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AIRCRAFT FUEL MEASUREMENT SYSTEM BASED ON HYDROSTATIC PRESSURE SENSORS

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Abstract—*The construction of an aircraft fuel-measuring system based on hydrostatic pressure sensors is considered, which makes it possible to determine the fuel residue in the aircraft tanks during its evolutions. With the evolution of aircraft, measuring the fuel residue in existing fuel metering systems using float and capacitive fuel level sensors has a rather complex electromechanical design and significant weight and size characteristics. This together affects the reliability of such systems as a whole and leads to significant methodological errors in determining the remaining fuel during maneuverable flight. The proposed system using hydrostatic pressure sensors and a computer can significantly increase the efficiency of existing fuel metering systems, and can also be used for calibration tests both on the ground and in flight.*

Index Terms—Aircraft fuel system; hydrostatic pressure sensor; parallelepiped; tetrahedron; plane equation; recalculation of vector in coordinate systems.

I. INTRODUCTION

The errors of float and capacitance fuel gauges consist of the following components:

- errors resulting from longitudinal and lateral rolls and accelerations of the airplane;
- errors resulting from inaccurate installation of fuel tanks and deviations of their dimensions from those obtained during calculation and calibration;
- temperature errors caused by changes in fuel temperature in the tank and change of fuel grade;
- errors caused by changes in the voltage of the power supply.

The first three groups of errors are methodical errors. And if the errors associated with inaccurate installation of fuel tanks and temperature errors can be compensated by introducing additional calibration schemes, then to compensate for errors caused by the slope of the "fuel mirror" arising from changes in the angular orientation of the aircraft and the action of acceleration on the aircraft are developed not always successful algorithmic compensators.

The analysis of existing widespread fuel measuring systems (FMS) of aircraft using float and capacitive fuel level sensors shows that these systems measure fuel level in aircraft tanks with sufficient accuracy only in horizontal flight. During aircraft evolutions, the measurement of fuel residue in such FMS leads to significant methodological errors. In addition, they have a rather complex electromechanical design and significant mass-size characteristics, which together affects overall reliability of such vehicles as a whole. Therefore, studies aimed at the development of fuel measurement systems minimizing such methodological errors are very relevant.

II. PROBLEM STATEMENT

The known methods of measuring the fuel level in the tanks of modern aircraft using float and capacitive fuel level sensors do not allow to determine its residue during maneuvering, when pitch and roll angles undergo significant changes. In the conditions of maneuvering flight the methodical error increases significantly, which during prolonged maneuvering can lead to undesirable consequences. In order to obtain stable fuel residue readings for the above reasons, it is necessary to use new approaches to the construction of the measuring system of fuel residue in the tanks of the aircraft with high reliability, as well as with the minimum mass and dimensional dimensions.

III. PROBLEM SOLUTION

Let us consider the principle of FMS operation on the basis of hydrostatic pressure sensors (HPS) on the example of determining the volume of fuel in the tank of an aircraft made in the form of a parallelepiped with edges a, b, c with fuel "mirror height" h_{fm} (Fig. 1). The base of the parallelepiped is tied to the horizontal geotopic triangle OENH, which in this case is considered as an instrumental one. The angles of heading Ψ, pitch ϑ and roll Υ of the aircraft are assumed for this case to be zero.

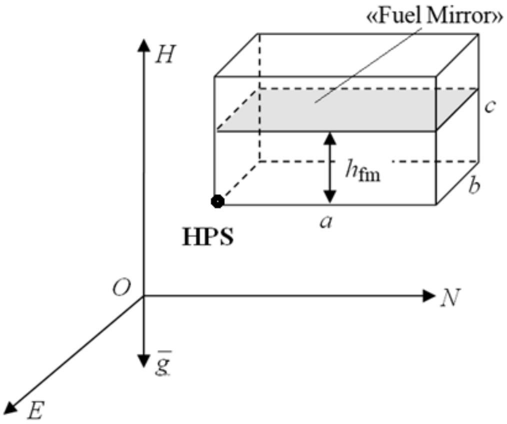

Fig. 1. Determining the volume of fuel in a parallelepiped-shaped tank

According to the readings of the HPS, which is located in one of the vertices of the base of the parallelepiped, we can calculate the height of the "fuel mirror" (FM) plane hfm. According to Pascal's law

$$P_{h_{\text{fm}}} = \rho \cdot g \cdot h_{\text{fm}}, \quad h_{\text{fm}} = \frac{P_{h_{\text{fm}}}}{(\rho \cdot g)}, \tag{1}$$

where $P_{h_{\text{fm}}}$ is hydrostatic pressure of a liquid with constant density in a homogeneous field of gravity: ρ is the density of the liquid; g is the acceleration of free fall.

