МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ

Факультет аеронавігації, електроніки та телекомунікацій

Кафедра аерокосмічних систем управління

ДОПУСТИТИ ДО ЗАХИСТУ

Завідувач кафедри АКСУ

___________Юрій МЕЛЬНИК

“_____” ____________ 2023 р.

КВАЛІФІКАЦІЙНА РОБОТА

(ПОЯСНЮВАЛЬНА ЗАПИСКА)

ВИПУСКНИКА ОСВІТНЬОГО СТУПЕНЯ

«БАКАЛАВР»

Тема: «Система початкової кутової стабілізації супутника»

Виконавець: студент групи СУ-402________________ Андрій ПОПОВИЧ

Керівник: ____Лев РИЖКОВ

Нормоконтролер:_________________________________Микола ДИВНИЧ

Київ 2023

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL AVIATION UNIVERSITY

Faculty of Air Navigation, Electronics and Telecommunications

Aerospace Control Systems Department

APPROVED FOR DEFENCE

Head of the ACS Department

____________ Yurii MELNYK

“_____” ____________ 2023 y.

QUALIFICATION WORK

(EXPLANATORY NOTE)

FOR THE ACADEMIC DEGREE OF BACHELOR

Title: «System of initial angular stabilization of the satellite»

Submitted by: student of group CS-402:____________ Andrii POPOVYCH.

Supervisor: Lev RYZHKOV

Standards inspector: Mykola DYVNYCH

Kyiv 2023

НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ

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Кафедра аерокосмічних систем управління

Спеціальність: 151 Автоматизація та комп’ютерно-інтегровані технології

ЗАТВЕРДЖУЮ

Завідувач кафедри АКСУ

________ Юрій МЕЛЬНИК

“ ____ ” __________ 2023 р.

ЗАВДАННЯ

на виконання кваліфікаційної роботи

Поповича Андрія Романовича

1. **Тема кваліфікаційної роботи:**«Система початкової кутової стабілізації супутника» затверджена наказом ректора від «13» квітня 2023 р. № 507/ст.
2. **Термін виконання роботи:** з 10.03.23 по 10.06.23.
3. **Вихідні дані роботи:** модель супутника з вихідними даними після відділення від ракети носія.
4. **Зміст пояснювальної записки:**

Розділ 1. Математична модель руху супутника;Розділ 2. Дослідження руху супутника в режимі початкової стабілізації;Розділ 3. Моделювання руху супутника в режимі початкової стабілізації.

5. **Перелік обов’язкового ілюстративного матеріалу:** Графіки результатів моделювання та розрахунків. Матеріали презентації в PowerPoint.

6. **Календарний план-графік**

№ пор.	Завдання	Термін виконання	Відмітка про виконання
1.	Отримання завдання	10.05.2023 – 11.05.2023	
2.	Формування мети та основних завдань дослідження	12.05.2023 – 13.05.2023	
3.	Аналіз існуючих методів початкової стабілізації супутника	14.05.2023 – 22.05.2023	
4.	Теоретичний розгляд рішення задачі	23.05-2023 – 26.05.2023	
5	Розробка моделі для симуляції початкової стабілізації супутника	27.05.2023-06.06.2023	
6	Підготовка презентації та роздаткового матеріалу	07.06.2023-11.06.2023	

7. **Дата видачі завдання:** “10” травня 2023 р.

Керівник кваліфікаційної роботи ________________ Лев РИЖКОВ
(підпис керівника)

Завдання прийняв до виконання ________________ Андрій ПОПОВИЧ
(підпис випускника)

NATIONAL AVIATION UNIVERSITY

Faculty of Air Navigation, Electronics and Telecommunications

Aerospace Control Systems Department

Specialty: 151 Automation and Computer-integrated Technologies

APPROVED BY

Head of the ACS Department

______ Yurii MELNYK

"____" __________2023

Qualification Paper Assignment for Graduate Student

Popovych Andrii Romanovych

1. **The qualification paper title** «System of initial angular stabilization of the satellite» was approved by the Rector's order of “ 13 ” April 2023 № 507/ст.
2. **The paper to be completed between: 10.05.23 and 10.06.23**
3. **Initial data for the paper:** a model of the satellite with the original data after separation from the launch vehicle.
4. **The content of the explanatory note:**

Chapter 1. Mathematical model of satellite movement; Chapter 2. Study of the movement of the satellite in the mode of initial stabilization; Chapter 3. Simulation of the movement of the satellite in the mode of initial stabilization.

5. **The list of mandatory illustrations:** Graphs of modeling results and calculations. Presentation materials in PowerPoint.

6. Timetable:

№	Assignment	Dates of completion	Completion mark
1.	Receiving a task	10.05.2023 – 11.05.2023	
2.	Formation of the purpose and main objectives of the study	12.05.2023 – 13.05.2023	
3.	Analysis of existing methods of initial satellite stabilization	14.05.2023 – 22.05.2023	
4.	Theoretical consideration of the problem solution	23.05-2023 – 26.05.2023	
5	Development of a model to simulate the initial stabilization of the satellite	27.05.2023-06.06.2023	
6	Preparing a presentation and handouts	07.06.2023-11.06.2023	

7. Assignment issue date: “10” May 2023 y.

Qualification paper supervisor ______________________ Lev RYZHKOV
(signature)

Issued task accepted ______________________ Andrii POPOVYCH
(signature)

РЕФЕРАТ

Текстова частина роботи: 56 с., 25 рис., 1 табл., 7 літературних джерела.

Об’єкт дослідження – процес початкової стабілізації супутника.

Предмет дослідження – система управління супутника. Початкова стабілізація.

Мета роботи – дослідження принципів і методів початкової стабілізації супутника. Створення симуляції початкової стабілізації.

Методи дослідження – теорія автоматичного керування рухомими об’єктами, математичне моделювання систем керування літальними апаратами та рухомими об’єктами; моделі динаміки керування літальними апаратами та рухомими об’єктами.

У роботі проведено дослідження системи початкової стабілізації супутника після відділення від ракети носія. Швидкість стабілізації залежить від схеми живлення супутника та його габаритів. Стабілізація може займати від кількох хвилин до кількох днів. У роботі проведені експериментальні дослідження початкової стабілізації.

ABSTRACT

Text part of the work: 56 p., 25 fig., 1 tables., 7 literature sources.

Object of research – the process of initial stabilization of the satellite.

Purpose of the work – satellite control system. Initial stabilization.

Purpose of the work – researching the principles and methods of initial satellite stabilization. Creating a simulation of initial stabilization.

Research methods – Theory of automatic control of moving objects, mathematical modeling of objects, mathematical modeling of control systems for aircraft and moving objects; models of the dynamics of control of aircraft and moving objects; models of aircraft and moving object control dynamics.

This paper investigates the system of initial stabilization of the satellite after separation from the launch vehicle. The speed of stabilization depends on the satellite's power supply scheme and its dimensions. Stabilization can take from several minutes to several days. In this work, experimental studies of initial stabilization were conducted.

Content

Introduction

Artificial satellites play a crucial role in modern space exploration and various scientific endeavors. These man-made objects orbit the Earth or other celestial bodies, performing a wide range of tasks and missions. Let’s explore some of the common tasks and missions of artificial satellites:

1. Communication: Communication satellites are among the most widely used artificial satellites. They facilitate global telecommunications by transmitting and relaying signals for television, telephone, internet, and other forms of communication. These satellites are strategically positioned in geostationary orbits, ensuring they remain stationary relative to Earth, enabling consistent coverage over specific regions.
2. Earth Observation: Satellites designed for Earth observation gather valuable data about our planet’s atmosphere, land surface, oceans, and climate patterns. They capture high-resolution images, monitor weather systems, track environmental changes, and study various phenomena such as deforestation, urban growth, and natural disasters. This data is crucial for weather forecasting, disaster management, agriculture, and environmental monitoring.
3. Navigation: Navigation satellites provide positioning, timing, and navigation services to users on the ground or in other vehicles. Systems like the Global Positioning System (GPS) rely on a constellation of satellites to accurately determine the location of receivers worldwide. These satellites continuously broadcast signals, allowing devices to calculate precise position coordinates, enabling applications in navigation, mapping, surveying, and timing synchronization.
4. Scientific Research: Satellites are deployed to conduct scientific experiments and gather data about space, celestial bodies, and the universe. They are equipped with instruments and sensors to observe distant stars, galaxies, and cosmic phenomena, helping astronomers and scientists deepen their understanding of the

cosmos. Satellites like the Hubble Space Telescope and the James Webb Space Telescope have revolutionized our knowledge of the universe.

5. Remote Sensing: Satellites equipped with remote sensing capabilities monitor and analyze the Earth's surface from space. They detect and measure electromagnetic radiation reflected or emitted by the planet, allowing us to study vegetation health, soil moisture, ocean currents, pollution levels, and other environmental factors. This information aids in agriculture, land-use planning, resource management, and disaster response.
6. Technology Demonstration: Some satellites are dedicated to testing and demonstrating new technologies or concepts in space. These missions often serve as precursors to more ambitious projects. They allow engineers and scientists to assess the feasibility, performance, and reliability of novel equipment and techniques in the harsh conditions of space.
7. Space Exploration: Satellites are instrumental in space exploration missions. They provide critical support for robotic probes and landers sent to other planets, moons, or asteroids. Communication satellites relay data and signals between these deep space missions and Earth, enabling real-time monitoring and control. Satellites also assist in mapping and surveying extraterrestrial bodies, studying their atmospheres, geology, and potential for human exploration.
8. Military and Defense: Satellites play a vital role in military and defense operations. They provide intelligence, surveillance, and reconnaissance capabilities, allowing military forces to monitor and gather information about potential threats, terrain, and enemy activities. These satellites enable secure communication networks for military personnel deployed in remote areas.

The tasks and missions of artificial satellites are diverse and continue to evolve with technological advancements. They have transformed numerous aspects of our lives, from global communications and weather forecasting to environmental monitoring and space exploration. As our understanding of the universe expands, satellites will undoubtedly play an even more critical role in furthering scientific research and exploration beyond our planet.

The System of Initial Angular Stabilization of a satellite plays a crucial role in ensuring the successful operation and functionality of spaceborne missions. Satellites are deployed in various orbits around the Earth for a wide range of purposes, such as communication, weather monitoring, scientific research, and surveillance. To accomplish their intended objectives, satellites must maintain a stable orientation and control their angular motion.

The System of Initial Angular Stabilization (SIAS) is designed to address the challenges associated with the initial phases of a satellite's deployment and operation. When a satellite is first launched into space, it experiences significant disturbances and uncertainties that can affect its attitude and stability. Factors such as residual atmospheric drag, release mechanisms, and launch vehicle-induced vibrations can cause unwanted rotations and oscillations, jeopardizing the satellite's mission objectives.

The primary objective of SIAS is to counteract these disturbances and establish a controlled and stable attitude for the satellite. It involves a combination of hardware and software components, including sensors, actuators, and control algorithms, to measure, analyze, and correct the satellite's angular motion. The system utilizes various techniques to achieve angular stabilization, such as reaction wheels, magnetic torques, thrusters, and momentum management.

One of the key components of SIAS is the attitude determination and control system (ADCS), which provides real-time information about the satellite's attitude and helps in maintaining the desired orientation. ADCS utilizes sensors such as sun sensors, star trackers, magnetometers, and gyroscopes to accurately measure the satellite's attitude with respect to Earth's reference frame. This information is then processed by onboard algorithms, which calculate the necessary corrections and commands to the satellite's actuators.

The SIAS also takes into account external factors such as solar radiation pressure, gravity gradients, and magnetic field interactions that can affect the satellite's stability.

These factors are continuously monitored, and appropriate control strategies are implemented to counteract their effects and maintain the desired angular stability.

The successful implementation of SIAS ensures that the satellite remains pointed toward its intended target, such as the Earth's surface, specific celestial objects, or other satellites, depending on its mission objectives. By achieving and maintaining the desired attitude, the satellite can carry out its primary functions effectively, such as data collection, communication, imaging, or scientific experiments.

In conclusion, the System of Initial Angular Stabilization (SIAS) is an essential component of satellite technology. It provides the necessary means to counteract disturbances and uncertainties during the initial phases of a satellite's deployment and operation, establishing a stable attitude and enabling the successful execution of the satellite's mission objectives. By incorporating advanced sensors, actuators, and control algorithms, SIAS plays a crucial role in ensuring the functionality and longevity of satellites in space.

CHAPTER 1. Mathematical model of satellite movement

1.1 Coordinate systems

Coordinate systems are very important for satellites. These systems are used to accurately determine and display the satellite's position and motion relative to the Earth.

Firstly, accurately determining the location of a satellite relative to a reference point on the Earth's surface is crucial for navigating our satellite, tracking it, and communicating with it if needed.

Secondly, satellites move in specific orbits around the Earth. Coordinate systems allow us to define orbital trajectories. It is thanks to coordinate systems that operators can calculate and predict the future positions, velocities, and orbital parameters of a satellite.

Thirdly, we can track the current location of the satellite in real-time. This information is crucial for controlling the satellite, adjusting its trajectory and ensuring that it stays on course.

Fourthly, we can use coordinate systems to locate ground stations relative to the satellite. Thanks to this, we can establish communication lines, direct antennas to ensure the most efficient data transmission between the satellite and the ground station.

Fifthly, coordinate systems help in coordination and cooperation between different satellite operators, space agencies and international organizations to coordinate their actions. This provides a common understanding of the position and movement of satellites.

So, we have analyzed why coordinate systems are extremely important for satellites, and we have analyzed the main points for which these systems are used.

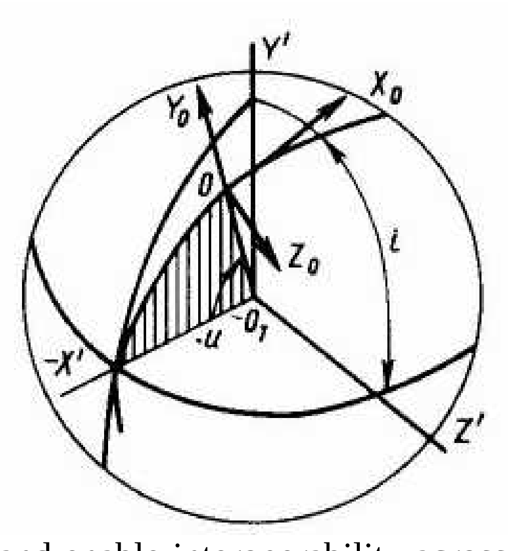
Satellites typically use two main coordinate systems: the Earth-centered, Earth-fixed (ECEF) coordinate system and the geocentric-equatorial coordinate system.

Earth-Centered, Earth-Fixed (ECEF) Coordinate System: The ECEF coordinate system is a Cartesian coordinate system fixed to the Earth's center. It serves as a reference frame for satellite navigation and positioning. In this system, the X-axis points towards the intersection of the prime meridian (0 degrees longitude) and the equator, the Y-axis extends along the prime meridian, and the Z-axis is aligned with the Earth's rotational axis (North Pole). The ECEF coordinates of a satellite define its position relative to the Earth's center, typically measured in meters. This system allows for precise calculations of satellite positions and is commonly used in navigation systems like GPS.

