P_F|_{t=t_0}=0$; and have already known the coefficients of alpha; i.e. Eq. (117): $\alpha_2^{(1)}=0$; $\alpha_1^{(1)}=0$; $\alpha_3^{(1)}=1$; Eq. (119): $\alpha_2^{(2,3)}=1$; Eq. (120): $\alpha_1^{(2,3)}=\dfrac{\mu_1}{\lambda_1+k_{2,3}}$; Eq. (127): $\alpha_3^{(2,3)}=\dfrac{\lambda_2}{k_{2,3}}$:

$$\left.\begin{array}{l} P_A=C_1\alpha_1^{(1)}e^{k_1t}+C_2\alpha_1^{(2)}e^{k_2t}+C_3\alpha_1^{(3)}e^{k_3t};\\ P_D=C_1\alpha_2^{(1)}e^{k_1t}+C_2\alpha_2^{(2)}e^{k_2t}+C_3\alpha_2^{(3)}e^{k_3t};\\ P_F=C_1\alpha_3^{(1)}e^{k_1t}+C_2\alpha_3^{(2)}e^{k_2t}+C_3\alpha_3^{(3)}e^{k_3t};\end{array}\right\}\Bigg|_{t_0=0} =$$

$$=\left\{\begin{array}{l} 1=C_1\cdot 0\cdot e^{k_1\cdot 0}+C_2\alpha_1^{(2)}e^{k_2\cdot 0}+C_3\alpha_1^{(3)}e^{k_3\cdot 0};\\ 0=C_1\cdot 0\cdot e^{k_1\cdot 0}+C_2\cdot 1\cdot e^{k_2\cdot 0}+C_3\cdot 1\cdot e^{k_3\cdot 0};\\ 0=C_1\cdot 1\cdot e^{k_1\cdot 0}+C_2\alpha_3^{(2)}e^{k_2\cdot 0}+C_3\alpha_3^{(3)}e^{k_3\cdot 0};\end{array}\right\}=$$

$$=\left\{\begin{array}{l} 1=0+C_2\alpha_1^{(2)}+C_3\alpha_1^{(3)};\\ 0=0+C_2+C_3;\\ 0=C_1+C_2\alpha_3^{(2)}+C_3\alpha_3^{(3)};\end{array}\right\}=\left\{\begin{array}{l} 1=0+C_2\dfrac{\mu_1}{\lambda_1+k_2}+C_3\dfrac{\mu_1}{\lambda_1+k_3};\\ 0=0+C_2+C_3;\\ 0=C_1+C_2\dfrac{\lambda_2}{k_2}+C_3\dfrac{\lambda_2}{k_3}.\end{array}\right\} \qquad (128)$$

From the second equation of the system of Eq. (128) it yields

$$C_2=-C_3. \qquad (129)$$

Substituting the values of Eq. (129) for the corresponding members into the first equation of the system of Eq. (128) we get

$$1=C_3\left(\frac{\mu_1}{\lambda_1+k_3}-\frac{\mu_1}{\lambda_1+k_2}\right); \qquad C_3=\frac{1}{\dfrac{\mu_1}{\lambda_1+k_3}-\dfrac{\mu_1}{\lambda_1+k_2}}. \qquad (130)$$

In order to make the notations shorter let us put down the indications with the alpha symbolizations:

$$1=-C_3\alpha_1^{(2)}+C_3\alpha_1^{(3)}=C_3\left[\alpha_1^{(3)}-\alpha_1^{(2)}\right]; \qquad C_3=\frac{1}{\alpha_1^{(3)}-\alpha_1^{(2)}}. \qquad (131)$$

Therefore

$$C_2=-\frac{1}{\alpha_1^{(3)}-\alpha_1^{(2)}}. \qquad (132)$$

From the third equation of the system of Eq. (128) we obtain

$$C_1=-C_2\alpha_3^{(2)}-C_3\alpha_3^{(3)}. \qquad (133)$$

Now, all coefficients are expressed through the given values, hence, the system of Eq. (56) is successfully solved.

It may be concluded. The obtained solutions results described with the mathematical expressions of (1)-(133) create a possibility for the students to model the situations relating with the reliability objective measures allowing successfully assessing the improvements of the A/C functional system M/T process considered in reference [112] on the basis of the methods discussed in [136, 140] in application to [1-144].

Further **PARTS** of the **METHOD GUIDE ON THE** SS problems are going to be intended for several other scientific components of the SS work on the academic subject. Background readings of the new theories [4-60, 64, 65, 89-109, 117, 118, 138] are welcome here in respect with the **CONTINUING AIRCRAFT AIRWORTHINESS (ICAO DOC 9760)** [1-144].

It is very attractive to use the calculus of variations theories [64, 65, 80, 85, 88, 95, 96, 117, 118] emphasizing the uncertainties of different kinds [4-60, 64, 65, 89-109, 117, 118, 138] for optimization of the problems stated in all references of [1-144].

