UDC 517.9

CATENARY ARCH

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A catenary arch is a type of architectural arch which is an inverted catenary curve. Catenary arches are the most stable ones because they redirect the vertical force of gravity into compression forces that press along the curve, holding the arch's building blocks in place, and since the internal compression forces are ideally compensated, they do not cause falling.

Why is a hanging chain described by the catenary equation?

Consider a freely hanging cable or string which is of uniform mass per unit of length and is acted upon solely by gravity. Let *(x,y)* be an arbitrary point on the cable, s the length along the arch of the chain from the lowest point to *(x,y)*, and w_0 be the linear density of the cable. The cable is acted by tension, drag, lift and weight forces and they lead the cable to satisfy the ODE :

$\frac{d^2y}{dx^2} = a\sqrt{1 + \frac{dy}{dx}^2}$, where $a = w_0/T_0$ and T_0 is the horizontal component of the tension force.

To solve this ODE let $p = \frac{dy}{dx}$, so it will be transformed to $\frac{dp}{dx} = a\sqrt{1+p^2}$ which can be solved by the separation of variables:

$$\int \frac{dp}{\sqrt{1+p^2}} = \int ax,$$

by integrating both sides we obtain: $\ln\sqrt{1+p^2} + p = ax + c_3$.

When $x = 0$, we have that $\frac{dy}{dx} = p = 0$ and so c_3 is zero and therefore : $\ln\sqrt{1+p^2} + p = ax$

which gives us $p = \frac{dy}{dx} = \frac{e^{ax} - e^{-ax}}{2}$. by derivative of the exponential function, we get:

$$y = \frac{e^{ax} + e^{-ax}}{2a} + c_4$$

We can shift the y-axis to make $c_4 = 0$. Thus, the equation of the curve is the following:

$$y = \frac{e^{ax} + e^{-ax}}{2a} = \frac{\cosh ax}{a}$$

The list of sources:

1. Paul Calter. Research «Gateway to Mathematics Equations of the St. Louis Arch». Nexus Network Journal – vol. 8, no. 2, 2006, p. 53-66.

Catenary arch. URL: https://en.wikipedia.org/wiki/Catenary_arch