Geocentric-Equatorial Coordinate System: The geocentric-equatorial coordinate system is based on the Earth's equatorial plane and is used to describe the positions of celestial objects and satellites relative to the Earth. It is an extension of the Earth's latitude and longitude system. In this coordinate system, the origin (0, 0) is the Earth's center, the X-axis is aligned with the intersection of the prime meridian and the equator, and the Z-axis is perpendicular to the equatorial plane, passing through the North Pole. The Y-axis completes the right-handed coordinate system.

Satellites' positions are specified using latitude (Φ), longitude (λ), and altitude (h) or radius (r) in this coordinate system. Latitude represents the angle from the equatorial plane (ranging from -90° to +90°), longitude represents the angle around the Earth from the prime meridian (ranging from -180° to +180°), and altitude or radius specifies the distance from the Earth's center.

These coordinate systems allow satellites to accurately determine their position, navigate, communicate, and synchronize time. They provide a standardized framework



for satellite operations and enable interoperability across different satellite systems and

Figure 1.1 Orbital coordinate system

applications.

As the basic coordinate system, we will choose the orbital coordinate system, the direction of the axes of which is shown in Figure 1.1. This figure also shows the coordinate system , which characterizes the position of the satellite's orbit relative to the geocentric coordinate system, and the parameters and , which are called the inclination of the orbit and the latitude argument, respectively, characterize the position of the satellite in the orbit. To determine the angular deviations of the satellite of the basic coordinate system, a coordinate system rigidly connected to the satellite is introduced. Both systems have their origin in the center of mass of the apparatus. If the spacecraft occupies the required position in space, the axes of the bound and orbital coordinate systems coincide.

The deviation of the satellite from this position will be determined the position of the axes of the related coordinate system relative to the axes of the base coordinate system.

This position is described by three Euler-Krylov angles g , ¤ and ¨ , which are called pitch, yaw (course) and roll angles (Table 1.1). From this figure, you can make a table of guide cosines.

Table 1.1

Roll angles

	X	Y	Z
X_0	cos¤ cosg	sin¨ sin¤ cosg – cos¨ sing	cos¨ sin¤ cosg + sin¨ sing
Y_0	cos¤ sing	sin¨ sin¤ sing + cos¨ cosg	cos¨ sin¤ sing – sin¨ cosg
Z_0	– sin¤	sin¨ cos¤	cos¨ cos¤

1.2 Setting the position of the satellite in space

Satellite positioning refers to the use of satellites and associated systems to determine the precise location of objects on Earth's surface or in space. It involves utilizing signals from Global Navigation Satellite Systems (GNSS) like GPS (Global

Positioning System), GLONASS, Galileo, and BeiDou. Satellite positioning serves several important purposes and offers numerous applications. Let’s analyze some applications.

Firstly, navigation. Satellite positioning is widely used for navigational purposes by people, vehicles, ships, aircraft, military purposes and civilian life. For example, GPS provides accurate location information that helps vehicles, aircraft, and drones plan their route. This revolutionized navigation and ensured the selection of the most efficient routes.

Secondly, satellite positioning plays an equally important role in geodesy, cartography and mapping. This system allows surveyors, geospatial specialists, and the military to identify various objects on the earth’s surface, such as landmarks, property boundaries, infrastructure, natural resources, various fields, military facilities, and equipment. For example, civilians need this information for urban planning, resource management, and infrastructure development. For the military, this is the definition of the concentration of the enemy, enemy equipment, headquarters and warehouses with ammunition for successful planning of their operations.

Thirdly, satellite positioning is fundamental to geodesy, the science of measuring the shape, gravitational field, and rotation of the Earth. By accurately tracking the position of satellites and analyzing their signals received on the ground, scientists can obtain valuable information about the structure of the Earth, tectonic movements, sea level changes and other geophysical phenomena. This helps to understand the Earth’s dynamics and track changes in the environment.

Fourthly, these systems provide highly accurate timing signals that are critical for timing purposes. Many sectors of our lives depend on satellite time synchronization, namely telecommunications, power grids and scientific research, ordinary people, and large IT companies. It is this system that ensures coordination and synchronization of activities, data transmission and network operation.

Fifthly, thanks to these systems, special services can instantly respond to emergency situations or natural disasters. Satellite positioning is extremely important for search and rescue operations, coordination of response to natural disasters and assessment of the consequences of an event. This allows emergency services to locate affected areas or cities, plan evacuation routes and deploy resources efficiently. This system helps control the movement of hazardous materials and predict the spread of polluting materials in the event of an environmental accident.

Sixthly, recently satellite positioning is increasingly used in agriculture. Thanks to precise placement of agricultural equipment, monitoring of crop growth and analysis of soil conditions, farmers can apply the most efficient farming methods. This leads to efficient use and management of resources, reduced environmental impact, higher yields and better farm productivity.

Seventhly, this system is gaining great popularity among the military. Thanks to this, they can monitor the movement of enemy equipment, personnel and ammunition in real-time. Thanks to that, they plan very thorough operations to eliminate the enemy, using the most optimal weapon for a specific task. This leads to a reduction in losses on their part and effective use of ammunition.

In general, satellite positioning is critical for navigation, surveying, mapping, geophysics, time synchronization, emergency response, agriculture, and military purposes. The system provides accurate location information, ensures efficient resource management, facilitates coordination and supports a wide range of programs that contribute to various sectors of the economy and society as a whole.

Orientation is the angular position of a solid or a system of solids relative to the specified reference directions. It is customary to speak of the orientation of an aircraft, structural elements of control objects, antennas, measuring devices, etc. In this case, the basic directions can be taken as the magnetic and gravitational field lines, characteristic directions associated with the Earth’s shape, etc.

In general, the concept of orientation is associated with the rotational motion of solids. The most common way to determine the orientation in the theoretical description of rotational motion is to set the body-related orthogonal coordinate system relative to the base (calculated) coordinate system associated with the selected reference directions. Parameters that uniquely determine their relative orientation are called orientation parameters.

The translation of the coordinate system associated with a solid from the initial position to the final position (e.g., to the position at which the axes of the associated trihedron coincide with the corresponding axes of the base trihedron) can be accomplished by three or one rotation around appropriately selected axes. Accordingly, three rotations can be made in a certain sequence around any three non-collinear axes. The angles of these three rotations are called Eulerian angles and represent three independent orientation parameters.

The position of a solid body in space (the position of the body-related coordinate system OXYZ relative to the reference coordinate system $O\;{}_oY_oZ_o$) is determined by the Euler angles ψ, θ ,φ (Figure. 1.2). In the MATLAB environment, such a sequence of rotations is denoted as "ZYX" (it can be omitted in standard MATLAB functions).

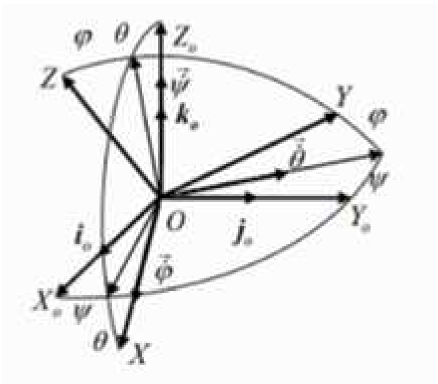


Figure 1.2 Euler angles

Matrix of direction cosines:

$$R = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \sin\varphi\sin\theta\cos\psi + \cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi & \cos\varphi\cos\theta \end{bmatrix}$$

Establishes a connection $E = R$ bases

$$I = \begin{bmatrix} i_0 \\ j_0 \\ k_0 \end{bmatrix}; E = \begin{bmatrix} i \\ j \\ k \end{bmatrix}$$

where i_0, j_0, k_0 ar i, j, k - unit vector of coordinate systems $O\;{}_oY_oZ_o$ write as:

$$r_0 = x_0 i_0 + y_0 j_0 + z_0 k_0 = [x_0 y_0 z_0] I = r_0^T I,$$

where $r_0 = [x_0 y_0 z_0]^T$.

$$r = x \ + y \ + z \ = [x \ ]I = r^T I,$$

where $r = [x\ y\ z]^T$.

there is a relationship

$$r = Rr_0 \qquad (1.1)$$

Formula 1.1 establishes a relationship between the projections of a constant vector in two coordinate systems when the axes are rotated. This is how the task of determining the orientation of the body is considered, where the vectors are stationary, for example, the gravitational acceleration vector, and the projections of these vectors on the axes of the stationary and moving coordinate systems are used to calculate the matrix of direction cosines. Another formulation of the problem is possible, when the vector rotates together with the body relative to a fixed coordinate system. This is how the problem is interpreted, for example, in the kinematics of a solid body when finding the velocity of a point according to Euler’s formula, where in a fixed coordinate system the rotation of a variable in the direction of the radius-vector, invariably bound by the body, is analyzed. The obtained conclusion can also be explained by the fact that the reason for the change in the output signal of the meter can be both the rotation of the vector it measures and the rotation of the meter itself.

1.3 Satellite motion equations based on Euler’s dynamic equations

Euler’s dynamic equations describe the rotational motion of a rigid body. The equations that describe the motion of an artificial satellite in space are usually the equations of orbital motion, which are derived from Newton’s laws of motion and the law of universal gravitation.

The movement of the satellite in orbit around the central body can be described using the following equations:Newton’s second law of motion:

$$F = m * a$$

where:

F is the gravitational force between the satellite and the central body, m is the mass of the satellite, and a is the acceleration of the satellite.

Gravitational force equation:

$$F = \frac{G * (m * M)}{r^2}$$

where:

G is the gravitational constant (approximately 6.67430 × 10^-11 m^3 kg^(-1) s^(-2)),

M is the mass of the central body, r is the distance between the satellite and the center of the central body.

Euler’s dynamic equations

In the program, the control moments are assumed to be zero. Initial angular velocities are set on the integrators.

$$\begin{aligned} I_x\dot{\mu}_x + \left(I_z - I_y\right)\mu_y\mu_z &= M_x; \\ I_y\dot{\mu}_y + \left(I_x - I_z\right)\mu_x\mu_z &= M_y; \\ I_z\dot{\mu}_z + \left(I_y - I_x\right)\mu_x\mu_y &= M_z. \end{aligned} \quad \Rightarrow \quad \begin{aligned} \mu_x &= \frac{1}{s}\frac{1}{I_x}\left[M_x - \left(I_z - I_y\right)\mu_y\mu_z\right]; \\ \mu_y &= \frac{1}{s}\frac{1}{I_y}\left[M_y - \left(I_x - I_x\right)\mu_x\mu_z\right]; \\ \mu_z &= \frac{1}{s}\frac{1}{I_z}\left[M_z - \left(I_y - I_x\right)\mu_x\mu_y\right]. \end{aligned}$$

The angles $\cdot, \bar{}, \ddot{}$ correspond to angular velocities $\dot{\cdot}, \dot{\bar{}}, \dot{\ddot{}}$ (Formula. 1.11), through which it is possible to express the projections of the angular velocity vector of a body on the axes of the coordinate system associated with it $OXYZ$ (Euler’s kinematic equations)

$$\begin{aligned} \mu_x &= \dot{\ddot{}} - \dot{\cdot}\, s\text{in}\,\bar{}; \\ \mu_y &= \dot{\bar{}}\cos\ddot{} + \dot{\cdot}\cos\bar{}\,\sin\ddot{}; \\ \mu_z &= \dot{\cdot}\cos\bar{}\,\cos\ddot{} - \dot{\bar{}}\sin\ddot{}. \end{aligned} \tag{1.11}$$

These deposits are known to be

$$\begin{aligned} \dot{\cdot} &= \frac{1}{\cos\bar{}}\left(\mu_z\cos\ddot{} + \mu_y\sin\ddot{}\right); \\ \dot{\bar{}} &= \mu_y\cos\ddot{} - \mu_z\sin\ddot{}; \\ \dot{\ddot{}} &= \mu_x + tg\,\bar{}\left(\mu_z\cos\ddot{} + \mu_y\sin\ddot{}\right). \end{aligned} \tag{1.12}$$

The orientation angles are found by integrating equations (1.12).

Using Euler's kinematic equations in quaternionic form

In matrix form, the kinematic Euler's equations (Poisson's equations) are as follows

$$\dot{\boldsymbol{R}} = -\boldsymbol{R}\,\mathrm{t}_E \qquad (5)$$

where

$$\mathrm{t}_E = \begin{bmatrix} 0 & -\mu_z & \mu_y \\ \mu_z & 0 & -\mu_x \\ -\mu_y & \mu_x & 0 \end{bmatrix}$$

The Simulink model for solving equation (5) is shown in Figure 1.3.

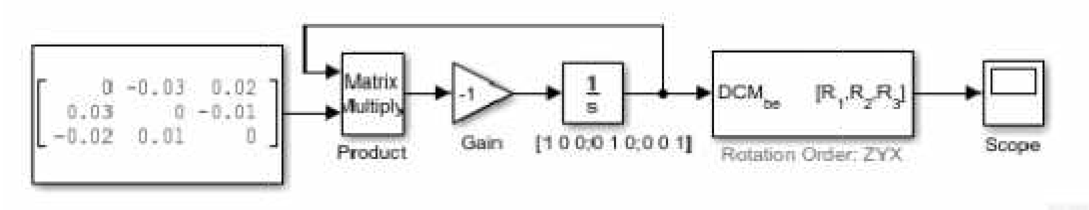


Figure 1.3 Simulink model

1.4 Structure of the satellite control system

The satellite control system, also known as the ground control system or ground segment, is a crucial component of satellite operations. It consists of various subsystems and components that are responsible for monitoring, controlling, and managing the satellite throughout its mission. The primary functions of the satellite control system include satellite tracking, telemetry data reception, command transmission, orbit determination, and overall mission planning.

The structure of a satellite control system can vary depending on the specific mission requirements and the complexity of the satellite itself.

The main modes of operation of the angular motion control system of the spacecraft are orientation and stabilization modes.

1. **Orientation** is the process by which the spacecraft takes a certain position or sequence of certain positions in space. As a rule, the orientation system, eliminating a large initial deviation, combines the coordinate system associated with the spacecraft with the reference (base) coordinate system; the latter is set on

board the spacecraft with the help of special devices and instruments and can be moved in inertial space or be stationary.

2. **Stabilization** is the process of eliminating angular deviations of the related coordinate system of the spacecraft, which inevitably occur in flight, from the reference coordinate system. The stabilization system gives the aircraft the ability, after a certain orientation in space, to restore its original position, disturbed by internal or external disturbing influences, or to resist the effects of disturbances.