The volume of fuel in the tank $V_{h_{\text{fm}}}$ at height h_{fm} is calculated from the formula of parallelepiped volume:

$$V_{h_{\text{fm}}} = abc = abh_{\text{fm}}. \tag{2}$$

Modern designs of aircraft fuel tanks are usually located in the wings and fuselage of the airplane and have shapes close to a parallelepiped. To solve the problem of determining the fuel volume in each of the tanks, it is proposed to divide its space into inscribed volume figures, the fuel volume in which can be calculated separately, and the total fuel volume can be found as the sum of the fuel volumes of the figures inscribed in the tank. In this article, the tetrahedron (T) is considered as an inscribed volume figure.

For example, let us consider the possibility of partitioning a parallelepiped into tetrahedrons. The parallelepiped, as can be seen from Fig. 2, is partitioned into four tetrahedrons:

- tetrahedron 1, with vertices: 1, 3, 4, 8;
- tetrahedron 2, with vertices: 1, 2, 3, 6;
- tetrahedron 3, with vertices: 3, 6, 7, 8;
- tetrahedron 4, with vertices: 1, 5, 6, 8.

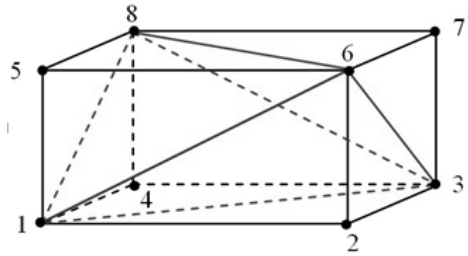

Fig. 2. Partitioning a parallelepiped into tetrahedrons

Fuel residue volume V_x for tetrahedron No.2 with vertices 1, 2, 3, 6 (Fig. 3) in the coordinate system of the instrument trihedron *OENH* and HPS located in one of the base vertices can be determined from the following relations:

1) Total volume of the tank V_0 (volume of the tetrahedron with vertices 1, 2, 3, 6 and two mutually perpendicular side planes of the tetrahedron):

$$V_0 = \frac{1}{3} Sh_0, \quad S = \frac{1}{2} ab, \quad V_0 = \frac{1}{6} abh_0,$$

where *a, b* is lengths of faces T; h_0 is the calculated height of T according to the information from HPS at vertex 2.

2) The volume of the unfilled part of the tank V_{up} with vertices in points 1', 2', 3', 6 is determined by the lengths of the segments of the edges of the unfilled part of the tetrahedron *a*' and *b*' cut off by the FM. The sought segments *a*' and *b*' are found from the relations for rectangular similar triangles. Through similar triangles formed by vertices 1, 2, 6 and 1', 2', 6' the segment *a*' is found, and through triangles with vertices 3, 2, 6 and 3', 2', 6' the segment *b*' is determined:

$$a' = h_{\text{fm}} \frac{b}{h_0}, \quad b' = h_{\text{fm}} \frac{a}{h_0},$$

$$V_{\text{up}} = \frac{1}{6} a'b' \left(h_0 - h_{\text{fm}}\right).$$

3) The desired fuel volume is defined as the difference between V_0 and the volume of the unfilled part of the tetrahedron V_{up}:

$$V_x = V_0 - V_{\text{up}}.$$

To find the fuel residue in a tetrahedron of arbitrary shape, to calculate the lengths of the segments obtained from the intersection of the fm with the edges of the tetrahedron, it is necessary to recalculate the parameters of the vector perpendicular to the FM plane $h_{\text{fm}}\{0, h_{\text{fm}}, 0\}$ on its inclined edges. Such a problem of computing the volume V_x in segments is solvable, but requires the use of a rather complex algorithm [1].