REFERENCES

1. **Airworthiness** Manual: ICAO Doc 9760, 3rd Edition. – Montreal, Quebec, Canada: ICAO, 2014. – 281 p.
2. **Continuing** aircraft airworthiness: manual / S. Dmitriev, O. Tuharinov, Yu. Kazarinov, O. Rugain. – Kyiv, Ukraine: Publishing House of the National Aviation University, 2015. – 104 p.
3. **Dhillon B. S.** Maintainability, maintenance, and reliability for engineers / B. S. Dhillon. – New York: Taylor & Francis Group, 2006. – 214 p.
4. **Goncharenko A.** Aircraft operation depending upon the uncertainty of maintenance alternatives / A. Goncharenko // Aviation. – 2017. Vol. 21(4). – pp. 126-131.
5. **Goncharenko A. V.** A concept of ballast water treatment on the basis of multi-alternativeness / A. V. Goncharenko, V. A. Evdokimova // Сучасні інформаційні та інноваційні технології на транспорті (MINTT-2014) [Збірка матеріалів VI Міжнародної науково-практичної конференції. (27-29 травня 2014 р., Херсон)]. – Херсон: Херсонська державна морська академія, 2014. – pp. 18-21.
6. **Goncharenko A. V.** A concept of entropy approach to the problem of multi-alternative operational modes control / A. V. Goncharenko // Науковий вісник ХДМА. – 2013. – № 2(9). – pp. 26-34.
7. **Goncharenko A. V.** A concept of multi-optional optimality at modeling ideal gas isothermal processes / A. V. Goncharenko // Electronics and control systems. – 2017. – № 2(52). – pp. 94-97.
8. **Goncharenko A. V.** A diagnostics problem of a-posterior probability determination via Bayes' formula obtained in the multi-optional hybrid functions entropy conditional optimization way / A. V. Goncharenko // Problems of friction and wear. – 2017. – № 4(77). – pp. 95-99.
9. **Goncharenko A. V.** A hybrid approach to the optimal aeronautical engineering maintenance periodicity determination / A. V. Goncharenko // Proceedings of the NAU. – 2017. – № 3(72). – pp. 42-47.
10. **Goncharenko A. V.** A hybrid pseudo-entropy function for a decision making in conditions of uncertainty / A. V. Goncharenko // Інтелектуальні системи прийняття рішень та проблеми обчислювального інтелекту: міжнародна наукова конференція, Залізний Порт, Україна, 25-28 травня 2015 р.: матеріали конф. – Херсон: Видавництво ХНТУ, 2015. – pp. 174-176. (ISBN 978-966-2207-24-8)
11. **Goncharenko A. V.** A particular case of a variational problem of control in an active aviation system / A. V. Goncharenko // Transactions of the institute of aviation. – 2013. – № 228, pp. 3-12.
12. **Goncharenko A. V.** Aeronautical and aerospace materials and structures damages to failures: theoretical concepts / A. V. Goncharenko //

International Journal of Aerospace Engineering. – Volume 2018 (2018), Article ID 4126085, 7 pages https://doi.org/10.1155/2018/4126085; 2018. – pp. 1-7.

13. **Goncharenko A. V.** Aeronautical engineering maintenance periodicity optimization with the help of subjective preferences distributions / A. V. Goncharenko // Proceedings of the NAU. – 2017. – № 2(71). – pp. 51-56.

14. **Goncharenko A. V.** Aircraft maximal distance horizontal flights in the conceptual framework of subjective analysis / A. V. Goncharenko // Proceedings of the NAU. – 2013. – № 4(57). – pp. 56-62.

15. **Goncharenko A. V.** Alternativeness of control and power equipment repair versus purchasing according to the preferences of the options / A. V. Goncharenko // Electronics and control systems. – 2016. – № 4(50). – pp. 98-101.

16. **Goncharenko A. V.** An alternative method of the main psychophysics law derivation / A. V. Goncharenko // Clin. and Exp. Psychol. – 2017. – 3: 155. – pp. 1-5. doi: 10.4172/2471-2701.1000155. (ISSN: 2471-2701)

17. **Goncharenko A. V.** An example of an alternative method of the normal distribution density derivation via a concept of a multi-optional optimality / A. V. Goncharenko // Electronics and control systems. – 2017. – № 3(53). – pp. 95-99. DOI: 10.18372/1990-5548.53.12149 (ISSN: 1990-5548)

18. **Goncharenko A. V.** An optional hybrid functions method of an ideal gas adiabatic process equation derivation / A. V. Goncharenko // Electronics and control systems. – 2017. – № 4(54). – pp. 109-112.

19. **Goncharenko A. V.** Applicable aspects of alternative UAV operation / A. V. Goncharenko // 2015 IEEE 3rd International Conference "Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD)" Proceedings. October, 13-15, 2015, Kyiv, Ukraine. – K.: Освіта України, 2015. – pp. 316-319.

20. **Goncharenko A. V.** Artificial versus natural intellect in control of optimality / A. V. Goncharenko // Інтелектуальні системи прийняття рішень та проблеми обчислювального інтелекту: міжнародна наукова конференція, Євпаторія, 20-24 травня 2013 р.: матеріали конф. – Херсон: ХНТУ, 2013. – pp. 20-22. (ISBN 978-966-8912-70-2)

21. **Goncharenko A. V.** Bayes criterion modified with subjective preferences functions densities distributions used at the choosing of the decision making thresholds / A. V. Goncharenko // Матеріали XIII міжнародної науково-технічної конференції "АВІА-2017". (19-21 квітня 2017 р., Київ). – К.: НАУ, 2017. – pp. 17.17-17.21.