There are also many systems for monitoring satellites, here are some of them:

1. Satellite Operations Center (SOC): The Satellite Operations Center serves as the command center for satellite operations. It is equipped with the necessary hardware, software, and communication systems to monitor and control the satellite. The SOC is staffed by operators, engineers, and mission controllers responsible for executing satellite commands, analyzing telemetry data, and ensuring the satellite's health, safety, and mission objectives.
2. Ground Control Station (GCS): The GCS serves as the primary control center for the satellite. It is usually located on Earth and is responsible for monitoring and controlling the satellite's operations. The GCS communicates with the satellite through various means, such as radio frequency (RF) links or data networks.
3. Telemetry, Tracking, and Command (TT&C) Subsystem: The TT&C subsystem handles the communication between the satellite and the ground control system. It consists of various components:
   - Antennas: Ground-based antennas are used to transmit commands to the satellite and receive telemetry data. Different types of antennas, such as parabolic or phased array antennas, are employed depending on the specific mission requirements.
   - Transceivers: Transceivers act as the communication interface between the satellite and the ground station. They transmit commands from the

ground control system to the satellite and receive telemetry data from the satellite.

- ♦ Telemetry Receivers: Telemetry receivers receive and demodulate the telemetry signals sent by the satellite. These signals contain vital information about the satellite's health, performance, attitude, orbit, and other operational parameters.
- ♦ Command Transmitters: Command transmitters encode and transmit commands from the ground control system to the satellite. These commands instruct the satellite to perform specific actions, such as adjusting its orbit, activating instruments, or changing operational modes.

4. Onboard Computer System: The satellite is equipped with an onboard computer system that manages various functions and subsystems of the satellite. It executes commands received from the GCS, controls the satellite's attitude and orbit, handles data storage and processing, and manages power distribution and subsystem interactions.
5. Attitude Determination and Control System (ADCS): The ADCS is responsible for determining and controlling the satellite's attitude (orientation) in space. It uses various sensors, such as sun sensors, star trackers, and gyroscopes, to measure the satellite's orientation relative to the Earth, the Sun, and other reference points. The ADCS then adjusts the satellite's attitude using reaction wheels, magnetic torquers, or thrusters to maintain the desired orientation.
6. Power and Thermal Control System: Satellites require a power supply to operate their systems and subsystems. The power and thermal control system manages the generation, storage, and distribution of electrical power onboard the satellite. It also regulates the satellite's temperature to ensure that its components remain within acceptable operating limits.
7. Payload Control Systems: Depending on the mission of the satellite, it may have one or more payload control systems. These systems are responsible for operating

and controlling the specific payload instruments or experiments carried by the satellite. Examples of payloads include cameras, spectrometers, communication antennas, or scientific instruments.

8. Data Handling and Storage: Satellites generate a significant amount of data that needs to be handled, processed, and stored. The data handling and storage subsystem manages the storage of telemetry data, payload data, and other mission-related information. It may include data compression algorithms, error correction codes, and data formatting mechanisms to optimize data transmission and storage.
9. Redundancy and Fault Management: Satellite control systems often incorporate redundancy to ensure reliability and fault tolerance. Redundant components, such as backup power supplies, processors, or communication links, are included to mitigate the impact of failures or malfunctions. Fault management systems are responsible for detecting anomalies, diagnosing problems, and implementing corrective actions to maintain the satellite's operational capabilities.
10. Mission Planning System: The mission planning system enables the creation and optimization of satellite mission plans. It takes into account mission objectives, operational constraints, and various factors such as satellite visibility, orbital dynamics, power limitations, and communication windows. The system generates schedules for data acquisition, instrument operations, orbit maneuvers, and other mission activities.
11. Data Processing and Analysis: The satellite control system includes data processing and analysis capabilities to interpret telemetry data received from the satellite. This involves decoding and analyzing the telemetry parameters to monitor the satellite's health, performance, and status. Algorithms and software tools are used for data processing, anomaly detection, trend analysis, and performance assessment.
12. Orbit Determination and Control: The orbit determination and control subsystem is responsible for accurately determining the satellite's orbit and making

necessary adjustments to maintain the desired orbit. This involves processing telemetry data, performing orbit calculations, predicting future orbits, and planning and executing orbit maneuvers. Precise orbit determination is crucial for satellite positioning, tracking, and ensuring mission objectives are met.

The satellite control system is used for overall satellite management, including satellite deployment, commissioning, routine operations, and end-of-life disposal. It provides the means to communicate with the satellite, collect vital telemetry data, monitor its health, and execute commands for orbit maintenance, payload operations, and other mission activities. The system enables continuous monitoring and control of the satellite to ensure its operational efficiency, longevity, and mission success.

1.5 Mathematical model of the Earth's magnetic field

Four models of the geomagnetic field will be considered in this paper: IGRF, oblique and straight dipole, and the average model. The geomagnetic induction vector will be defined in several coordinate systems, which are commonly used when considering the angular motion of a satellite.

$O_a Y_1 Y_2 Y_3$- inertial system, , where O_a– center of mass of the Earth, axis $O_a Y_3$ is directed along the Earth's rotation axis, $O_a Y_1$ - lies in the plane of the Earth's equator and is directed to the ascending node of the satellite's orbit (we assume a circular orbit), $O_a Y_2$ - complements the system to the right.

$O_a J_1 J_2 J_3$- inertial system J2000, axis $O_a J_3$ directed along the axis of the Earth's rotation, $O_a J_1$ lies in the plane of the Earth's equator and is directed to the vernal equinox of the era 2000.0 , $O_a J_2$ complements the system to the right. Migration from the system $O_a J_1 J_2 J_3$ to the system $O_a Y_1 Y_2 Y_3$ is set by rotation to Greenwich Mean Time t_g about the $O_a J_3$ axis, if the precession of the Earth's rotation axis is not taken into account.

$O_a Z_1 Z_2 Z_3$ is an inertial system obtained from the system $O_a Y_1 Y_2 Y_3$ by rotating it by some angle θ around the axis $O_a Y_1$. The value of this angle will be determined using a median model.

$O_aS_1S_2S_3$ - system associated with the position of the satellite's orbit in inertial space. Axis $O_aS_3$is directed along the normal to the plane orbit, $O_aS_1$is directed to the ascending orbital node, O_aS_2 complements the system to the right. Transition between systems $O_uY_1Y_2Y_3$ and $O_uS_1S_2S_3$is set by rotating the angle i (orbital inclination) with respect to the axis O_uY_1, but between $O_uS_1S_2S_3$and $O_uZ_1Z_2Z_3$ on angle θ - i along the axis O_uS_1. The inertial systems coordinate systems are shown in Figure 1.4

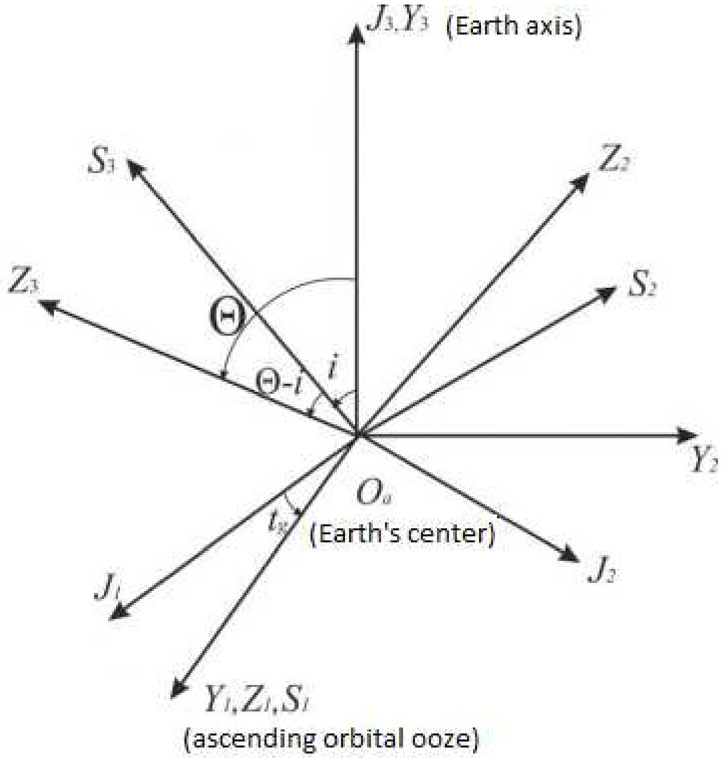


Figure 1.4 Inertial coordinate system

$OX_1X_2X_3$ - orbital coordinate system centered at the center of mass of the satellite, axis OX_1 lies in the plane of the orbit and is normal to the radius-vector in the direction of the satellite's motion, making an acute angle with the velocity vector of its center of mass (in a circular orbit, the axis direction coincides with the direction of the satellite's forward motion), axis OX_2 is directed along the radius-vector of the satellite's center of mass, OX_3 supplements system to the right.

$OL_1L_2L_3$ - system associated with the kinetic momentum of the satellite. Axis OL_3 is directed along the vector of the satellite's kinetic momentum, OL_2 - perpendicular to OL_3 and lies in the plane parallel to the plane of the first two axes of the inertial system relative to which the motion of and passing through O, OL_1 completes the system to the right.

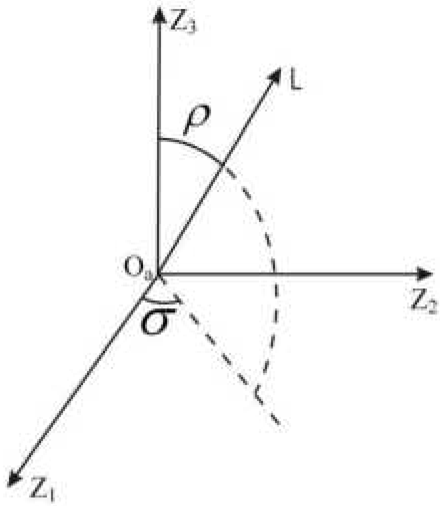


Figure 1.5 Orientation of kinetic momentum in inertial space

The International Geomagnetic Reference Field (IGRF) is a standard mathematical description of the large-scale structure of the Earth's main magnetic field and its secular variation. It was created by fitting parameters of a mathematical model of the magnetic field to measured magnetic field data from surveys, observatories and satellites across the globe.

The World Magnetic Model (WMM) is a large spatial-scale representation of the Earth's magnetic field. It was developed jointly by the US National Geophysical Data Center and the British Geological Survey. The data and updates are issued by the US National Geospatial-Intelligence Agency and the UK Defence Geographic Centre.

1.6 Conclusion

In this chapter, we have considered various aspects of satellites. Their mathematical model, coordinate systems, determination of position in space, equations describing the motion of the satellite based on Euler's dynamic equations, the structure of satellite control systems, and a mathematical model of the Earth's magnetic field. Using this knowledge and applying the concepts, researchers, engineers and enthusiasts can gain valuable knowledge that lays the foundation for further development of this technology and space exploration.

CHAPTER 2. Study of the movement of the satellite in the mode of initial stabilization

2.1 Analysis of the movement of the satellite as a solid body at the initial stage of movement

Analyzing the motion of a satellite as a rigid body at its initial stage of motion is important for several reasons.

Firstly, the stability assessment. By treating the satellite as a solid body, engineers can estimate the stability of its motion, analyze the satellite's mass distribution, moments of inertia, and the effects of external forces such as gravity, atmospheric drag, and solar radiation pressure. Thanks to this analysis, we can be sure that the initial movement of the satellite is stable and does not lead to unwanted oscillations or overturning.

Secondly, it allows you to determine the position related to its orientation in space. By analyzing the motion as a whole, it is possible to determine the initial position of the satellite and understand how it will change over time due to external torques or control maneuvers. Position determination is of high importance for various satellite operations, instrument guidance, communication support, and stabilization of imaging systems.

Third, it is the analysis of the satellite as a solid body that helps to design the control system. The control system itself may use various mechanisms to stabilize and control the satellite, such as engines, jet wheels, or other mechanisms. By understanding the initial motion of the satellite, it is possible to develop an effective algorithm and mechanism to control and counter external interference and maintain the desired motion characteristics.

Fourthly, by analyzing, we can optimize existing algorithms and mechanisms for efficient use of resources or to reduce the size or price of the satellite itself. By considering satellite mass distribution and moments of inertia, the satellite can be designed to achieve desired characteristics such as stability, maneuverability, and efficient use of power resources. The analysis helps determine the optimal spacecraft configuration and control strategy.

Variants of satellite analysis at the initial stage of movement include the following procedures:

1. Mathematical modeling. It is necessary to develop mathematical models that describe the motion of the satellite as a solid body, using the principles of mechanics, dynamics and control theory. The models must take into account the properties of the satellite’s mass, external forces and moments, allowing it to simulate and analyze its motion.

2. Physical tests. A model or prototype of a satellite is specially created for such tests. We need to create the closest conditions to real ones and observe the satellite’s behavior. Conduct analysis of motion and damping of initial angular velocity. This option is quite expensive because if the test is unsuccessful, the layout may break, and it will be necessary to repair or create a new one.

3. Computational modeling. Computer simulation can be used to simulate the movement of the satellite. Using special software, we can enter the satellite’s initial conditions, mass properties, and external forces and observe the movement in real-time. Simulations provide insight into the satellite’s behavior, stability, and response to various scenarios. Using this approach, we save money that would have been spent on a physical test.

In general, analyzing the motion of a satellite as a rigid body at the initial stage helps engineers assess stability, determine the attitude, design control systems, optimize performance, and ensure mission success. This provides a basis for understanding and controlling the movement of the satellite throughout its lifetime.

Analysis of the motion of the satellite as a solid body at the initial stage includes consideration of such factors as initial velocity, initial orientation, moments of inertia, and conservation laws.

1. **Initial speed and orientation**: at the initial stage of the satellite’s movement, it is important to consider its initial speed and orientation. The initial velocity can be determined by launching the satellite using a rocket or other launch vehicle.

Orientation is determined by the position and orientation of the satellite relative to the Earth or another celestial body.

2. **Moments of inertia**: the satellite is considered a rigid body, so the moments of inertia play a crucial role in its motion. The moment of inertia relative to the axis of rotation depends on the mass of the body and the distribution of mass around this axis. Knowing the moments of inertia helps to study the motion of the satellite.
3. **Conservation Laws**: At the initial stage of satellite motion, certain conservation laws, such as the conservation of kinetic energy and conservation of angular momentum, can be applied for analysis. Conservation of kinetic energy states that the sum of kinetic and potential energy remains constant in the absence of external forces. Conservation of angular momentum states that the sum of angular momentum remains constant if there are no external torques.

2.2 Mathematical model of a satellite with a control system

A mathematical model of a satellite with a control system is essential.

First of all, a mathematical model allows engineers and scientists to design and analyze the behavior of a satellite system before it is built and launched. By simulating the satellite dynamics and control system in a mathematical model, they can evaluate its performance, stability, and response to various inputs and disturbances. This helps to optimize the design, identify potential problems, and ensure that the satellite will fulfill its mission.

Secondly, satellites often require sophisticated control systems to maintain the desired orbit, attitude (orientation) and stability. A mathematical model allows engineers to develop and tune control algorithms to regulate the satellite's behavior. By understanding the dynamics of the system and using the principles of control theory, they can design controllers that provide accurate and reliable control, compensating for disturbances and maintaining the desired state of the satellite.