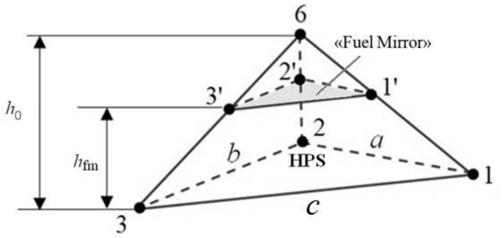

Fig. 3. Volume of fuel residue for the tetrahedron

One of the variants of another solution of the problem of fuel residue determination for an arbitrary tetrahedron can be realized with the transition from volume calculation in segments to volume calculation by coordinates of fm intersection with tetrahedron edges.

In general, the equation of the plane, linear with respect to Cartesian rectangular coordinates is [1]:

$$Ax + By + Cz + D = 0,$$

where A, B and C not equal to zero simultaneously define the plane.

The equations of three side planes of a tetrahedron of arbitrary shape with one HPSS in one of the vertices of the base, which coincides with the horizontal plane of the instrumental tetrahedron OENH, can be represented in the following form:

- plane with vertices 1, 6, 3

$$A_1 x_{ENH} + B_1 y_{ENH} + C_1 z_{ENH} + D_{1ENH} = 0; \quad (3)$$

- plane with vertices 1, 6, 2

$$A_2 x_{ENH} + B_2 y_{ENH} + C_2 z_{ENH} + D_{2ENH} = 0; \quad (4)$$

- plane with vertices 2, 6, 3

$$A_3 x_{ENH} + B_3 y_{ENH} + C_3 z_{ENH} + D_{3ENH} = 0, \quad (5)$$

where

$$D_{1ENH} = Ax_{10ENH} + By_{10ENH} + Cz_{10ENH},$$
$$D_{2ENH} = Ax_{20ENH} + By_{20ENH} + Cz_{20ENH},$$
$$D_{3ENH} = Ax_{30ENH} + By_{30ENH} + Cz_{30ENH}.$$

The parameters of the planes with respect to the *OENH* coordinate system are assumed to be known.

The equation of the FM plane passing through one of its points and perpendicular to the vector $\mathbf{N}\{A, B, C\}$ can be represented in the following form

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0. \quad (6)$$

In the case of parallelism of the FM plane to the *OENH* plane ($A = 0$, $C = 0$), equation (6) will take the following form:

$$B(y - y_0) = 0,$$

whence at $\mathbf{N}\{0, h_{\text{fm}}, 0\}$

$$D = By_0,$$
$$y = y_0 = h_{\text{fm}ENH}, \quad (7)$$

for all points of the FM.

The x and y coordinates of the FM for points 1', 2', 3' (Fig. 3) are found as the intersection of the tetrahedron planes (3) … (5) with the FM plane (7).

For the coordinates of points 1', 2', 3', respectively, we have three systems of equations and their solution with respect to the three points of intersection of the FM with the tetrahedron:

- for coordinates of point 1'(x'_{1ENH} , h_{fm}1', z'_{1ENH}):

$$A_1 x_{1ENH} + h_{\text{fm}} + C_1 z_{1ENH} + D_{1ENH} = 0,$$
$$D_{1ENH} = Ax_{01ENH} + By_{01ENH} + Cz_{01ENH},$$
$$A_2 x_{1ENH} + h_{\text{fm}} + C_2 z_{1ENH} + D_{2ENH} = 0,$$
$$D_{2ENH} = Ax_{02ENH} + By_{02ENH} + Cz_{02ENH},$$
$$z'_{1ENH} = -\frac{A_2 D_{1ENH} - D_{2ENH} h_{\text{fm}}}{C_{1ENH}},$$
$$x'_{1ENH} = \frac{h_{\text{fm}}(C_{2ENH} - C_{1ENH}) + C_2 D_{1ENH} - C_1 D_{2ENH}}{A_2 C_{1ENH} - A_{21} C2_{ENH}}; \quad (8)$$