22. **Goncharenko A. V.** Concentrations formula conditional optimality with respect to their entropy / A. V. Goncharenko // Problems of friction and wear. – 2018. – № 1(78). – pp. 85-88. (ISSN 0370-2197)

23. **Goncharenko A. V.** Conflictability of operational situations in terms of entropy paradigm / A. V. Goncharenko // Сучасні інформаційні та інноваційні технології на транспорті (MINTT-2013) [Збірка матеріалів V Міжнародної науково-практичної конференції. У 2-х тт. (28-30 травня 2013 р., Херсон)]. – Т. 1. – Херсон: Херсонська державна морська академія, 2013. – pp. 115-118.

24. **Goncharenko A. V.** Control of flight safety with the use of preferences functions / A. V. Goncharenko // Electronics and control systems. – 2013. – № 3(37). – pp. 113-119. (ISSN: 1990-5548)

25. **Goncharenko A. V.** Distinguishing minimal engineering diagnosis risks via preferences functions / A. V. Goncharenko // Proceedings of The Seventh World Congress "Aviation in the XXI-st Century" "Safety in Aviation and Space Technologies". Kyiv, Ukraine, September 19-21, 2016: матеріали конгр. – Київ, NAU; 2016. – pp. 1.2.6-1.2.10.

26. **Goncharenko A. V.** Expediency of an improvement for a diesel-geared propulsion with respect to subjectively preferred operational factors / A. V. Goncharenko // Науковий вісник ХДМІ. – 2011. – № 1(4). – pp. 30-41.

27. **Goncharenko A. V.** Expediency of unmanned air vehicles application in the framework of subjective analysis / A. V. Goncharenko // 2013 IEEE 2nd International Conference "Actual Problems of Unmanned Air Vehicles Developments" Proceedings. October, 15-17, 2013, Kyiv, Ukraine. – K.: Освіта України, 2013. – pp. 129-133.

28. **Goncharenko A. V.** Exponential distribution density derived with the help of the multi-optional hybrid functions entropy conditional optimization / A. V. Goncharenko // Problems of friction and wear. – 2017. – № 4(77). – pp. 90-94. (ISSN 0370-2197)

29. **Goncharenko A. V.** Extremality of control and preferences distributions "goodness" / A. V. Goncharenko // Electronics and control systems. – 2014. – № 4(42). – pp. 84-90. (ISSN: 1990-5548)

30. **Goncharenko A. V.** Fuel oil atomization characteristics smoothed by a logarithm normal distribution for marine diesel engines / A. V. Goncharenko // Двигатели внутреннего сгорания. – 2010. – № 2. – pp. 34-40. (ISSN 0419-8719)

31. **Goncharenko A. V.** Generalization for the degrading state maximal probability in the framework of the hybrid-optional entropy conditional optimality doctrine / A. V. Goncharenko // Problems of friction and wear. – 2018. – № 1(78). – pp. 89-92. (ISSN 0370-2197)

32. **Goncharenko A. V.** Horizontal flight for maximal distance at presence of conflict behavior (control) of the aircraft control system active element / A. V. Goncharenko // Матеріали XI міжнародної науково-

технічної конференції “АВІА-2013”. (21-23 травня 2013 р., Київ). – Т. 4. – К.: НАУ, 2013. – pp. 22.30-22.33.

33. **Goncharenko A. V.** Human factor aspect applicably to aeronautical engineering maintenance / A. V. Goncharenko // Матеріали XIII міжнародної науково-технічної конференції “АВІА-2017”. (19-21 квітня 2017 р., Київ). – К.: НАУ, 2017. – pp. 17.9-17.13.

34. **Goncharenko A. V.** Mathematical modeling of the ship's main engine random operational process / A. V. Goncharenko // Двигатели внутреннего сгорания. – 2012. – № 2. – pp. 117-125. (ISSN 0419-8719)

35. **Goncharenko A. V.** Measures for estimating transport vessels operators' subjective preferences uncertainty / A. V. Goncharenko // Науковий вісник ХДМА. – 2012. – № 1(6). – pp. 59-69.

36. **Goncharenko A. V.** Measures for estimating transport vessels operators' subjective preferences uncertainty / A. V. Goncharenko // Transactions of the institute of aviation. – 2013. – № 228 – pp. 13-21.

37. **Goncharenko A. V.** Modeling aviation legislation influence upon airworthiness support technologies via preferences functions / A. V. Goncharenko // Proceedings of The Seventh World Congress “Aviation in the XXI-st Century” “Safety in Aviation and Space Technologies”. Kyiv, Ukraine, September 19-21, 2016: матеріали конгр. – Київ, NAU; 2016. – pp. 1.2.11-1.2.15.

38. **Goncharenko A. V.** Multi-optional hybrid effectiveness functions optimality doctrine for maintenance purposes / A. V. Goncharenko // 14th IEEE International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET-2018). – February, 20-24, 2018, Lviv-Slavske, Ukraine. – 2018. – pp. 771-775.

39. **Goncharenko A. V.** Navigational alternatives, their control and subjective entropy of individual preferences / A. V. Goncharenko // 2014 IEEE 3rd International Conference “Methods and Systems of Navigation and Motion Control (MSNMC)” Proceedings. October, 14-17, 2014, Kyiv, Ukraine. – К.: Освіта України, 2014. – pp. 99-103.