Third, the mathematical model allows us to predict the future behavior of the satellite system under various conditions. By simulating the model, we can predict the satellite’s trajectory, changes in its position, and other important parameters. This prediction capability is crucial for planning satellite maneuvers, coordinating operations, and optimizing mission objectives.

Fourthly, mathematical models of satellites are used to train and test the performance of control systems and algorithms. By implementing control algorithms in a simulation and evaluating their behavior in various scenarios, engineers can fine-tune control parameters and ensure system reliability and safety before implementing them on a real satellite.

Fifth, the mathematical model can serve as a benchmark for detecting anomalies and diagnosing faults in the satellite system. By comparing real-time measurements from the satellite with model predictions, discrepancies can be identified that indicate potential malfunctions or deviations from expected behavior. This allows operators to take corrective action and solve problems quickly.

After separation from the launch vehicle, the satellite may have significant angular velocities that need to be dampened for the satellite to continue functioning. This movement is called "detumbling," which means "tumbling."

To generate control torques, we will generate control signals proportional to the satellite’s angular velocities.

Let’s consider two options for building a control scheme.

In the first option (detumbling control), we will use three angular velocity sensors, which involves installing them in the satellite.

In the second variant (B-dot control), these signals are formed as derivatives of the three output signals of the magnetometer, which implies the installation of the magnetometer in the satellite.

In both cases, we will use a magnetic satellite control system.

Let’s write the dynamic Euler equations of satellite motion with control moments

$$M_x = -k_x \mu_x;\ M_y = -k_y \mu_y;\ M_z = -k_z \mu_z \qquad (1)$$

where $k_x,\ k_y,\ k_z$ - coefficients.

$$I_x\dot{\mu}_x + (I_z - I_y)\mu_y\mu_z = -k_x\mu_x;$$
$$I_y\dot{\mu}_y + (I_x - I_z)\mu_z\mu_x = -k_y\mu_y; \qquad (2)$$
$$I_z\dot{\mu}_z + (I_y - I_x)\mu_y\mu_x = -k_z\mu_z$$

For a symmetric satellite, the feasibility of such control is obvious since system (2) decomposes into three independent equations

$$I\dot{\mu}_x + k_x\mu_x = 0;$$
$$I\dot{\mu}_y + k_y\mu_y = 0; \qquad (3)$$
$$I\dot{\mu}_z + k_z\mu_z = 0$$

with solutions

$$\mu_x = \mu_{xo}\left(1 - e^{-\frac{k_x}{I}t}\right);\ \mu_y = \mu_{yo}\left(1 - e^{-\frac{k_y}{I}t}\right);\ \mu_z = \mu_{zo}\left(1 - e^{-\frac{k_z}{I}t}\right), \qquad (4)$$

where μ_{xo}, μ_{yo}, μ_{zo} - the initial angular velocities.

Let's show the effectiveness of such control in the general case.

Let's multiply the first equation by μ_x, the second equation by μ_y, the third equation by μ_z, and add the resulting expressions. We have

$$\frac{d}{dt}\left(\frac{I_x\mu_x^2 + I_y\mu_y^2 + I_z\mu_z^2}{2}\right) = -\left(k_x\mu_x^2 + k_y\mu_y^2 + k_z\mu_z^2\right) \qquad (5)$$

That is, the kinetic energy of the satellite $T = \frac{I_x\mu_x^2 + I_y\mu_y^2 + I_z\mu_z^2}{2}$ decreases

$$\frac{dT}{dt} < 0$$

Thus, controlling the satellite according to the laws of (1) ensures that the satellite is stabilized in space.

A more specific result occurs when

$$\frac{I_y}{I_x} = \frac{k_y}{k_x} = a_1;\ \frac{I_z}{I_x} = \frac{k_z}{k_x} = a_2 \qquad (6)$$

In this case, expression (5) can be written as follows

$$\frac{I_x}{2}\frac{d}{dt}\left(\mu_x^2 + a_1\mu_y^2 + a_2\mu_z^2\right) + k_x\left(\mu_x^2 + a_1\mu_y^2 + a_2\mu_z^2\right) = 0$$

That is,

$$\dot{\Omega} + 2\frac{k_x}{I_x}\Omega = 0$$

where $\Omega = \mu_x^2 + a_1\mu_y^2 + a_2\mu_z^2$.

The solution to this equation is as follows

$$\Omega = \Omega_o e^{-2\frac{k_x}{I_x}t}$$

That is, $\Omega \to 0$, which is possible in the case of $\mu_x \to 0,\ \mu_y \to 0,\ \mu_z \to 0$. That is, the solution of the satellite motion stabilization problem is guaranteed.

The nonlinear system (1) does not have a general solution, so it is advisable to consider some special cases.

Let us first assume an uncontrolled system for $I_x = I_z$. We have a system of linear equations

$$I_x\dot{\mu}_x + n(I_z - I_y)\mu_z = 0;$$
$$I_z\dot{\mu}_z + n(I_y - I_x)\mu_x = 0,$$

where $n_o = \mu_{yo} = const.$

Let's write the characteristic equation of this system

$$\Delta = \begin{vmatrix} I_x s & n(I_z - I_y) \\ n(I_y - I_x) & I_z s \end{vmatrix} = I_x I_z s^2 + n^2(I_z - I_y)(I_x - I_y) = 0$$

which is determined by n That is, there are non-damped oscillations of angular velocities μ_x, μ_z with a frequency initial angular speed of rotation μ_{yo}

$$r = n_o\sqrt{\frac{(I_z - I_y)(I_x - I_y)}{I_x I_z}}$$

For the stability of these oscillations, the moment of inertia I_y must be either minimal $(I_y < I_z,\ I_y < I_x)$ or maximal ($I_y > I_z,\ I_y > I_x$).

Let's take into account the presence of the component $-k_y\mu_y$. Then from the $I_y\dot{\mu}_y + k_y\mu_y = 0$ equation we obtain the solution $n = \mu_y = \mu_{yo}e^{-\frac{k_y}{I_y}t}$. If the frequency n

changes little during the oscillation period, then we can assume that there are undamped oscillations of angular velocities μ_x, μ_z with a frequency decreasing with time.

One of the effective methods of approximate analysis of nonlinear systems is the method of successive approximations, according to which (the main content of the method) the components that are smaller than the ones that are retained are discarded. Let us use this approach.

Let us write system (2) in the form

$$
\begin{aligned}
&I_x\dot{\mu}_x + k_x\mu_x + (I_z - I_y)\mu_y\mu_z = 0;\\
&I_y\dot{\mu}_y + k_y\mu_y + (I_x - I_z)\mu_z\mu_x = 0;\\
&I_z\dot{\mu}_z + k_z\mu_z + (I_y - I_x)\mu_y\mu_x = 0
\end{aligned}
$$

If we assume that the maximum values of angular velocities are less than one, we will discard the components with products of angular velocities in each equation. Then we have a system of linear equations

$$
\begin{aligned}
&I_x\dot{\mu}_x + k_x\mu_x = 0;\\
&I_y\dot{\mu}_y + k_y\mu_y = 0;\\
&I_z\dot{\mu}_z + k_z\mu_z = 0
\end{aligned}
$$

with solutions

$$\mu_x = \mu_{xo}\left(1 - e^{-\frac{k_x}{I_x}t}\right);\ \mu_y = \mu_{yo}\left(1 - e^{-\frac{k_y}{I_y}t}\right);\ \mu_z = \mu_{zo}\left(1 - e^{-\frac{k_z}{I_z}t}\right),$$

These expressions can be used as an approximate estimate of the problem solutions.

It is also advisable to use them for preliminary selection of the controller coefficients $k_x,\ k_y,\ k_z$. Since the transient time ^{2} for the aperiodic link $I\dot{\mu} + k\mu = 0$ is approximately equal to $^2 = 3T$, where $T = \frac{I}{k}$, then, given the transient time for each equation, we can choose the time constants $T_x,\ T_y,\ T_z$, and then find the coefficients $k_x,\ k_y,\ k_z$.

If the dependencies (7) are fulfilled, the equations of motion take the form

$$I_x\dot{\mu}_x + k_x\mu_x + (\mathrm{a}_2 - \mathrm{a}_1)I_x\mu_y\mu_z = 0;$$
$$\mathrm{a}_1 I_x\dot{\mu}_y + \mathrm{a}_1 k_x\mu_y + (1-\mathrm{a}_2)I_x\mu_z\mu_x = 0;$$
$$\mathrm{a}_2 I_x\dot{\mu}_z + \mathrm{a}_2 k_x\mu_z + (\mathrm{a}_1 - 1)I_x\mu_y\mu_x = 0$$

or

$$\dot{\mu}_x + {\circ}\,\mu_x = -(\mathrm{a}_2 - \mathrm{a}_1)\mu_y\mu_z;$$
$$\dot{\mu}_y + {\circ}\,\mu_y = -\frac{1-\mathrm{a}_2}{\mathrm{a}_1}\mu_z\mu_x;$$
$$\dot{\mu}_z + {\circ}\,\mu_z = -\frac{\mathrm{a}_1 - 1}{\mathrm{a}_2}\mu_y\mu_x,$$

where ${\circ} = \dfrac{k_x}{I_x}$.

That is, all linearized equations are the same.

Mathematical model of a system with magnetic coils

Let us obtain a mathematical model of a detumbling-controlled system in which magnetic coils are used as torque sensors.

The equation of motion of the satellite is written in the form

$$I_x\dot{\mu}_x + (I_z - I_y)\mu_y\mu_z = M_x;$$
$$I_y\dot{\mu}_y + (I_x - I_z)\mu_z\mu_x = M_y;$$
$$I_z\dot{\mu}_z + (I_y - I_x)\mu_y\mu_x = M_z.$$

For the dipole model of the Earth's magnetic field, the vector of induction of the Earth's magnetic field in the orbital coordinate system is

$$\boldsymbol{B} = \begin{bmatrix} B_{x_0} & B_{y_0} & B_{z_0} \end{bmatrix}^T = h\boldsymbol{a},$$

where $h = \dfrac{«_e}{r^3}$, $\mu_e = 7{,}812 \cdot 10^6 k\ \ ^3 \cdot k\ \ \cdot s^{-2} \cdot A^{-1}$- the earth's magnetism; $r = 7000$ km - distance from the center of the Earth to the satellite $\boldsymbol{a} = \begin{bmatrix}\cos u \sin i & \cos i & -2\sin u \sin i\end{bmatrix}^T$; u - the breadth argument; $i = 98^0$ - orbit inclination angle.

The magnetic moment of the coils will be formed in the form

$$\boldsymbol{m} = k\frac{\mu \times \_b}{\|\_b\|^2}$$

where k - coefficient; μ - vector of measured angular velocities of the satellite; $\_{b} = \boldsymbol{RB}$ - the vector of induction of the Earth's magnetic field in the coordinate system associated with the satellite; $\boldsymbol{R}$ - is a matrix of guide cosines for the transition from the orbital coordinate system to the coordinate system associated with the satellite.

The matrix of guide cosines $\boldsymbol{R}$ is sought as a solution to Poisson's equation

$$\dot{\boldsymbol{R}} = -\boldsymbol{R}\,t$$

where

$$t \;\; Z \begin{bmatrix} 0 & -\mu_z & \mu_y \\ \mu_z & 0 & -\mu_x \\ -\mu_y & \mu_x & 0 \end{bmatrix}.$$

The torque flown by the coils is equal to

$$\boldsymbol{M} = \boldsymbol{m} \times \_{b}$$

2.3 Theoretical analysis of satellite motion in the initial stabilization mode

There are various methods of stabilizing the position in space, some passive and determined by the physical characteristics of the satellite, and some active, such as stabilization by magnetic coils and torque flywheel.

Passive stabilization methods

The passive methods chosen for the calculation are: gravity gradient, rotational stabilization, and passive magnets.

A common property of passive stabilization is that the satellite can be stabilized in only two axes.

Gravity gradient uses the Earth's gravity to stabilize the satellite. A feature of this type of stabilization is the location of the satellite's center of gravity.

Stabilization by satellite rotation is constant when the satellite rotates around its axis with a large moment of inertia. A passive magnet can stabilize the satellite due to the Earth's magnetic fields, and the satellite's motion will follow the magnetic fields.

Depending on the location of the center of gravity, the gravity gradient may not have a stabilizing effect. For example, for CubeSat satellites (small satellites), the center of gravity should be no further than 2 cm from the center. This can be used to distribute the moments of inertia in a way that makes the gravity gradient a stabilizing factor.

Rotational stabilization depends on the moments of inertia. Ideally, only the tensor of the moments of inertia should have a value different from zero and this should contribute to a more significant difference from each other. This will make it easier to control the satellite, since the rotational moment (torque) applied along one axis will not affect the other. In the design, the axis of rotation is defined, and the mass should be distributed mainly on the other two axes. Any other arrangement will have a disturbing rather than a stabilizing effect.

A passive magnet would align the satellite with the Earth's magnetic field, but this is undesirable for most performance purposes, as the satellite will eventually become a point with sufficient poleward accuracy. Moreover, this will make it much tougher to use a magnetometer to determine the spatial position, since the magnetic fields of the passive magnets are much stronger than the Earth's magnetic field and thus can saturate magnetic sensors.

The effect of rotational stabilization is preferable to use, as it can provide passive stability along the axis where the camera is placed. Usually, a rotational speed of 30-90 rpm is used to stabilize the satellite, but in order not to distort the image, the rotational speed can be as low as 2-3 rpm, and there is practically no stabilizing effect.

Active stabilization methods

Active methods take into account stabilization with magnetic coils and a torque flywheel. Active stabilization makes it possible to control three axes. Each actuator (transducer) is effective on only one axis, so 3 independent transducers are needed for 3-axis control. In most cases, 3 types of transducers are used: magnetic torque coils (magnetorquers), pulse (amount of motion) rotations, and reactive control system motor (jack, pusher).

A magnetic torque sensor is a coil of wire used to induce an electromagnetic field. This field interacts with the Earth's magnetic field to produce torque, allowing the satellite to be stabilized and oriented.

The torques produced are greatest when the magnetic torque sensor and the Earth's magnetic field are parallel, and they have no effect when they are perpendicular.

Torque flywheel (amount of rotation motion)

The torque flywheel has unnecessary characteristics that do not carry information regarding the magnetic field that needs to be controlled.

The stabilizing effect of the torque flywheel is similar to passive rotation stabilization, however, it is possible to control the torque by the amount of rotation. The torque value is limited, so it determines the need for reactive control motors (thruster) or a torque sensor for compensating torque.

Double (coupled) rotation

Dual rotation stabilization is a modification of the passive rotation stabilization mentioned above. In coupled (double) rotation, only part of the satellite rotates, as opposed to passive rotation stabilization, where the whole satellite is rotating. Dual rotation can be performed by either of the two parts of the satellite and can support the rotation of the stationary payload platform or set the rotation momentum along an axis that is stabilizing.