- for coordinates of point 2'(x'_{2ENH} , h_{fm}2', z'_{2ENH}):

$$A_2 x_{2ENH} + h_{\text{fm}} + C_2 z_{2ENH} + D_{2ENH} = 0,$$
$$D_{2ENH} = Ax_{02ENH} + By_{02ENH} + Cz_{02ENH},$$
$$A_3 x_{3ENH} + h_{\text{fm}} + C_3 z_{3ENH} + D_{3ENH} = 0,$$
$$D_{3ENH} = Ax_{03ENH} + By_{03ENH} + Cz_{03ENH},$$
$$z'_{2ENH} = -\frac{A_3 D_{2ENH} - D_{3ENH} h_{\text{fm}}}{C_{2ENH}},$$
$$x'_{2ENH} = \frac{h_{\text{fm}}(C_{3ENH} - C_{2ENH}) + C_3 D_{2ENH} - C_2 D_{3ENH}}{A_3 C_{2ENH} - A_2 C_{3ENH}}. \quad (9)$$

- for coordinates of point 3'(x'_{3ENH} , h_{fm}3', z'_{3ENH}):

$$A_1 x_{1ENH} + h_{\text{fm}} + C_1 z_{1ENH} + D_{1ENH} = 0,$$
$$D_{1ENH} = Ax_{01ENH} + By_{01ENH} + Cz_{01ENH},$$
$$A_3 x_{2ENH} + h_{\text{fm}} + C_3 z_{2ENH} + D_{3ENH} = 0,$$
$$D_{3ENH} = Ax_{03ENH} + By_{03ENH} + Cz_{03ENH},$$
$$z'_{3ENH} = -\frac{A_3 D_{1ENH} - D_{3ENH} h_{\text{fm}}}{C_{1ENH}},$$
$$x'_{3ENH} = \frac{h_{\text{fm}}(C_{3ENH} - C_{1ENH}) + C_3 D_{1ENH} - C_1 D_{3ENH}}{A_3 C_{1ENH} - A_1 C_{3ENH}}. \quad (10)$$

According to the found coordinates of FM and the known coordinates of the tetrahedron vertex $x_{V\,ENH}$,

$y_{V\,ENH}$, $z_{V\,ENH}$ the volume of the unfilled part of fuel in the tank is determined by the formula [2]:

$$V_{\text{up}} = \frac{1}{6}\begin{vmatrix} x_{V\,ENH} & y_{V\,ENH} & z_{V\,ENH} & 1 \\ x_1' & y_1' & z_1' & 1 \\ x_2' & y_2' & z_2' & 1 \\ x_3' & y_3' & z_3' & 1 \end{vmatrix}. \qquad (11)$$

The volume of the remaining fuel is found as the difference between the a priori known volume of the tetrahedron V_0 and the calculated unfilled part V_{up}.

The obtained solution of the fuel determination problem was considered in the *OENH* coordinate system. At the same time, in the conditions of a fixed base and during the aircraft evolutions, the position of the instrument triangle *OXYZ* will change in accordance with the changes in its angular position relative to the triangle *OENH*, i.e., with the changes in the angles of heading ψ, pitch ϑ, and roll γ of the aircraft. It follows that for finding the coordinates 1', 2', 3' of the FM intersection with the edges of the tetrahedron it is necessary to take into account the inclination of the fm plane relative to the movable instrumental trihedral *OXYZ* (Fig. 4).

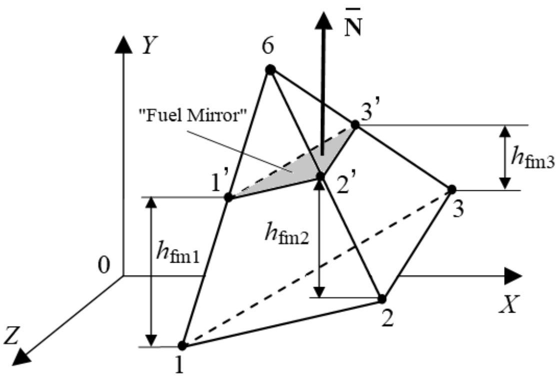

Fig. 4. Tilt of the "fuel mirror" plane relative to the movable instrumental trihedral

