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Application of partial derivatives in finding coefficients of elasticity of the production function

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Often, when solving various economic problems, we are given several factors (for example: the living labor costs and, embodied labor cost) and it is necessary to find out exactly how they will affect the final result (for example: the amount of production). To get an answer to such questions, it is worth using the production function, which, in fact, reflects the dependence between the result and the factors that affect it. Each of the factors affects the result in its own way and, for example, the living labor costs all costs associated with the payment of labor can affect the amount of production much more strongly, and the costs of embodied labor all costs associated with the use of various resources, created by man, is significantly weaker. An indicator that measures how a change in one or another factor affects the value of a function is called elasticity. If the costs of living labor affect the amount of production more strongly, and the costs of embodied labor less, then the function is more elastic with respect to the change in living labor costs than with respect to the costs of embodied labor. [1]

Let $z = f(x,y)$ be the amount of production, where x is the cost of living labor, y is the cost of embodied labor. To determine the elasticity of the quantity of production relative to live labor and embodied labor, we use the following formulas: $E_x(z) = \frac{x}{z}\frac{\delta z}{\delta x}$ and $E_y(z) = \frac{y}{z}\frac{\delta z}{\delta y}$

The formula on the left indicates an approximate percentage increase in the amount of production corresponding to a 1% increase in living labor costs, provided that the costs of embodied labor do not change. The formula on the right indicates an approximate percentage increase in the amount of production corresponding to a 1% increase in embodied labor costs, provided that the costs of living labor do not change.

Depending on the percentage growth of the production function relative to the increase in the living labor costs (or relative to the increase in the cost of embodied labor), it can be divided into 3 types:

- Elastic provided that with an increase in living labor costs by 1%, the amount of production will increase by more than 1%.

- Neutral provided that with an increase in living labor costs by 1%, the amount of production will increase by exactly 1%.
- Inelastic provided that with an increase in living labor costs by 1%, the amount of production will increase by less than 1%. [1, 2]

Example. Let the production function $z = 7x^3y + 5xy^2 + x^4$, where x is living labor costs, y is embodied labor costs be given. Elasticity $E_x(z)$ and $E_y(z)$ should be found at the points (1; 0), (1;1).

The elasticity of the production function $z = f(x, y)$ relative to the factors x and y is determined by the formulas:

$$E_x(z) = \frac{x}{z}\frac{\delta z}{\delta x} \text{ and } E_y(z) = \frac{y}{z}\frac{\delta z}{\delta y} \quad (1)$$

Let's define the partial derivatives of function z with respect to x and with respect to y:

$$\frac{\delta z}{\delta x} = 21x^2y + 5y^2 + 4x^3; \qquad \frac{\delta z}{\delta y} = 7x^3 + 10xy. \quad (2)$$

After substitution of the obtained partial derivatives (2) into the formulas (1) elasticity acquires the form:

$$E_x(z) = \frac{x(21x^2y+5y^2+4x^3)}{7x^3y+5xy^2+x^4}; \qquad E_y(z) = \frac{y(7x^3+10xy)}{7x^3y+5xy^2+x^4}$$

Let's determine the elasticity $E_x(z)$ and $E_y(z)$ at the points (1;0), (1;1).

$$E_x(z)|_{(1;0)} = 4; \qquad E_y(z)\big|_{(1;0)} = 0.$$

If the costs of living labor increase by 1%, the amount of production increases by 4% (it is elastic), and if there is no increase in the costs of embodied labor - it will not change (is inelastic).

$$E_x(z)|_{(1;1)} = 2{,}31; \qquad E_y(z)\big|_{(1;1)} = 1{,}31.$$

If the cost of living labor increases by 1%, the amount of production increases by approximately 2.31% (is elastic), and if the cost of embodied labor increases by 1%, it increases by approximately 1.31% (is elastic).

Conclusion. Summarizing all the above we can conclude that if dealing with production functions, the influence of factors on it should be carefully determined. By finding the factor that contributes to the better elasticity of the production function, we can understand where it is better to direct the available resources in order to increase the value of this function more efficiently. This analysis of elasticity is possible thanks to partial derivatives, which proves the relevance of their use in economic problems.

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