40. **Goncharenko A. V.** One theoretical aspect of entropy paradigm application to the problems of tribology / A. V. Goncharenko // Problems of friction and wear. – 2017. – № 1(74). – pp. 78-83. (ISSN 0370-2197 print)

41. **Goncharenko A. V.** Operational reliability measures for marine propulsion diesel engines / A. V. Goncharenko // Сучасні енергетичні установки на транспорті і технології та обладнання для їх обслуговування: Всеукраїнська науково-практична конференція, Херсон, 12-14 жовтня 2011 р.: матеріали конф. – Херсон, 2011. – pp. 23-27.

42. **Goncharenko A. V.** Optimal controlling path determination with the help of hybrid optional functions distributions / A. V. Goncharenko // Radio Electronics, Computer Science, Control. – 2018. – № 1(44). – pp. 149-158.

43. **Goncharenko A. V.** Optimal dividing between purchasing and fabrication / A. V. Goncharenko // Сучасні енергетичні установки на транспорті, технології та обладнання для їх обслуговування: Республіканська науково-практична конференція, Херсон, 5-7 жовтня 2010 р.: матеріали конф. – Херсон, 2010. – pp. 54-55.

44. **Goncharenko A. V.** Optimal maintenance periodicity for aeronautical engineering operation determined on the theoretical platform of subjective analysis / A. V. Goncharenko // XIIIth International Conference “AVIA-2017”. (April 19-21, 2017, Kyiv). – Kyiv: National Aviation University, 2017. – pp. 17.29-17.33.

45. **Goncharenko A. V.** Optimal managerial and control values for active operation / A. V. Goncharenko // Electronics and control systems. – 2016. – № 3(49). – pp. 112-115. (ISSN: 1990-5548)

46. **Goncharenko A. V.** Optimal UAV maintenance periodicity obtained on the multi-optional basis / A. V. Goncharenko // 2017 IEEE 4th International Conference “Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD)” Proceedings. Section A. Unmanned aerial vehicle design. – October, 17-19, 2017, Kyiv, Ukraine. – 2017. – pp. 65-68.

47. **Goncharenko A. V.** Preferences distributions densities for a common continuous alternative / A. V. Goncharenko // Науковий вісник ХДМА. – 2014. – № 2(11). – pp. 22-27. (ISSN 2313-4763)

48. **Goncharenko A. V.** Prospects of alternative sources of energy and engines used in ships propulsion and power plants / A. V. Goncharenko // Сучасні енергетичні установки на транспорті, технології та обладнання для їх обслуговування: Республіканська науково-практична конференція, Херсон, 5-7 жовтня 2010 р.: матеріали конф. – Херсон, 2010. – pp. 115-116.

49. **Goncharenko A. V.** Rational modes of operation for a four-arm tiller electro-hydraulic steering gear with respect to multi-alternativeness and preferences / A. V. Goncharenko // Науковий вісник ХДМА. – 2013. – № 1(8). – С. 28-34. (ISSN 2077-3617)

50. **Goncharenko A. V.** Research of operational effectiveness changes / A. V. Goncharenko // Сучасні енергетичні установки на транспорті, технології та обладнання для їх обслуговування: Республіканська науково-практична конференція, Херсон, 5-7 жовтня 2010 р.: матеріали конф. – Херсон, 2010. – pp. 20-23.

51. **Goncharenko A. V.** Safe maneuvering of a ship in a multi-alternative operational situation / A. V. Goncharenko // Bezpieczeństwo na lądzie, morzu i w powietrzu w XXI wieku. – 2014. – pp. 207-210. (ISBN 978-83-61520-02-3)

52. **Goncharenko A. V.** Safety and its entropy measures of certainty or uncertainty / A. V. Goncharenko // Безпека життєдіяльності на транспорті і виробництві – освіта, наука, практика (SLA-2014) [збірник матеріалів Міжнародної науково-практичної конференції. (18-19 вересня 2014 р.,

Херсон)]. – Херсон: Херсонська державна морська академія, 2014. – pp. 44-46.

53. **Goncharenko A. V.** Several models of artificial intelligence elements for aircraft control / A. V. Goncharenko // 2016 IEEE 4th International Conference "Methods and Systems of Navigation and Motion Control (MSNMC)" Proceedings. October, 18-20, 2016, Kyiv, Ukraine. – K.: Освіта України, 2016. – pp. 224-227.

54. **Goncharenko A. V.** Several models of physical exercise subjective preferences / A. V. Goncharenko // Clin. and Exp. Psychol. – 2016. – 2: 121. – pp. 1-6. doi:10.4172/2471-2701.1000121. (ISSN: 2471-2701 CEP)

55. **Goncharenko A. V.** Some identities of subjective analysis derived on the basis of the subjective entropy extremization principle by Professor V. A. Kasianov / A. V. Goncharenko // Automatic Control and Information Sciences. – 2014. – Vol. 2, No. 1. – pp. 20-25.

56. **Goncharenko A. V.** Subjective entropy extremization principle as a tool of an aircraft maximal duration horizontal flight control / A. V. Goncharenko // Авиационно-космическая техника и технология. – 2013. – Вып. 8(105). – pp. 229-234.