The relationship between the coordinates of the *OENH* triangles in *OXYZ* at the current moment of time can be found through the rectangular matrix of directional cosines

$$\mathbf{B} = L(\psi)L(\vartheta)L(\gamma) \qquad (12)$$

in the form

$$\mathbf{B} = \begin{bmatrix} \cos\psi\cos\vartheta & \begin{array}{c}\sin\psi\sin\gamma - \\ -\cos\psi\sin\vartheta\cos\gamma\end{array} & \begin{array}{c}\sin\psi\cos\gamma + \\ +\sin\psi\cos\vartheta\sin\gamma\end{array} \\ \sin\vartheta & \cos\vartheta\cos\gamma & -\cos\vartheta\sin\gamma \\ -\sin\psi\cos\vartheta & \begin{array}{c}\cos\psi\sin\gamma + \\ +\sin\psi\sin\vartheta\cos\gamma\end{array} & \begin{array}{c}\cos\psi\cos\gamma - \\ -\sin\psi\sin\vartheta\sin\gamma\end{array} \end{bmatrix}.$$

In this case, the coordinates of the tetrahedron planes in the axes of the tetrahedron *OENH* will have the form

$$\begin{bmatrix} x_{V\,ENH} \\ y_{V\,ENH} \\ z_{V\,ENH} \end{bmatrix} = \mathbf{B}\begin{bmatrix} x \\ y \\ z \end{bmatrix}. \qquad (13)$$

The angles ψ, ϑ, and γ of the aircraft can be obtained from the inertial navigation system (INS), the heading and vertical system and other meters available on board.

In these cases, the FM plane at points 1',2',3' in the case of its inclination relative to the moving instrument triangle *OXYZ* is defined not by one vector perpendicular to the FM plane $N\{0, h_{\text{fm}}, 0\}$, as it was at zero heading angles ψ, ϑ, and γ of the aircraft, but by three vectors $\{0, h_{\text{fm}}1', 0\}$ as it was at zero heading angles ψ, ϑ and γ of the aircraft, but three:

$$\{0, \text{hfm}1', 0\}, \{0, \text{hfm}2', 0\}, \{0, \text{hfm}3', 0\}.$$

Thus, to calculate the heights $h_{\text{fm}}1'$, $h_{\text{fm}}2'$ and $h_{\text{fm}}3'$, it is necessary to have information from three HPS located at the vertices of the base of the tetrahedron (see Fig.4).

Thus, to solve the problem of determining the fuel residue in the mobile coordinate system at the current moment of time, it is necessary, in addition to the calculated values of $h_{\text{fm}}1'$, $h_{\text{fm}}2'$ and $h_{\text{fm}}3'$, to have information about the current coordinates of the vertices of the tetrahedron x_i, y_i, z_i and the coefficients of its planes $A_i, B_i, C_i, D_i \quad i=\overline{1\ldots4}$ in the coordinate system *OXYZ*. These parameters at the current moment of time can be determined on the basis of the inverse matrix of directional cosines, obtained from the direct square matrix **B** through the known angles ψ, ϑ, γ:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \mathbf{B}^{-1}\begin{bmatrix} x_{i\,ENH} \\ y_{i\,ENH} \\ z_{i\,ENH} \end{bmatrix}, \qquad (14)$$

$$\begin{bmatrix} A_i \\ B_i \\ C_i \end{bmatrix} = \mathbf{B}^{-1}\begin{bmatrix} A_{i\,ENH} \\ B_{i\,ENH} \\ C_{i\,ENH} \end{bmatrix}, \qquad (15)$$

$$D_i = A_i x_{i0} + B_i y_{i0} + C_i z_{i0}, \quad i = \overline{1...3}. \qquad (16)$$