Basic principles of magnetic stabilization

As a result of the interaction between the magnetic fields of the planet and the spacecraft, an external moment arises, which is used to control the angular position of the satellite. Both active and passive control systems can be used. In active systems, the actuating elements are electromagnets, and in passive systems - permanent magnets.

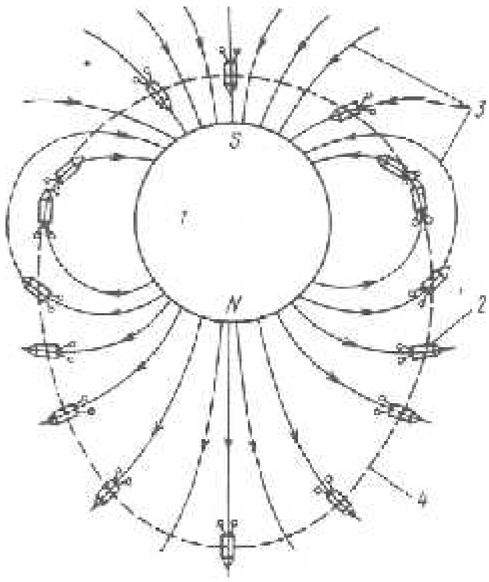


Figure 2.1 Satellite orientation along magnetic field lines.

Passive stabilization of the satellite by the vector of the magnetic field strength of the planet is very desirable for a number of scientific experiments. With the passive method of control, a permanent magnet is rigidly attached to the body of the satellite along the axis of symmetry. An artificial satellite with passive magnetic stabilization is always oriented along magnetic field lines, so their magnetic dipole agrees with the local direction of the magnetic field lines of the planet.

As the distance from the Earth's surface increases, its magnetic field becomes much weaker (in proportion to the cube of the distance from the Earth's center). In first approximation, it can be approximated quite accurately by the magnetic field of a dipole whose axis passes through the center of the Earth and is deflected from its axis of rotation to the plane of the Earth's equator by a small constant angle. When obtaining qualitative estimates, it can be assumed that the dipole axis and the Earth's rotation axis practically coincide. This assumption is accurate enough to consider the passive magnetic stabilization system of the satellite.

The force lines of such a dipole have the shape shown in Figure 2.1. If the Earth's axis 1 lies in the plane of orbit 4, then satellite 2, tracing the direction of the magnetic field force lines 3, makes two complete rotations around its axis in one revolution.

The moment arising from the interaction between the Earth's magnetic field and a magnetic rod that has a magnetic dipole moment $\vec{L}$, is defined by the equation

$\vec{M} = \vec{L} \times \vec{H}$, where $\vec{H}$ - the strength of the Earth's magnetic field. The modulus of the moment vector is $M = LH \sin \pm$, where $\pm$ - angle between the axis of the magnetic rod and the magnetic field vector of the Earth. The magnetic dipole moment of a ferromagnetic rod is equal to the product of the induction and the volume of the rod: *V:* $L = BV$. If the rod is a permanent magnet, its induction is approximately constant B_0 and the momentum arising from the interaction between the permanent magnet and the magnetic field of the Earth is $M = HVB_0 \sin \pm$. This moment is used as a restoring moment for passive stabilization of the satellite position. The permanent magnet tends to align its axis with the local direction of the Earth's magnetic field strength.

When tracking the symmetry axis of the satellite, the angular velocity of the Earth's magnetic field strength vector does not remain constant. Therefore, if it is necessary to orient the satellite precisely along the force lines and to have satisfactory transients, it is necessary to calculate the passive magnetic stabilization system taking into account not only the moments arising due to deflection from the force lines, but also the moments that depend on deflection derivatives, i.e. damping moments due to the nonuniformity of Earth magnetic field strength vector movement. The last fact is very important because satellites have practically no natural damping. Damping can be obtained by using hysteresis energy losses in special ferromagnetic rods.

In passive magnetic stabilization systems, the damping of angular oscillations of the satellite is carried out mainly by using hysteresis re-magnetization in rods made of special magnetic materials with high magnetic permeability.

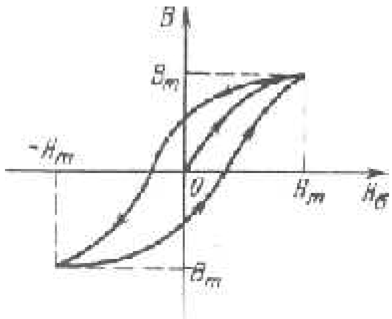


Figure 2.2 Approximation of the hysteresis loop.

Their action is based on the fact that the satellite oscillations are reduced as a result of energy losses on hysteresis. The energy loss is proportional to the area inside the closed hysteresis magnetization curve $B = f(H)$ (Figure 2.2). Since the hysteresis characteristic is ambiguous, it is difficult to write down an analytical expression for the exact time dependence of damped oscillations. The presence of hysteresis damping in combination with damping due to eddy currents was confirmed by tests on a number of artificial satellites of the Earth.

The hysteresis energy loss is maximal when the magnetizing force is both positive and negative at its highest values. Therefore, the vibration damping will be most effective if the rod with high magnetic permeability is oriented perpendicular to the lines of force, which also means it is perpendicular to the permanent magnet axis. The energy of the hysteresis losses depends on the value of the maximum field strength, the material, the geometric dimensions of the rods, their location relative to each other, the magnetic permeability of the rods and, finally, on the magnetic fields generated by the various devices inside the satellite. The rods should be arranged in such a way that they produce high hysteresis losses, which are maximum possible when moving in the geomagnetic field. On the other hand, they should create a minimum of magnetic disturbances inside the satellite.

Controllability of a magnetically actuated satellite

A magnetically actuated satellite has a limitation in control, since such a satellite can only be controlled on two axes simultaneously. This is because of the torque produced by the interaction between the Earth's geomagnetic field and the magnetic field of the magnetic torque sensors. The torque produced is always perpendicular to the geomagnetic field. Thus, one axis will always be parallel to the geomagnetic field, and therefore not controllable.

The geomagnetic field changes orientation when the satellite changes its position in orbit. This field interacts with the Earth's magnetic field, generating torque to reorient the satellite. The torques produced are greatest when the magnetic torque sensor and the Earth's magnetic field are parallel, and they have no effect when they are perpendicular.

In general, the instruments used on board as sensors for the attitude determination system can be divided into two classes - position sensors and inertial angular velocity sensors.

At present, it is inexpedient to use inertial sensors in their pure form for the orientation determination system of a small spacecraft. Analysis of the literature shows that in classical orientation determination systems, inertial sensors are supplemented by position sensors, making measurements every in their spectrum of angular velocity changes.

Position sensors measure the direction of the reference direction or its (direction) projection on the axes of the associated coordinate system. An example of the former is a sun sensor, an example of the latter is a magnetometer measuring three components of the magnetic induction vector. At least two independently measured vectors are required to determine the orientation of the three axes.

Main sources of position sensor errors:

- own measurement errors;
- errors of models describing the required reference direction

In this sense, the star chamber is the most accurate meter, because it uses a CCD with a sufficiently high resolution, and the position of stars is also known with a very high accuracy. The positions of the Sun and the Earth are known with worse accuracy (the accuracy of the model calculating the Sun's position is 0.006 deg).

The solar sensor includes photosensitive elements that measure one or two angles of sunlight incidence on the surface of the cells. The signal is then digitized via an ADC and output to an onboard computer.

Solar sensors are reliable and simple in construction, but require an open field of view, and in addition, they are of little use in the shade. For example, in one of the literature sources the error of determining the position of the Sun by a solar sensor is in the range of 0.005°r - 4°r, and the Czech slot sensors of the Sun, offered in their time, had an error of 0.2°r. In general the accuracy achievable with them may change with time as a function of many parameters, e.g. relative position of the Earth and the Sun, etc.

Horizon sensors are infrared sensors that record the temperature contrast between space and the Earth's atmosphere. Some nadir-targeted satellites use wide-angle sensors that cover the entire Earth in their field of view. Typical accuracy of sensor measurements is from 0.1 to 0.25 degrees Celsius, and in some cases can be increased to 0.03 degrees Celsius.

The use of GPS receiver to determine the orientation of the spacecraft gives accuracy of orientation of the order of 0.5...1 degree for spacecraft with geometric dimensions less than 1 m.

Magnetometers are simple and reliable sensors. However, their use is limited to areas where the direction of the magnetic induction vector is known with high accuracy.

Sources of errors of magnetometers are:

- field perturbations under the action of on-board electronics;
- errors of the magnetic field model;
- magnetic field perturbations associated, for example, with currents in the ionosphere.

2.4 Conclusion

Theoretically examined the motion and stabilization of the satellite, including the analysis of the behavior of the satellite as a solid, the development of mathematical models that include the control system, and the analysis of the satellite during initial stabilization. With this knowledge, we can effectively build the control system and initial stabilization for the satellite.

CHAPTER 3. Simulation of the movement of the satellite in the mode of initial stabilization

3.1 General characteristics of the computer model

Let's look at some of the advantages and disadvantages of creating computer models.

First of all, when creating such a model, we have the opportunity to simulate complex systems and phenomena, which help us understand how they work and predict their behavior in different situations. This is especially useful for engineers.

Second, conducting physical experiments or observations in the real world can be time-consuming, expensive, or even impossible in certain situations. Computer models offer a cost-effective alternative because they allow researchers to virtually simulate experiments and scenarios, reducing the need for physical resources.

Third, computer models can be easily modified and adjusted to explore different parameters, conditions, or assumptions. This flexibility allows researchers to explore a wide range of possibilities and conduct virtual experiments that would be difficult or impractical to replicate in the real world.

Fourth, modeling a system or process on a computer can be much faster than waiting for real-time events to occur. Models can provide results and insights quickly, allowing researchers to iterate, refine, and optimize their approaches more quickly.

Fifth, computer models allow researchers to assess potential risks and their consequences before implementing solutions or making important decisions. For example, in engineering, models can help identify potential flaws or weaknesses in a design before physical prototypes are created.

Despite their many advantages, computer models also have certain limitations and drawbacks.

First of all, models require simplifications and assumptions to reflect complex real-
world phenomena. These simplifications can lead to inaccuracies or limitations in the

model's predictions, potentially resulting in less accurate or incomplete results.

Second, developing accurate models often depends on high-quality data for calibration and validation. Obtaining complete and reliable data can be difficult or expensive, limiting the accuracy and scope of the model.

Third, building and using computer models can be a complex process and require knowledge of mathematics, programming, and industry-specific expertise. Adequate knowledge and experience are necessary to create reliable models and correctly interpret their results.

Fourth, ensuring the accuracy and reliability of computer models can be challenging. Models must be thoroughly tested and validated with real data and experiments to establish their reliability.

Now let's look at our computer model built with Simulink.

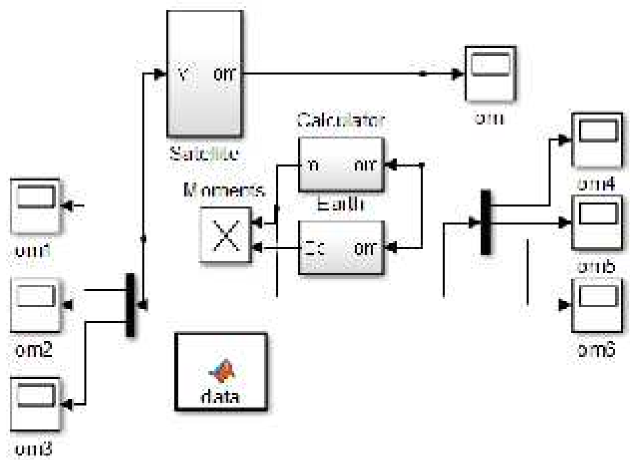


Figure 3.1 Main simulink model.

Let's analyze our computer model in more detail. We can distinguish 4 main blocks:

1. Satellite - our satellite system.
2. Calculator - the satellite control system.
3. Earth - the earth's magnetic field system.
4. data – block with initial data in our system.

After processing the data from the satellite's sensors, they are sent to the Calculator, which calculates the forces required to stabilize the satellite. After that, the data from the Calculator and Earth blocks create a torque, which is then sent to the satellite.

This system does not use a standard block but rather an Embedded MATLAB Function. When you can use this block:

- You have an existing MATLAB function that models custom functionality, or it is easy for you to create such a function.
- Your model requires custom functionality that is not or cannot be captured in the Simulink graphical language.
- You find it easier to model custom functionality by using a MATLAB function than by using a Simulink block diagram.
- The custom functionality that you want to model does not include continuous or discrete dynamic states. To model dynamic states, use S-functions.

For my model, I used it to store initial values. This is very convenient in that you do not need to constantly use two files (Simulink model and Matlab file). This system is built in such a way that for different situations you do not need to change the system itself and the values in the blocks, just change them in the data block and paste them into the Command window.

3.2 Implementation of separate blocks and systems

Let's take a closer look at the structure of the main systems.

The satellite system is shown in Figure 2. We have constant values that are set in the data block. This is all processed by the function, and we get oscillations as an output.