57. **Goncharenko A. V.** Subjective entropy maximum principle for preferences functions of alternatives given in the view of logical conditions / A. V. Goncharenko // Штучний інтелект. – 2013. – № 4(62). – 1 G. pp. 4-9.

58. **Goncharenko A. V.** Subjective preferences for optimal economy continuous rating of MaK 9M453C / A. V. Goncharenko // Сучасні інформаційні та інноваційні технології на транспорті: III Міжнародна науково-практична конференція, Херсон, 23-25 травня 2011 р.: матеріали конф. У 2-х тт. Том 2. – Херсон: Видавництво Херсонського державного морського інституту, 2011. – pp. 114-119.

59. **Goncharenko A. V.** The optimal commercial speed of a transport vessel with respect to operators' subjective preferences / A. V. Goncharenko // Науковий вісник ХДМІ. – 2011. – № 2(5). – pp. 12-20.

60. **Goncharenko A. V.** The optimal internal "shadow" taxation on condition of a firm external economic activity / A. V. Goncharenko, O. A. Zaporozchenko // Proceedings of the NAU. – 2013. – № 2(55). – pp. 251-257.

61. **Jaynes E. T.** Information theory and statistical mechanics / E. T. Jaynes // Physical review. – U.S.A. – 1957. – Vol. 106, № 4. – pp. 620-630.

62. **Jaynes E. T.** Information theory and statistical mechanics. II / E. T. Jaynes // Physical review. – U.S.A. – 1957. – Vol. 108, № 2. – pp. 171-190.

63. **Jaynes E. T.** On the rationale of maximum-entropy methods / E. T. Jaynes // Proceedings of the IEEE. – 1982. – Vol. 70. – pp. 939-952.

64. **Kasianov V.** Subjective entropy of preferences. Subjective analysis: monograph / V. Kasianov. – Warsaw, Poland: Institute of Aviation Scientific Publications, 2013. – 644 p. (ISBN 978-83-63539-08-5)

65. **Kasianov V. A.** Extremal principle of subjective analysis. Light and shadow. Proportions of shadow economy. Entropy approach: monograph / V. A. Kasianov, A. V. Goncharenko. – Kyiv, Ukraine: Publishing House "Kafedra", 2017. – 90 p. (ISBN 978-617-7301-41-6).

66. **Kroes M. J.** Aircraft maintenance and repair: 7th ed. / M. J. Kroes, W. A. Watkins, F. Delp, R. Sterkenburg. – New York, New York, USA, McGraw-Hill, Education, 2013. – 736 p.

67. **Kroes M. J.** Aircraft powerplants: 7th ed. / M. J. Kroes, T. W. Wild. – New York, New York, USA: GLENCOE Macmillan/McGraw-Hill, International Editions, 1994. – 694 p.

68. **Lind D. A.** Basic statistics for business and economics: 2nd ed. / D. A. Lind, R. D. Mason. – Boston, Massachusetts, USA: Irwin/McGraw-Hill, 1997. – 597 p.

69.**Manuals** on flight operations of aircraft.

70. **Random** House Webster's unabridged dictionary: 2nd ed. – New-York, USA: Random House, 1999. – 2230 p.

71.**Schedules** and technological guidance materials on aircraft scheduled works performances.

72. **Smith D. J.** Reliability, maintainability and risk. Practical methods for engineers / D. J. Smith. – London: Elsevier, 2005. – 365 p.

73. **Sushchenko O.** Design of robust systems for stabilization of unmanned aerial vehicle equipment / O. Sushchenko, A. Goncharenko // International Journal of Aerospace Engineering. – Volume 2016 (2016), Article ID 6054081. – 2016. – pp. 1-10.

74.**Technical** descriptions of aircraft.

75. **Wild T. W.** Aircraft powerplants: 8th ed. / T. W. Wild, M. J. Kroes. – New York, New York, USA: McGraw-Hill, Education, 2014. – 756 p.

76. **Барлоу Р.** Математическая теория надежности: пер с англ. под. ред. Б. В. Гнеденко / Р. Барлоу, Ф. Прошан. – М.: Сов. Радио, 1969. – 288 с.

77. **Баруча-Рид А. Т.** Элементы теории марковских процессов и их приложения / А. Т. Баруча-Рид. – М.: Наука, 1969. – 551 с.

78. **Боровков А. А.** Вероятностные процессы в теории массового обслуживания / А. А. Боровков. – М.: Наука, 1972. – 367 с.

79. **Бронштейн И. Н.** Справочник по математике для инженеров и учащихся втузов / И. Н. Бронштейн, К.А. Семендяев. – М.: Наука, 1986. – 544 с.

80. **Буслаев В. С.** Вариационное исчисление: учеб. пособие / В. С. Буслаев. – Л.: Изд-во Ленингр. ун-та, 1980. – 288 с.

