GEOMETRY VIA SPRAY ON FRÉCHET MANIFOLDS

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For a Fréchet manifold by employing sprays we will construct connection maps, linear symmetric connections on tangent and second-order tangent bundles. We characterize linear symmetric connections on tangent bundles by using the bilinear symmetric mappings associated with a given spray on a manifold. Moreover, we give another characterization of linear symmetric connections on tangent bundles using tangent structures. We show that there is a bijective correspondence between linear symmetric connections on tangent bundles and connection maps induced by sprays on a manifold. Let M be a Fréchet manifold and $\mathbf{S}$ a given spray on M. We denote by TM, T^2M and $T(TM)$ its tangent bundle, second-order tangent bundle, and the tangent bundle over TM, receptively.

Theorem 1. [1] *Let ∇ be the covariant derivative associated with $\mathbf{S}$. Then there exists a unique vector bundle morphism (called a connection map) $K : T(TM) \to TM$ such that $\nabla = K \circ T$, and for all C^{k-1}-vector fields X, Y on M the following diagram is commutative:*

$$\begin{array}{ccc} TM & \xrightarrow{TX} & T(TM) \\ {\scriptstyle Y}\uparrow & & \downarrow{\scriptstyle K} \\ M & \xleftarrow[\nabla_Y X]{} & TM \end{array}$$

Theorem 2. [1] *There exists a unique linear symmetric connection on TM which is fully characterized by the associated symmetric bilinear mappings of $\mathbf{S}$. Conversely, if C is a linear symmetric connection on TM, then there exists a unique spray on M whose associated connection map is determined by C.*

Theorem 3. [1] *Any linear symmetric connection on TM induces a linear symmetric connection on T^2M, and vice versa. Moreover, any linear symmetric connection on the tangent bundle determines a connection map and vice versa.*

References

[1] Eftekharinasab K. (2023). Geometry via spray on Fréchet manifolds. *https://arxiv.org/abs/2307.15955*.

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