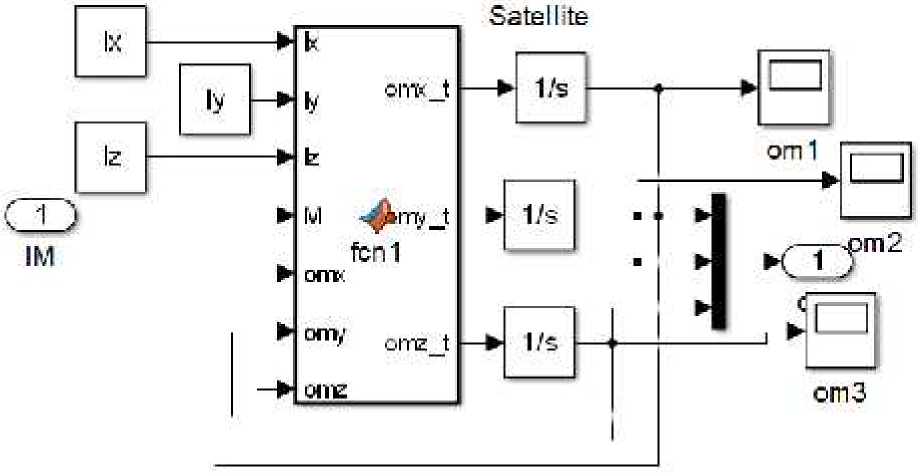


Figure 3.2 Satellite system

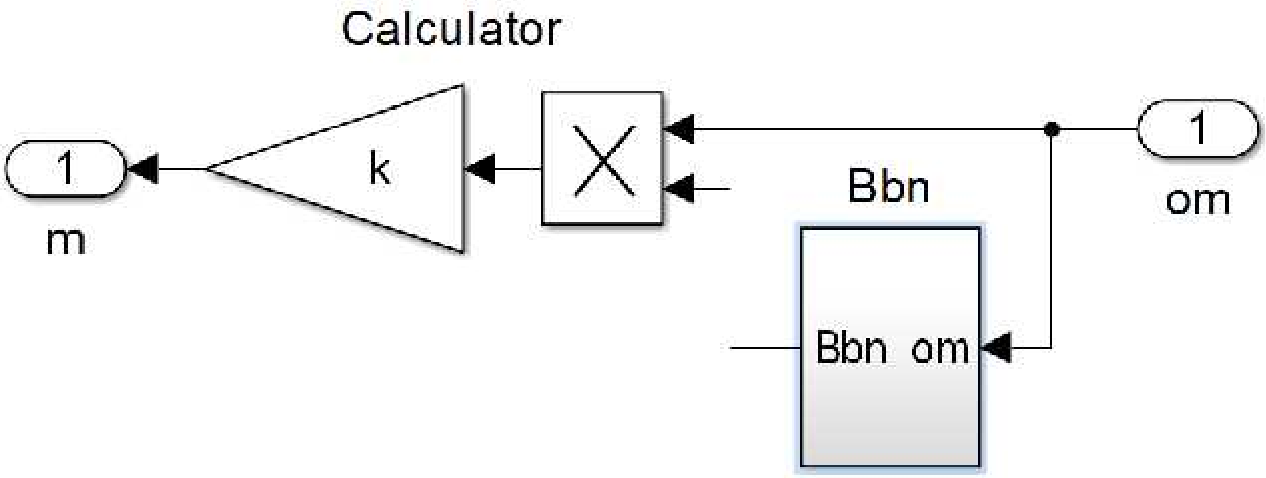


Figure 3.3 Formation of magnetic torque

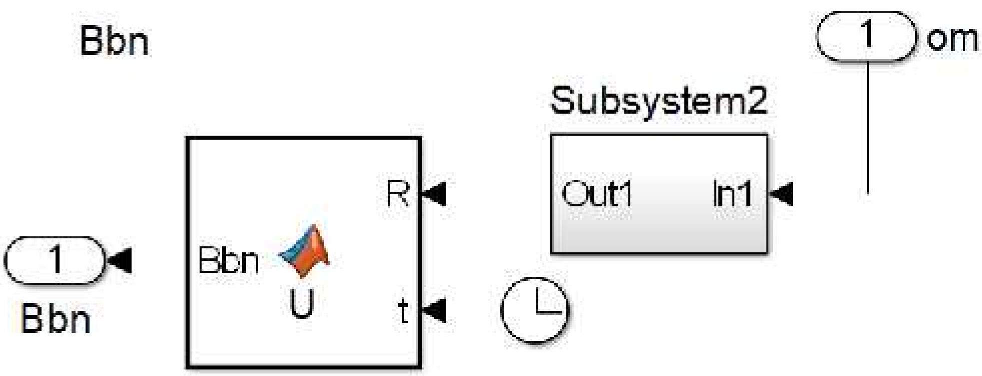


Figure 3.4 Bbn block in Calculator system

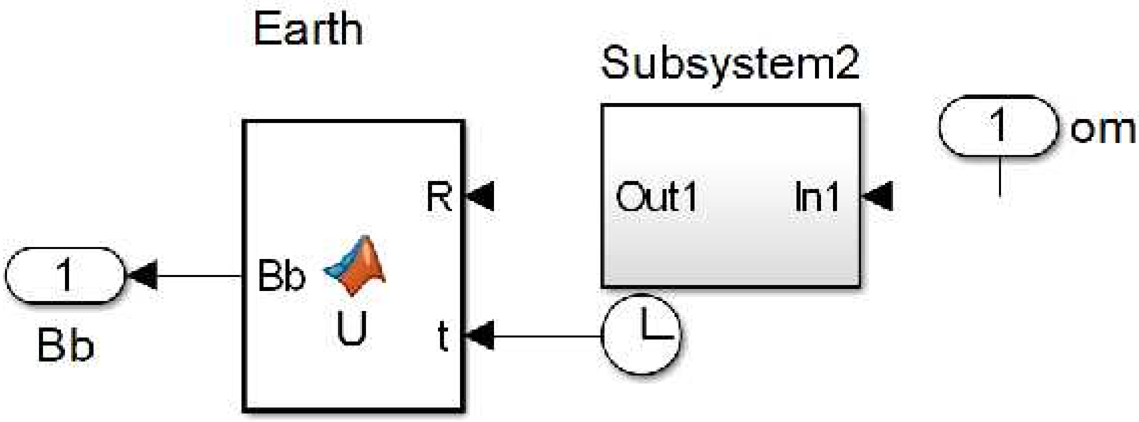


Figure 3.5 Earth system

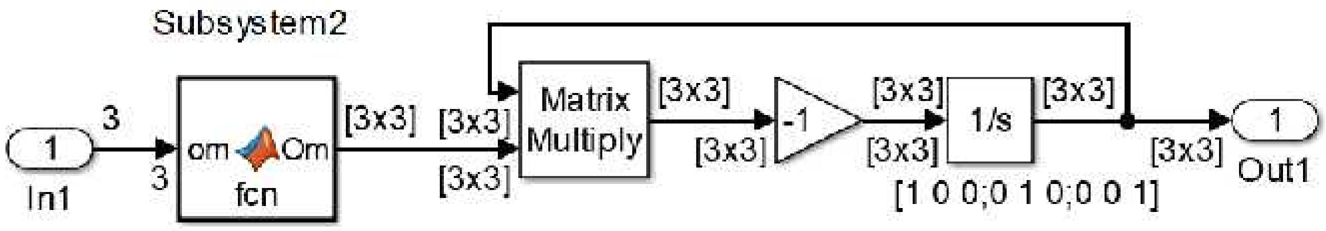


Figure 3.6 Subsystem2 block in Calculator and Earth systems

EmbeddedMATLAB Function2

```
function data

Ix=0.4; Iy=0.7; Iz=0.3;
k=2e-4;
omn=0.1;
```