Based on relations (14) – (16) and calculated values of heights $h_{\text{fm}}1'$, $h_{\text{fm}}2'$, $h_{\text{fm}}3'$ according to the readings of three FMSs, the system of equations for calculating the coordinates of FM (8) … (10) in the axes of the moving trihedron *OXYZ* is as follows:

$$z'_{1_{XYZ}} = -\frac{A_2 D_{1XYZ} - D_{2XYZ} h_{\text{fm}}}{C_{1XYZ}},$$
$$x'_{1_{XYZ}} = \frac{h_{\text{fm}}(C_{2XYZ} - C_{1XYZ}) + C_2 D_{1XYZ} - C_1 D_{2XYZ}}{A_2 C_{1XYZ} - A_1 C_{2XYZ}}, \quad (17)$$

$$z'_{2_{XYZ}} = -\frac{A_3 D_{2XYZ} - D_{3XYZ} h_{\text{fm}}}{C_{2XYZ}},$$
$$x'_{2_{XYZ}} = \frac{h_{\text{fm}}(C_{3XYZ} - C_{2XYZ}) + C_3 D_{2XYZ} - C_2 D_{3XYZ}}{A_3 C_{2XYZ} - A_2 C_{3XYZ}}, \quad (18)$$

$$z'_{3_{XYZ}} = -\frac{A_3 D_{1XYZ} - D_{3XYZ} h_{\text{fm}}}{C_{1XYZ}},$$
$$x'_{3_{XYZ}} = \frac{h_{\text{fm}}(C_{3XYZ} - C_{1XYZ}) + C_3 D_{1XYZ} - C_1 D_{3XYZ}}{A_3 C_{1XYZ} - A_1 C_{3XYZ}}. \quad (19)$$

Based on relations (12) and (14)…(19), the structure of the algorithm for determining the fuel residue at the current moment of time for a tank in the form of an arbitrary tetrahedron for a mobile aircraft can be represented as follows (Fig. 5).

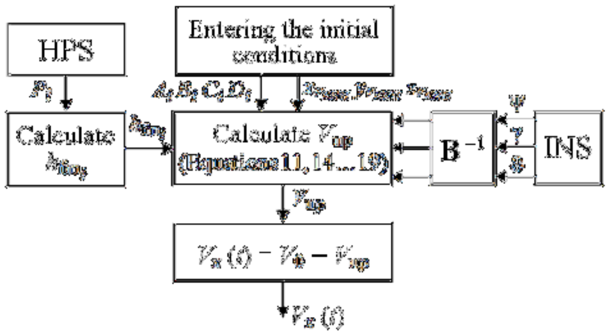

Fig. 5. Structure of the algorithm for determining the fuel residue at the current moment of time

When realizing the method of measuring the fuel residue in the tanks of aircraft with the division of tank volumes into tetrahedrons of arbitrary shape, there is a methodological error associated with the oscillations of the fuel surface at the occurrence of aircraft acceleration. For estimation of sensor measurements in the selected time interval of pressure measurement it is supposed to use methods of optimal measurement processing.

At mismatch of planes of tetrahedrons inscribed in the tank volume with real planes of tanks for compensation of methodical error the alignment is required. The alignment procedure can be realized by measuring by means of FMS the readings of current fuel consumption at change of heading angles ψ, ϑ and γ of the aircraft and comparing them with the real readings of fuel residue in the tank. The difference in readings can be taken into account in flight in the form of corrections coming from the calculator.

IV. CONCLUSIONS

The use of hydrostatic pressure sensors in the system of fuel residue measurement and calculator allows to solve the problem of increasing the accuracy of the aircraft fuel measuring system in all flight modes and to reduce its mass and dimensional characteristics.

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О. І. Смірнов, М. К. Філяшкін. Система вимірювання палива літаків на основі датчиків гідростатичного тиску

Розроблено побудову паливно-вимірювальної системи літака на основі датчиків гідростатичного тиску, що дає змогу визначити залишок палива в баках літака під час його еволюцій. При еволюціях літаків вимірювання залишку палива в існуючих системах вимірювання палива за допомогою поплавкових і ємнісних датчиків рівня палива має досить складну електромеханічну конструкцію та значні масогабаритні характеристики. Це в сукупності впливає на надійність таких систем в цілому і призводить до значних методичні похибки у визначенні залишку палива при маневреному польоті. Запропонована система з використанням датчиків гідростатичного тиску та комп'ютера може значно підвищити ефективність існуючих систем вимірювання палива, а також може використовуватися для калібрувальних випробувань як на землі, так і в польоті.

Ключові слова: паливна система літака; датчик гідростатичного тиску; паралелепіпед; тетраедр; рівняння площини; перерахунок вектора в системах координат.

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