81. **Введение** в математическое моделирование: учебное пособие / В. Н. Ашихмин и др. Под ред. П. В. Трусова. – М.: Интермет Инжиниринг, 2000. – 275 с.
82. **Вентцель Е. С.** Теория вероятностей / Е. С. Вентцель. – М.: В. Школа, 2000. – 458 с.
83. **Вирченко Н. А.** Графики функций: справочник / Н. А. Вирченко, И. И. Ляшко, К. И. Швецов. – К.: Наукова думка, 1979. – 320 с.
84. **Волков Л. И.** Надежность летательных аппаратов: учеб. пособие для авиационных вузов / Л. И. Волков, А. М. Шишкевич. – М.: В. Школа, 1975. – 296 с.
85. **Гельфанд И. М.** Вариационное исчисление / И. М. Гельфанд, С. В. Фомин. – М.: Государственное издательство физико-математической литературы, 1961. – 228 с.
86. **Гнеденко Б. В.** Введение в теорию массового обслуживания / Б. В. Гнеденко, И. Н. Коваленко. – М.: URSS, 2013. – 400 с.
87. **Гнеденко Б. В.** Математические методы в теории надежности / Б. В. Гнеденко, Ю. К. Беляев, А. Д. Соловьев. – М.: Наука, 1965. – 524 с.
88. **Гноенский Л. С.** Математические основы теории управляемых систем / Л. С. Гноенский, Г. А. Каменский, Л. Э. Эльсгольц. – М.: Наука, 1969. – 512 с.
89. **Гончаренко А. В.** Аналіз гвинтової характеристики двигуна 5 ДКРН 70/226,8 за питомою витратою палива / А. В. Гончаренко // Науковий вісник ХДМІ. – 2009. – № 1(1). – С. 16-24.
90. **Гончаренко А. В.** Аналіз параметричних досліджень реологічних властивостей водовугільних суспензій для застосування у суднових двигунах внутрішнього згоряння / А. В. Гончаренко // Авиационно-космическая техника и технология. – 2009. – № 8(65). – С. 90-95.
91. **Гончаренко А. В.** Варіанти вибору стратегії підтримання безпеки функціонування транспортної системи / А. В. Гончаренко // Вісник НАУ. – 2009. – № 2(39). – С. 30-35.
92. **Гончаренко А. В.** Вибір оптимальної комерційної швидкості транспортного судна / А. В. Гончаренко // Науковий вісник ХДМІ. – 2010. – № 1(2). – С. 41-49.
93. **Гончаренко А. В.** Вплив суб'єктивних переваг на показники роботи суднової енергетичної установки / А. В. Гончаренко // Автоматика, автоматизация, электротехнические комплексы и системы. – 2008. – № 2(22). – С. 105-111.
94. **Гончаренко А. В.** Дослідження характеристики двигуна стосовно зміни оптимальних значень / А. В. Гончаренко // Сучасні інформаційні та інноваційні технології на транспорті (MINTT-2009). [Збірка наукових праць у п'яти томах. (25-27 травня 2009 р., Херсон)]: Матеріали Міжнародної науково-практичної конференції. – Том 4. – Херсон: Видавництво Херсонського державного морського інституту, 2009. – С. 45-48.
95. **Гончаренко А. В.** Експлуатація активних транспортних систем в умовах багатоальтернативності та невизначеності: автореф. … докт. техн. наук: 05.22.20 / А. В. Гончаренко. – К.: НАУ, 2016. – 39 с.
96. **Гончаренко А. В.** Експлуатація активних транспортних систем в умовах багатоальтернативності та невизначеності: дис. … докт. техн. наук: 05.22.20 / А. В. Гончаренко. – К., 2016. – 328 с.
97. **Гончаренко А. В.** Керування підтриманням безпеки польотів через технічні та витратні чинники: автореф. … канд. техн. наук: 05.13.03 / А. В. Гончаренко. – К.: НАУ, 2005. – 20 с.
98. **Гончаренко А. В.** Керування підтриманням безпеки польотів через технічні та витратні чинники: дис. … канд. техн. наук: 05.13.03 / А. В. Гончаренко. – К., 2005. – 198 с.
99. **Гончаренко А. В.** Методичні рекомендації до дипломного проектування / А.В. Гончаренко. – Херсон: Видавництво ПП Тріфонов, друкарня «Графіка», 2010. – 63 с. (ISBN: 978-966-2997-06-4)
100. **Гончаренко А. В.** Моделювання впливу ентропії суб'єктивних переваг на прийняття рішень стосовно ремонту суднової енергетичної установки / А. В. Гончаренко // Автоматика, автоматизация, электротехнические комплексы и системы. – 2009. – № 1(23). – С. 123-131.
101. **Гончаренко А. В.** Моделювання впливу профілактичних замін на показники безпеки польотів / А. В. Гончаренко // Вісник НАУ. – 2004. – № 3(21). – С. 74-77.
102. **Гончаренко А. В.** Обоснование величины инвестиций в безопасность полетов с учетом коэффициента технического использования / А. В. Гончаренко // НАУКА І МОЛОДЬ. ПРИКЛАДНА СЕРІЯ: Збірник наукових праць. – К.: НАУ, 2004. – Вип. 4. – С. 15-18.
103. **Гончаренко А. В.** Обоснование величины инвестиций в безопасность полетов с учетом коэффициента технического использования / А. В. Гончаренко // ПОЛІТ: Матеріали IV Міжнародної наукової конференції студентів та молодих учених. – К.: НАУ, 2004. – С. 19.
104. **Гончаренко А. В.** Оптимальне внутрішнє тіньове оподаткування за умови зовнішньоекономічної діяльності фірми / А. В. Гончаренко, О. А. Запорожченко // Вісник НАУ. – 2013. – № 2(55). – С. 251-257.
105. **Гончаренко А. В.** Показники безпеки функціонування транспортної системи в умовах зростання цін на пальне / А. В. Гончаренко // Вісник НАУ. – 2009. – № 1(38). – С. 35-39.