Figure 3.7 data block

3.3 Simulation of the movement of the satellite

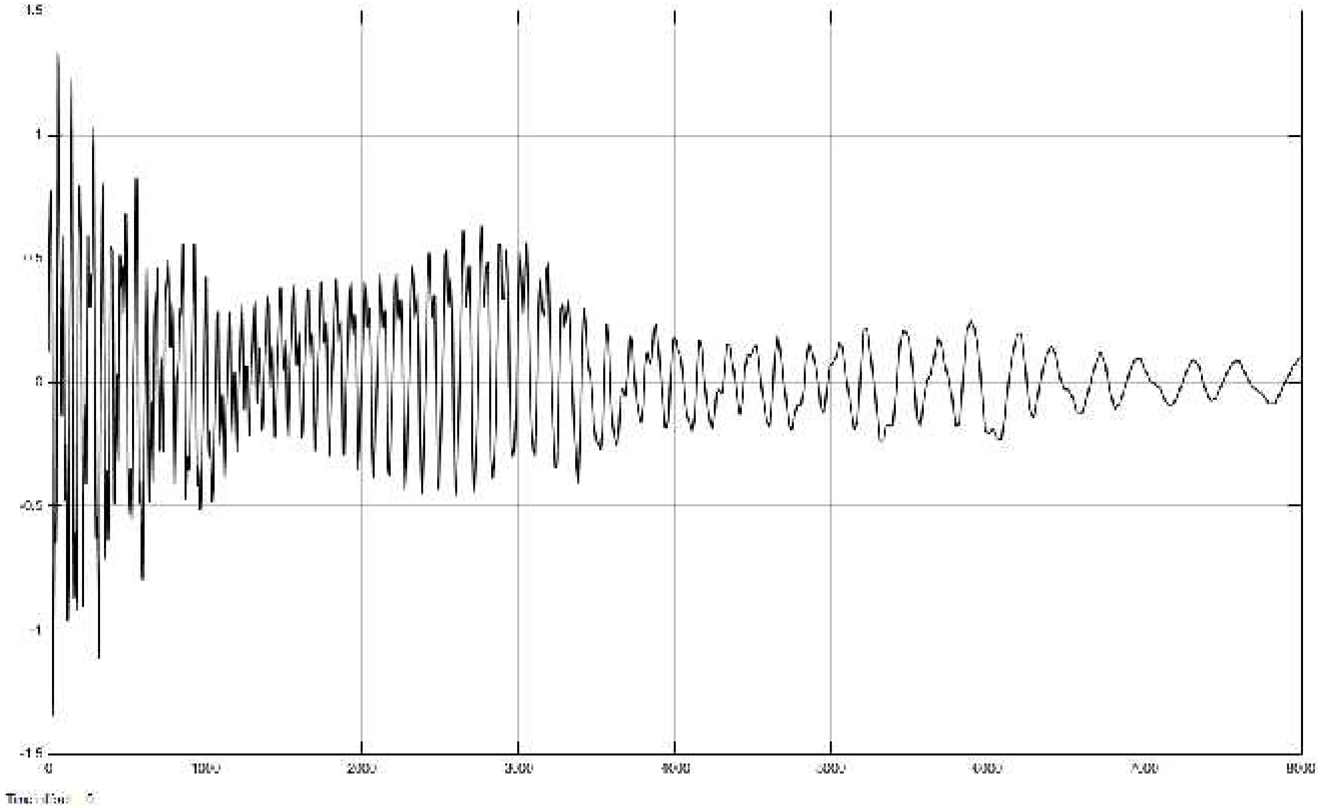


Figure 3.8 Earth's magnetic field 1

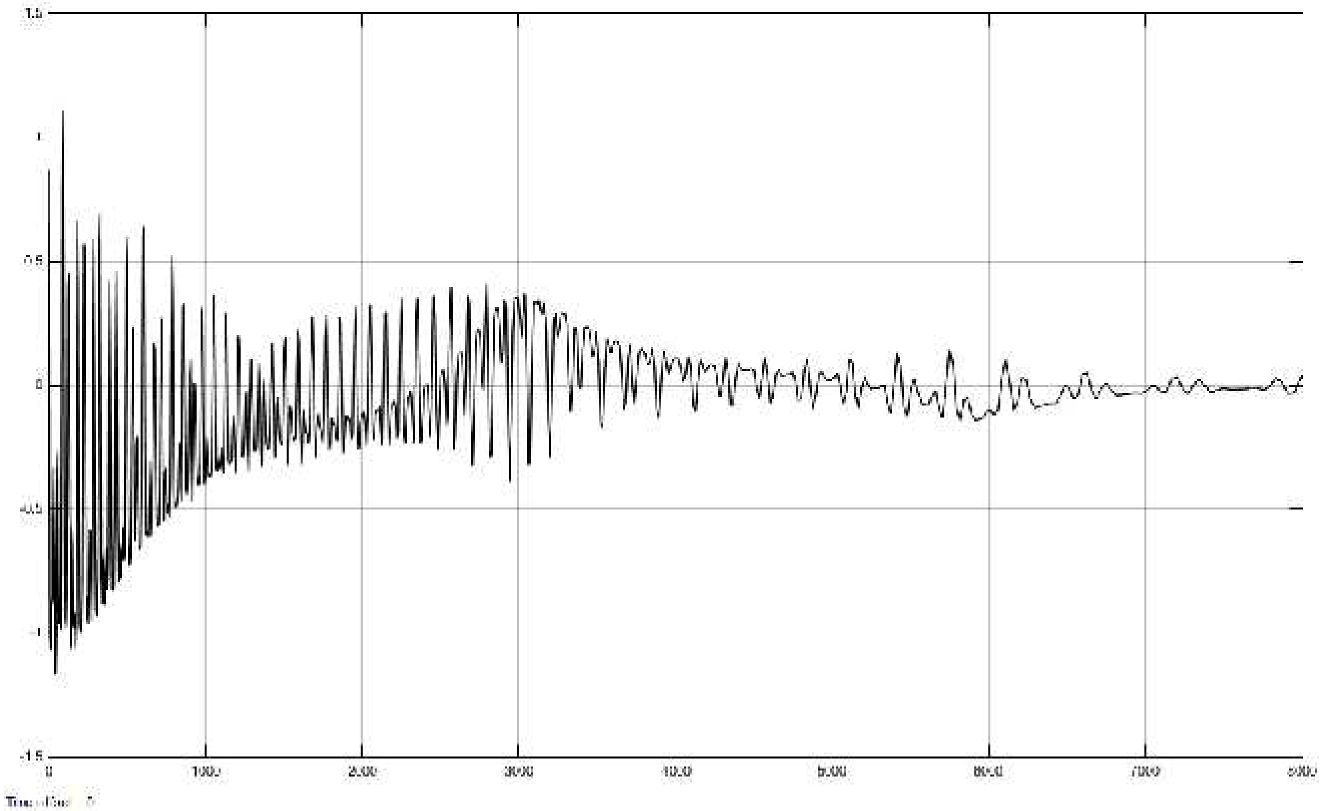


Figure 3.9 Earth's magnetic field 2

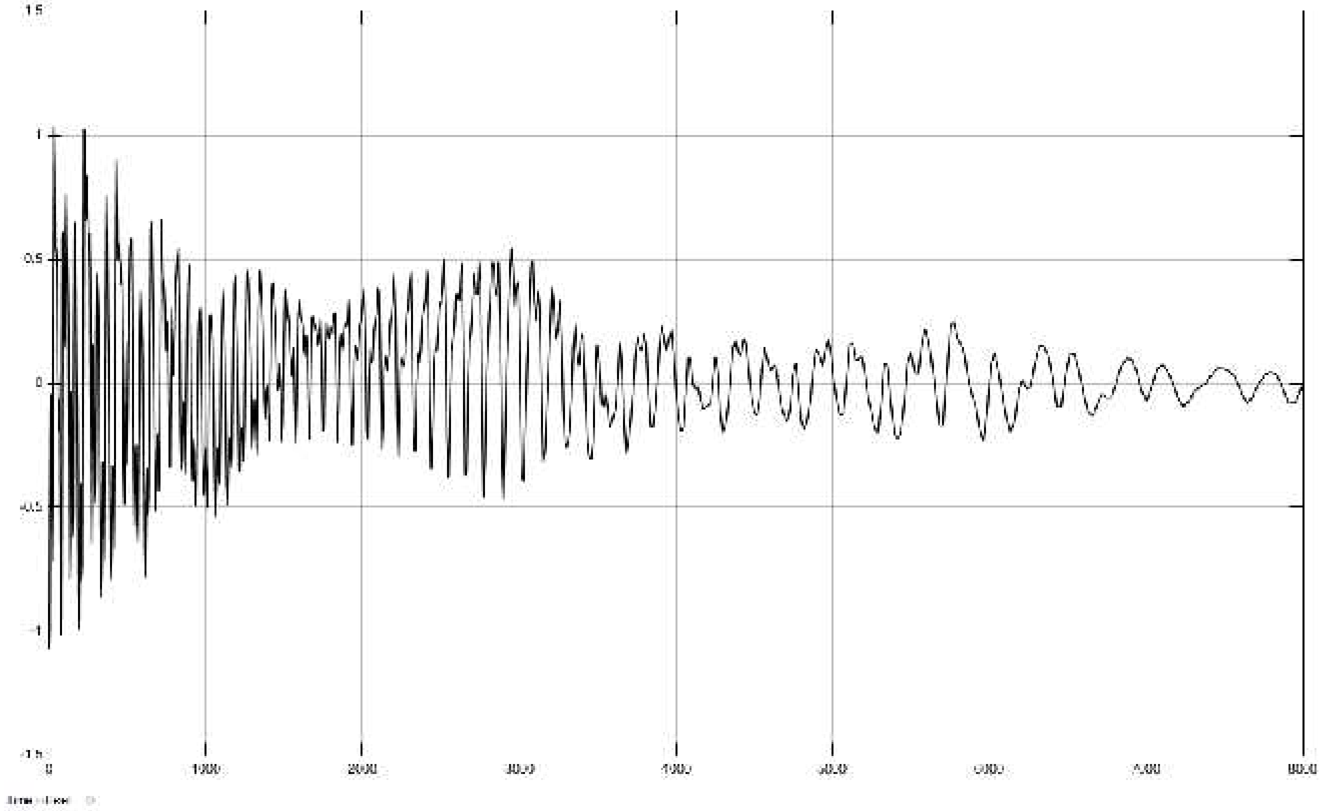


Figure 3.10 Earth's magnetic field 3

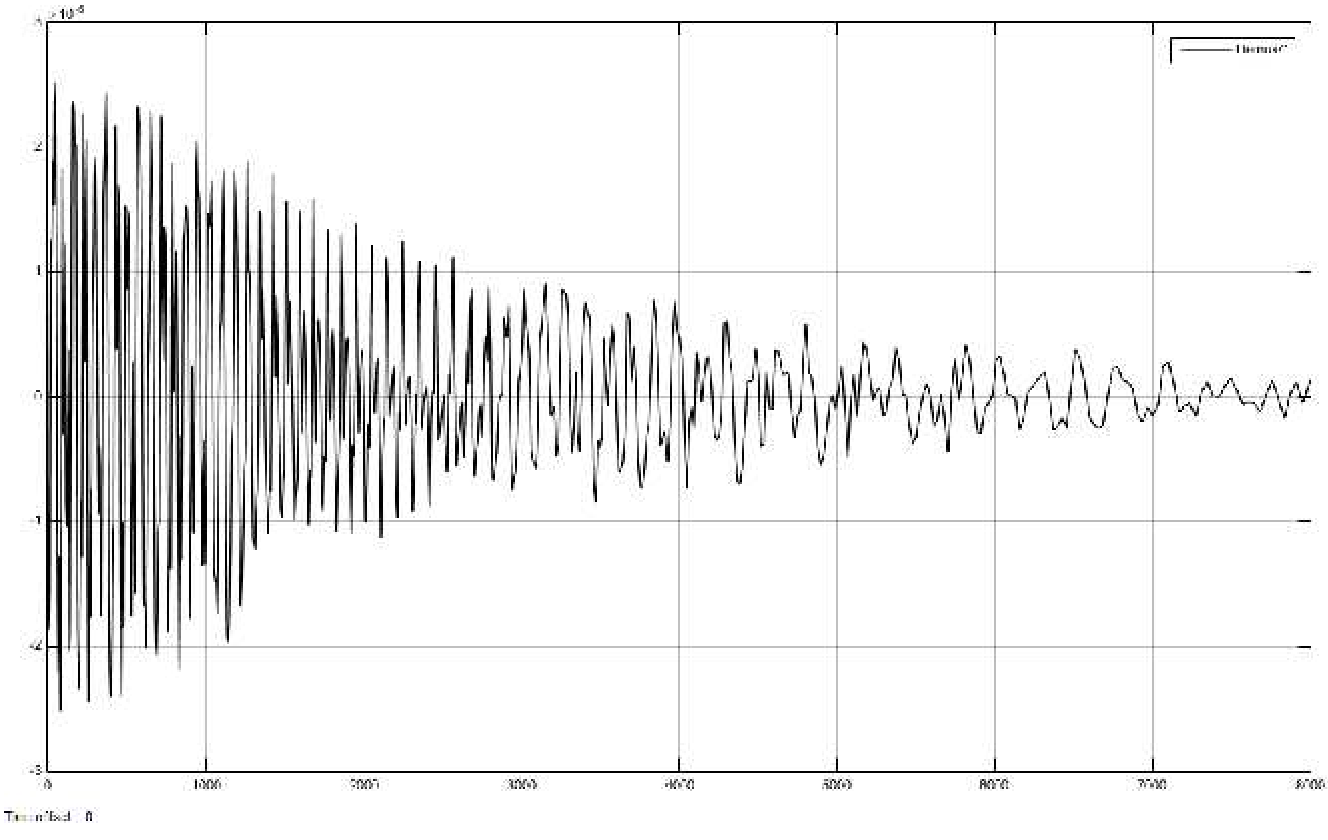


Figure 3.11 Moment 1

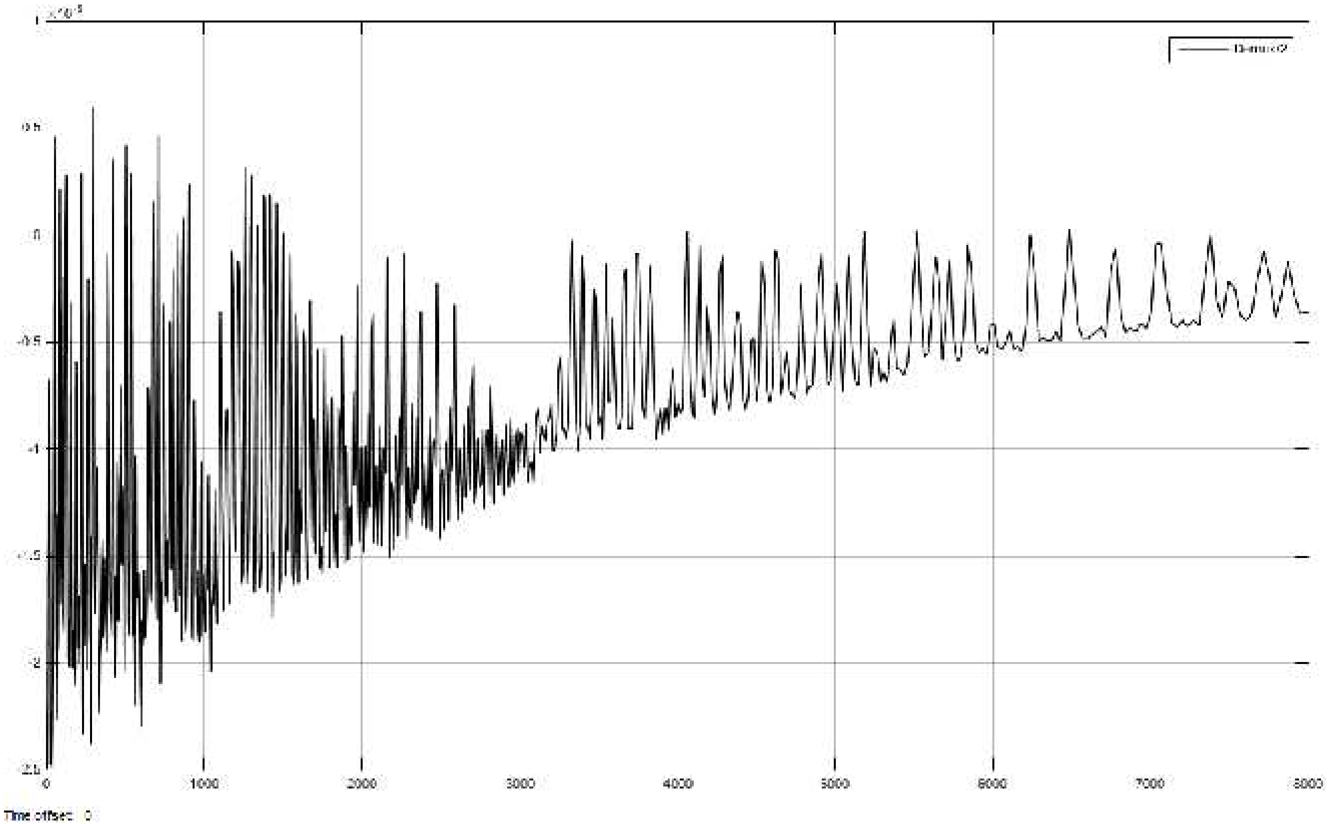


Figure 3.12 Moment 2

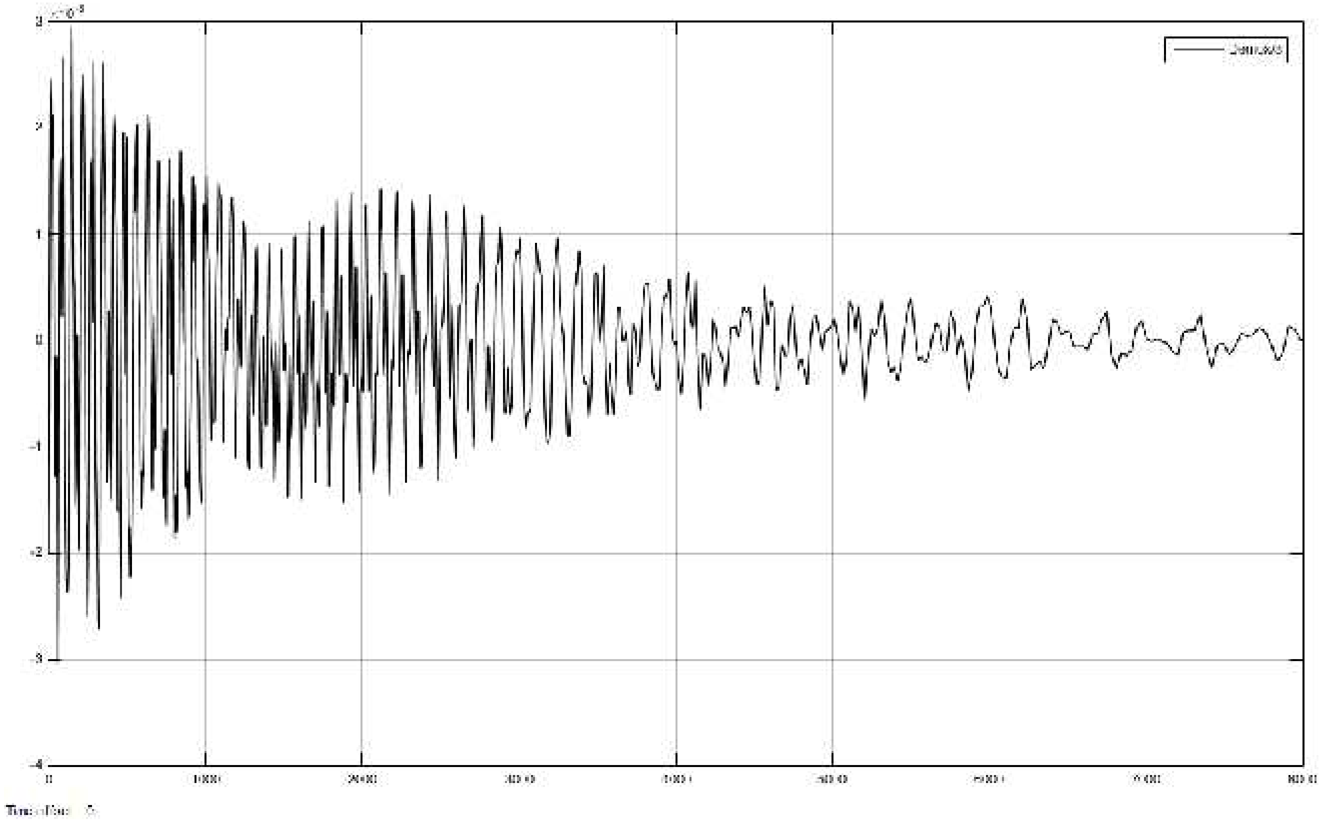


Figure 3.13 Moment 3

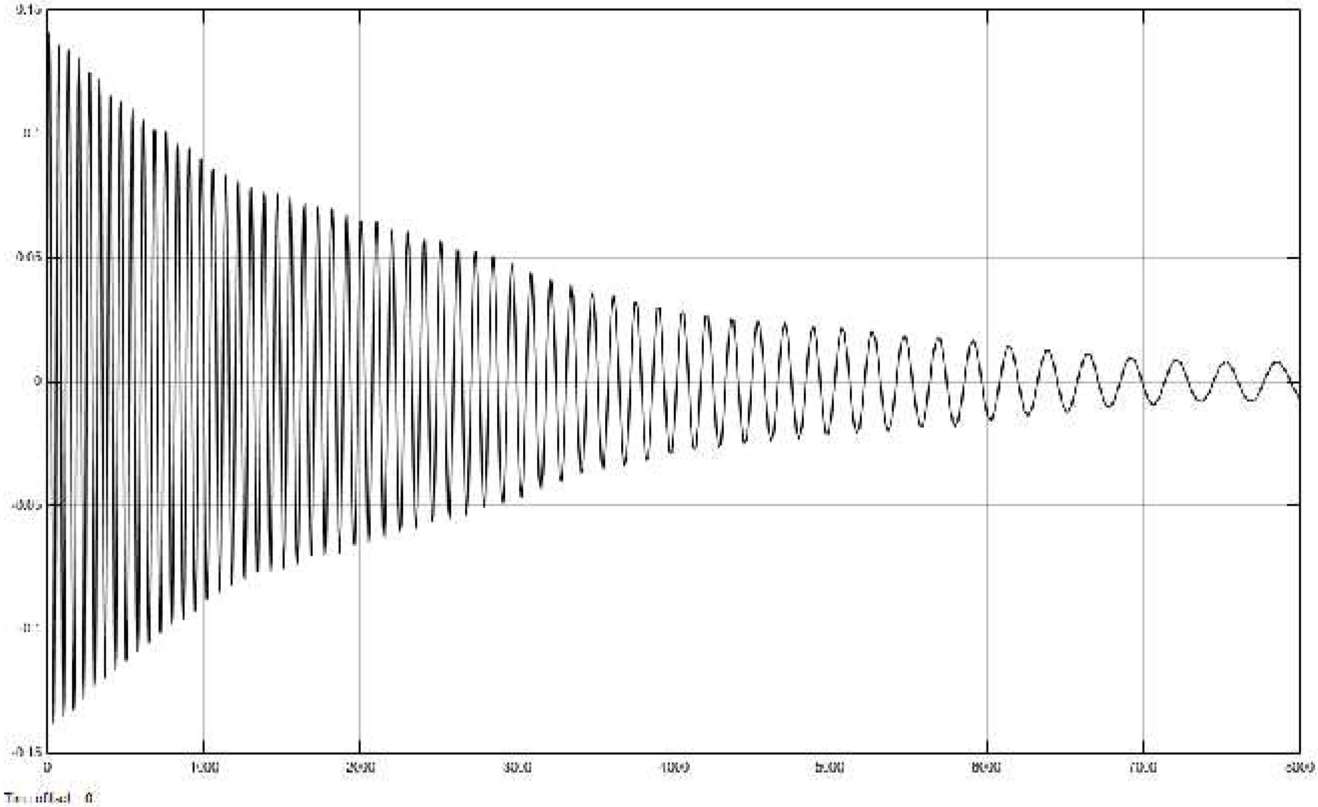


Figure 3.14 Angular velocity 1 – satellite

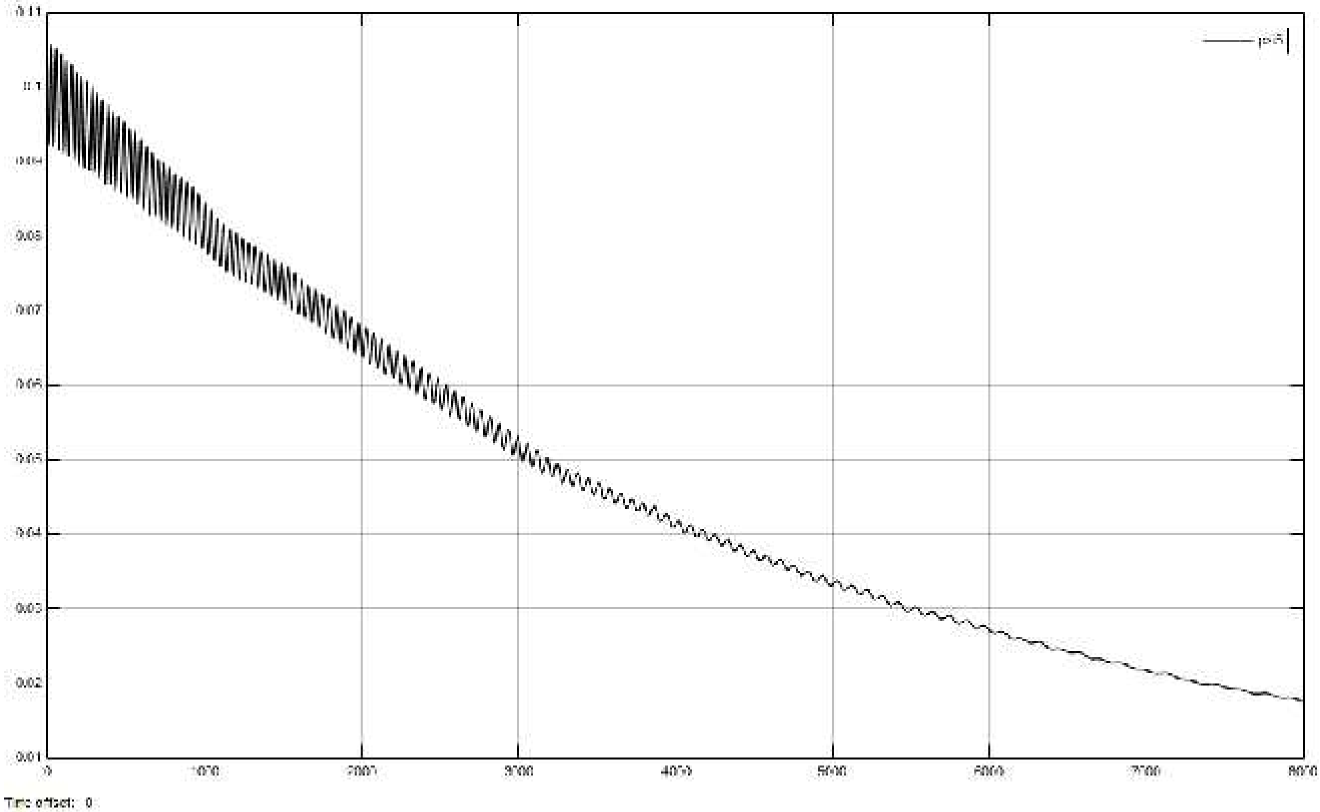


Figure 3.15 Angular velocity 2– satellite

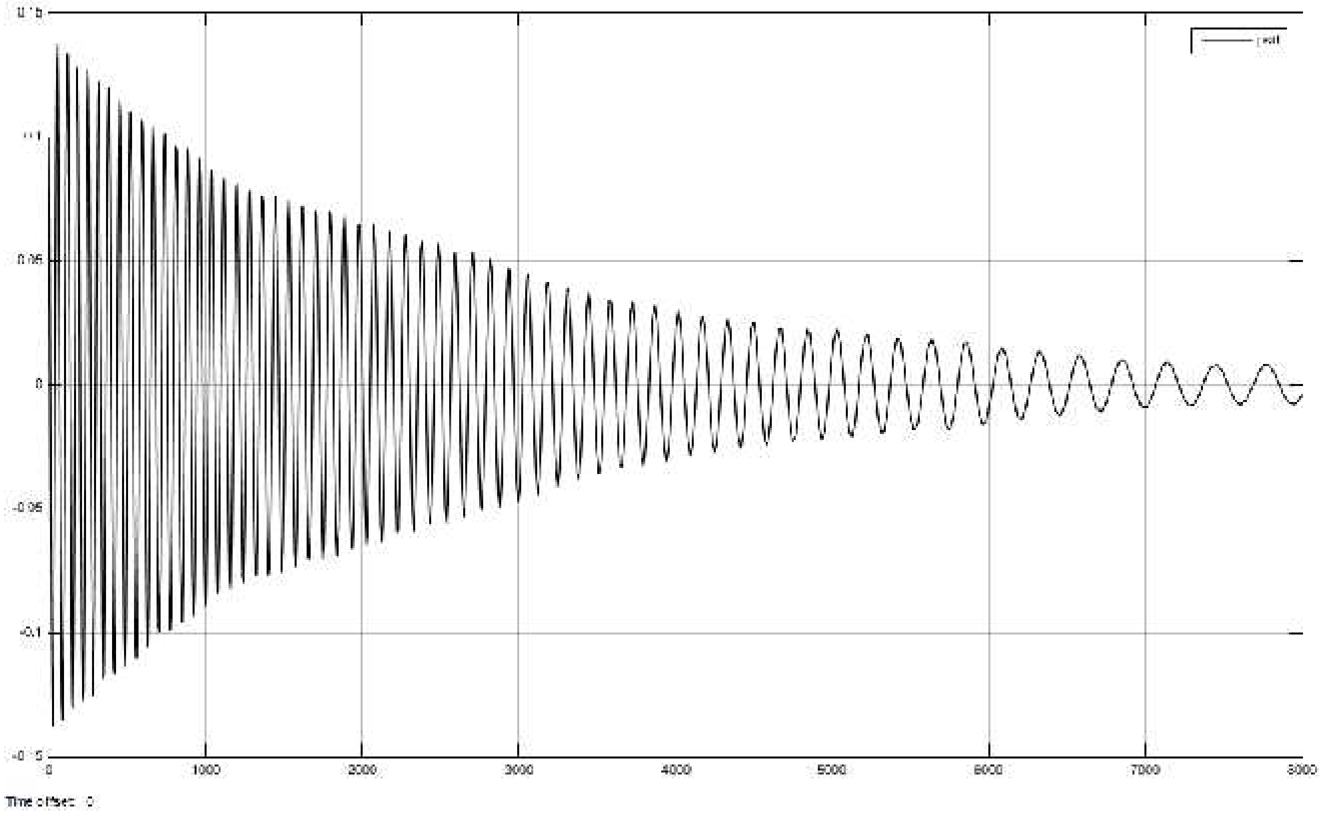


Figure 3.16 Angular velocity 3– satellite

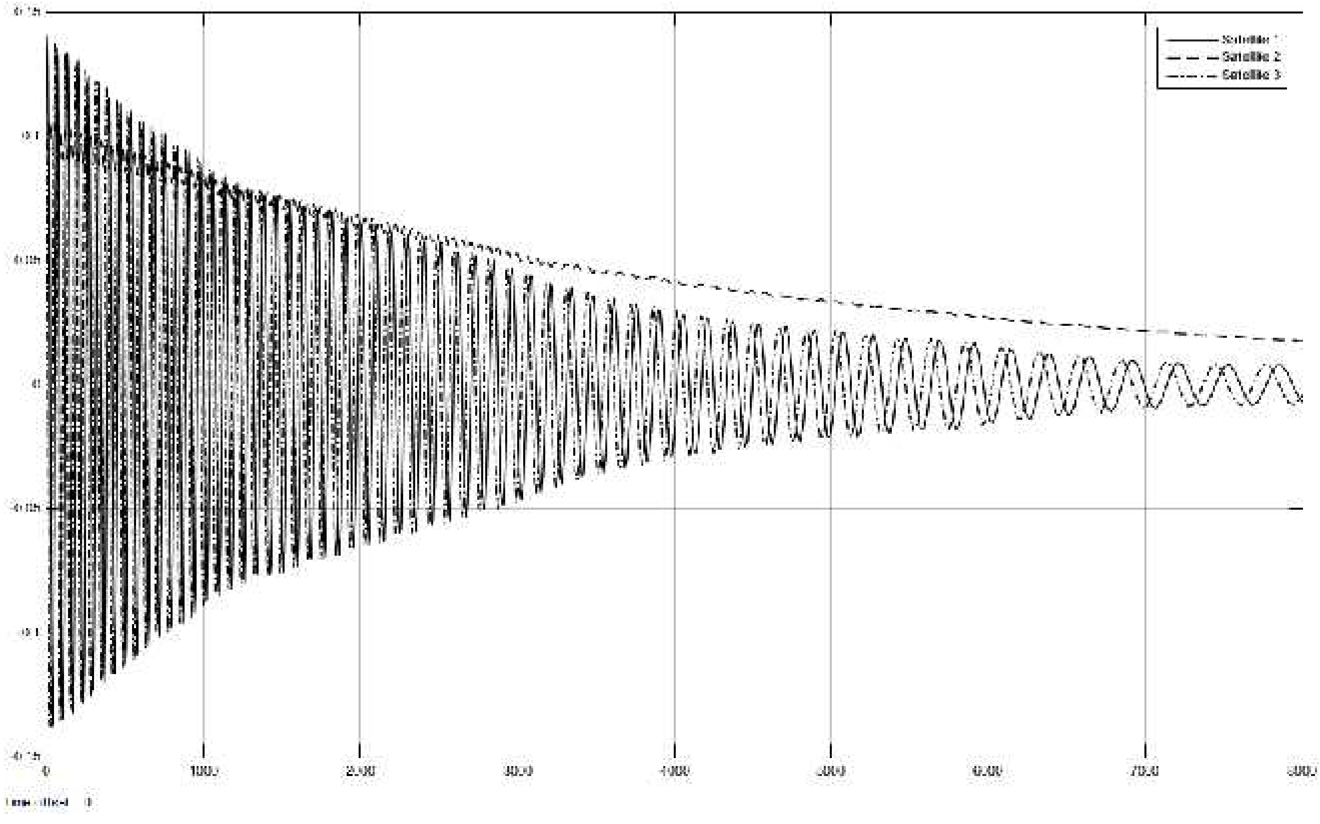


Figure 3.17 Angular velocities for k = 2e-4

If you increase the variable *k* in the **data** block, the satellite will fade faster, and vice versa, if you decrease this parameter, the satellite will stabilize longer.

For example, k = 4e-4;

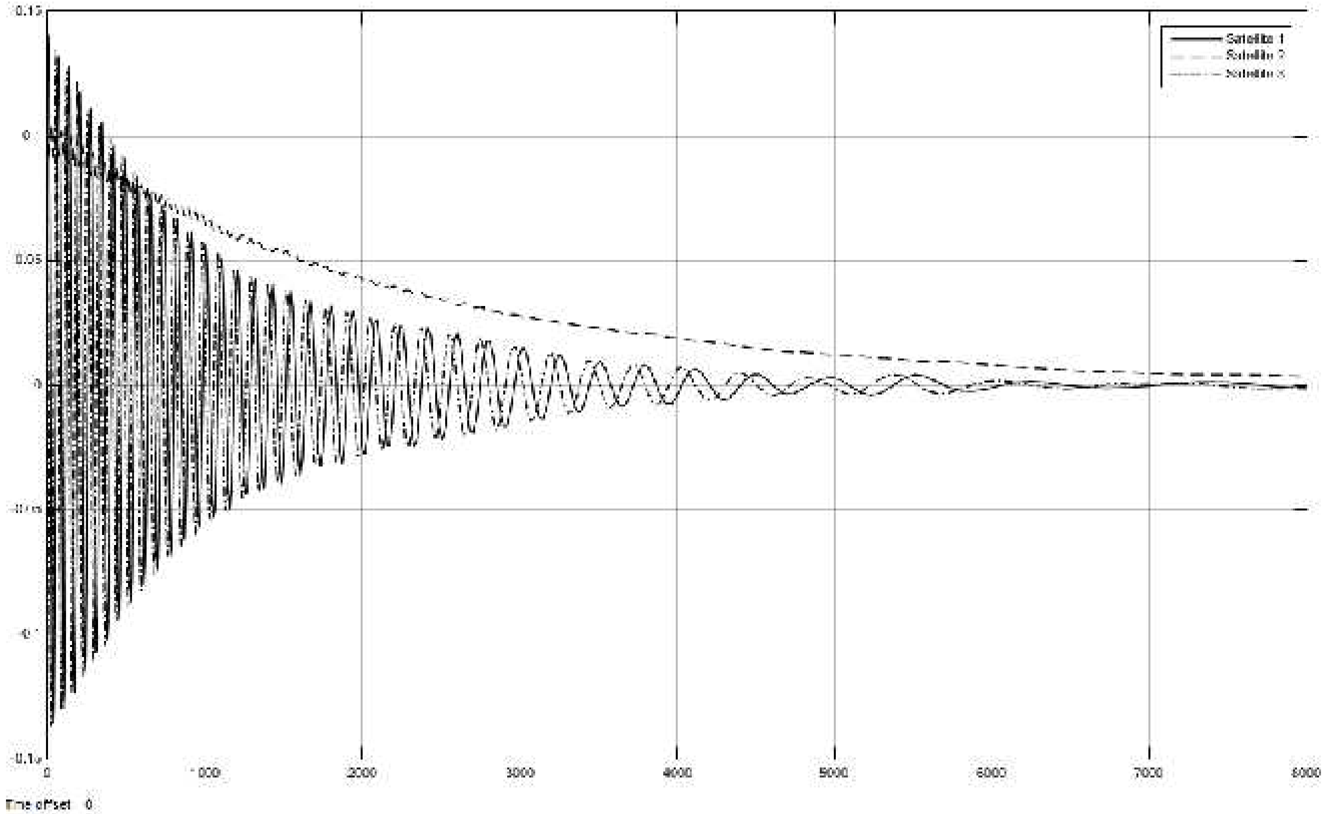


Figure 3.18 Angular velocities for k = 4e-4

We can clearly see that our satellite has stabilized faster.

3.4 Conclusion

In this section, we developed a computer model to simulate the satellite's motion at the initial stabilization stage. We showed an example of the model, its main blocks and formulas. We have successfully simulated the initial stabilization taking into account the Earth's magnetic field. The angular velocities of the satellite during initial stabilization are given.

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