106. **Гончаренко А. В.** Постановка задачі про вибір оптимального рівня витрат на підтримання безпеки польотів / А. В. Гончаренко // НАУКА І МОЛОДЬ: Матеріали міжнародної наукової конференції. – К.: НАУ, 2001. – С. 110.

107. **Гончаренко А. В.** Принципові питання змісту та методики виконання дипломної роботи за спеціальністю «Експлуатація суднових енергетичних установок» усіх форм навчання: навчальний посібник для ВНЗ / А. В. Гончаренко. – Херсон: Видавництво ПП Тріфонов, друкарня «Графіка», 2010. – 165 с. (ISBN: 978-966-2997-08-8)

108. **Гончаренко А. В.** Типи задач рекомендованих до опрацювання при виконанні дипломної роботи за спеціальністю «Експлуатація суднових енергетичних установок» усіх форм навчання: навчальний посібник для ВНЗ / А. В. Гончаренко. – Херсон: Видавництво ПП Тріфонов, друкарня «Графіка», 2010. – 192 с. (ISBN: 978-966-2997-07-1)

109. **Гончаренко В. С.** Моделювання процесу гасіння пожеж у машинно-котельних відділеннях / В. С. Гончаренко // Сучасні інформаційні та інноваційні технології на транспорті (MINTT-2009). [Збірка наукових праць у п'яти томах. (25-27 травня 2009 р., Херсон)]: Матеріали Міжнародної науково-практичної конференції. – Том 2. – Херсон: Видавництво Херсонського державного морського інституту, 2009. – С. 36-40.

110. **ГОСТ 27.002–89.** Надежность в технике. Основные понятия. Термины и определения. – введ. 01.07.1990. – М.: Изд-во стандартов, 1990. – 36 с.

111. **Единые** нормы летной годности гражданских транспортных самолетов. – М.: ЦАГИ, 1985. – 470 с.

112. **Збереження** льотної придатності повітряних суден: методичні вказівки до виконання курсового проекту / уклад.: В. Г. Докучаєв, Ю. Г. Кордянін, І. М. Новіков, та ін. – К.: НАУ, 2003. – 56 с.

113. **Ивченко Г. И.** Теория массового обслуживания / Г. И. Ивченко, В. А. Каштанов, И. Н. Коваленко. – М.: URSS, 2015. – 304 с.

114. **Игнатов В. А.** Элементы теории оптимального обслуживания технических изделий / В. А. Игнатов, Г. Г. Маньшин, В. В. Костановский. – Минск: Наука и техника, 1974. – 192 с.

115. **Камке Э.** Справочник по обыкновенным дифференциальным уравнениям / Э. Камке. – М.: Наука, 1961. – 703 с.

116. **Капур К.** Надежность и проектирование систем: пер. с англ. Е. Г. Коваленко; под ред. д-ра техн. наук, проф. И. А. Ушакова / К. Капур, Л. Ламберсон. – М.: Мир, 1980. – 604 с.

117. **Касьянов В. А.** Вариационные принципы субъективного анализа. Модифицированный вариационный принцип Эйлера-Лагранжа. Энтропийный подход: монография / В. А. Касьянов, А. В. Гончаренко. – К.: ДП НВЦ «Пріоритети», 2015. – 112 с. (ISBN 978-966-8809-67-5)

118. **Касьянов В. А.** Свет и тень. Пропорции теневой экономики. Энтропийный подход: монография / В. А. Касьянов, А. В. Гончаренко. – К.: Кафедра, 2013. – 86 с. (ISBN 978-966-2705-36-2)

119. **Кельберт М. Я.** Вероятность и статистика в примерах и задачах. Основные понятия теории вероятностей и математической статистики / М. Я. Кельберт, Ю. М. Сухов. – М.: изд. МЦНМО, 2007. – 455 с.

120. **Коваленко И. Н.** Анализ редких событий при оценке эффективности и надежности систем / И. Н. Коваленко. – М.: Сов. радио, 1980. – 209 с.

121. **Коваленко И. Н.** О двухциклической системе обслуживания / И. Н. Коваленко // Кибернетика и системный анализ. – 2015. – № 1. – С. 59-65.

122. **Корн Г.** Справочник по математике для научных работников и инженеров. Определения, теоремы, формулы: 5-е изд. / Г. Корн, Т. Корн. – М: Наука, 1984. – 831 с. [**Korn G. A.** Mathematical handbook for scientists and engineers. Definitions, theorems and formulas for reference and review: second, enlarged and revised edition / G. A. Korn, T. M. Korn. – USA: McGraw-Hill Book Company, 1968.]

123. **Кунцевич В. М.** Управление в условиях неопределенности: гарантированные результаты в задачах управления и идентификации: монография / В. М. Кунцевич. – К.: Наукова думка, 2006. – 264 с.

124. **Математическая** энциклопедия; гл. ред. И. М. Виноградов, в 5-ти томах. – М.: Советская Энциклопедия, 1984. – Т. 5. – 1248 стб.

125. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1986. – Т. 1: Методология. Организация. Терминология / под ред. А. И. Рембезы. – 224 с.

126. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1987. – Т. 2: Математические методы в теории надежности и эффективности / под ред. Б. В. Гнеденко. – 280 с.

127. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1988. – Т. 3: Эффективность технических систем / под общ. ред. В. Ф. Уткина, Ю. В. Крючкова. – 328 с.

128. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1988. – Т. 5: Проектный анализ надежности / под ред. В. И. Патрушева и А. И. Рембезы. – 316 с.

129. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1989. – Т. 6: Экспериментальная отработка и испытания / под общ. ред. Р. С. Судакова, О. И. Тескина. – 376 с.

130. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1990. – Т. 8: Эксплуатация и ремонт / под ред. В. И. Кузнецова и Е. Ю. Барзиловича. – 320 с.

131. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1990. – Т. 9: Техническая диагностика / под общ. ред. В. В. Клюева, П. П. Пархоменко. – 1987. – 352 с.

132. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1990. – Т. 10: Справочные данные по условиям эксплуатации и характеристикам надежности / под ред. В. А. Кузнецова. – 1990. – 336 с.

133. **Надежность** машин: учеб. пособие; под ред. Д. Н. Решетова / Д. Н. Решетов, А. С. Иванов, В. З. Фадеев. – М.: В. Школа, 1988. – 238 с.

134. **Наставление** по производству полетов в гражданской авиации (НПП ГА-85). – М.: Воздушный транспорт, 1985. – 286 с.

135. **Наставление** по штурманской службе ГА. – М.: Воздушный транспорт, 1985. – 254 с.

136. **Овчаров Л. А.** Прикладные задачи теории массового обслуживания / Л. А. Овчаров. – М.: Машиностроение, 1969. – 324 с.

137. **Организация** и проведение полетов самолетов ГА. – М.: Воздушный транспорт, 1986. – 232 с.

138. **Пат. 94181** Україна, МПК B63H 25/00. Спосіб вибору оптимальної комбінації режимів експлуатації суднової рульової машини / А. В. Гончаренко; заявник та власник патенту Національний авіаційний університет. – № u 2013 09054; заявл. 19.07.2013; опубл. 10.11.2014, Бюл. № 21.

139. **Пискунов Н. С.** Дифференциальное и интегральное исчисления для втузов / Н. С. Пискунов, т. 1: Учебное пособие для втузов. 13-е изд. – М.: Наука, Главная редакция физико-математической литературы, 1985. – 432 с.

140. **Пискунов Н. С.** Дифференциальное и интегральное исчисления для втузов / Н. С. Пискунов, т. 2: Учебное пособие для втузов. 13-е изд. – М.: Наука, Главная редакция физико-математической литературы, 1985. – 560 с.

141. **Смирнов Н. Н.** Методические указания к курсовому проектированию по дисциплине «Техническая эксплуатация летательных

аппаратов» для студентов специальности 1610 / Н. Н. Смирнов, Ю. М. Чинючин. – М.: РИО МИИГА, 1985. – 56 с.

142. **Смирнов Н. Н.** Обслуживание и ремонт авиационной техники по состоянию / Н. Н. Смирнов, А. А. Ицкович. – М.: Транспорт, 1987. – 272 с.

143. **Техническая** эксплуатация летательных аппаратов: учебник для вузов / под ред. Н. Н. Смирнова. – М.: Транспорт, 1990. – 423 с.

144. **Эльсгольц Л. Э.** Дифференциальные уравнения / Л. Э. Эльсгольц. – М.: Государственное издательство технико-теоретической литературы, 1957. – 271 с.

Навчальне видання

ПІДТРИМАННЯ ЛЬОТНОЇ ПРИДАТНОСТІ ПОВІТРЯНИХ СУДЕН (ICAO Doc 9760)

Частина I

ПОКАЗНИКИ НАДІЙНОСТІ ДЛЯ ОЦІНКИ УДОСКОНАЛЕННЯ ПРОЦЕСУ ТЕХНІЧНОГО ОБСЛУГОВУВАННЯ ПОВІТРЯНОГО СУДНА

Методичні рекомендації
до виконання самостійної роботи
для студентів 1-го курсу галузі знань
27 «Транспорт», спеціальності 272 «Авіаційний
транспорт», спеціалізації 01 «Технічне
обслуговування та ремонт повітряних
суден і авіадвигунів»

(Англійською мовою)

Укладач ГОНЧАРЕНКО Андрій Вікторович

В авторській редакції

Технічний редактор *А. І. Лавринович*
Комп'ютерна верстка *Н. С. Ахроменко*

Підп. до друку 27.06.2018. Формат 60х84/16. Папір офс.
Офс. друк. Ум. друк. арк. 2,79. Обл.-вид. арк. 3,0.
Тираж 100 пр. Замовлення № 93-1.

Видавець і виготівник
Національний авіаційний університет
03680. Київ-58, проспект Космонавта Комарова, 1

Свідоцтво про внесення до Державного реєстру ДК № 977 від 05.07.2002