МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ

Факультет аеронавігації, електроніки та телекомунікацій

Кафедра аерокосмічних систем управління

ДОПУСТИТИ ДО ЗАХИСТУ

Завідувач кафедри АКСУ

___________Юрій МЕЛЬНИК

“_____” ____________ 2024 р.

КВАЛІФІКАЦІЙНА РОБОТА

(ПОЯСНЮВАЛЬНА ЗАПИСКА)

ВИПУСКНИКА ОСВІТНЬОГО СТУПЕНЯ

«БАКАЛАВР»

Тема: «Система визначення орієнтації з автокомпенсацією похибок вимірювачів»

Виконавець: студент групи СУ-404________________ Артем КОГУТ

Керівник: ____Лев РИЖКОВ

Нормоконтролер: __________________________________Микола ДИВНИЧ

Київ 2024

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL AVIATION UNIVERSITY

Faculty of Air Navigation, Electronics and Telecommunications

Aerospace Control Systems Department

APPROVED FOR DEFENCE

Head of the ACS Department

____________ Yurii MELNYK

“_____” ____________ 2024 y.

QUALIFICATION WORK

(EXPLANATORY NOTE)

FOR THE ACADEMIC DEGREE OF BACHELOR

Title: «Orientation detection system with auto-compensation of measurement errors»

Submitted by: student of group CS-404:____________ Artem KOHUT.

Supervisor: ________________________ Lev RYZHKOV

Standards inspector: ________________________ Mykola DYVNYCH

Kyiv 2024

НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ

Факультет аеронавігації, електроніки та телекомунікацій

Кафедра аерокосмічних систем управління

Спеціальність: 151 Автоматизація та комп’ютерно-інтегровані технології

ЗАТВЕРДЖУЮ

Завідувач кафедри АКСУ

________ Юрій МЕЛЬНИК

“ ____ ” __________ 2024 р.

ЗАВДАННЯ

на виконання кваліфікаційної роботи

Когута Артема Віталійовича

Тема кваліфікаційної роботи:« Система визначення орієнтації з автокомпенсацією похибок вимірювачів» затверджена наказом ректора від «13» квітня 2024 р. № 507/ст.

1. **Термін виконання роботи:** з 10.03.24 по 16.06.24.
2. **Вихідні дані роботи:** зведена комп’ютерна модель, яка містить моделі у відповідності кінематичних рівнянь.
3. **Зміст пояснювальної записки:**

Розділ 1. Математична модель системи визначення орієнтації;Розділ 2. Дослідження визначення орієнтації з автокомпенсацією похибок вимірювання;Розділ 3. Реалізація комп'ютерних моделей в середовищі MATLAB-Simulink для визначення орієнтації на основі кінематичних рівнянь руху тіла.

4. **Перелік обов’язкового ілюстративного матеріалу:** Графіки результатів моделювання та розрахунків. Матеріали презентації в PowerPoint.

5. **Календарний план-графік**

№ пор.	Завдання	Термін виконання	Відмітка про виконання
1.	Отримання завдання	10.05.2024 – 11.05.2024	
2.	Формування мети та основних завдань дослідження	12.05.2024 – 13.05.2024	
3.	Аналіз існуючих методів авто компенсації похибок	14.05.2024 – 22.05.2024	
4.	Теоретичний розгляд рішення задачі	23.05-2024 – 26.05.2024	
5	Розробка моделі для системи визначення орієнтації	27.05.2024-06.06.2024	
6	Підготовка презентації та роздаткового матеріалу	07.06.2024-13.06.2024	

6. **Дата видачі завдання:** "10" травня 2024 р.

Керівник кваліфікаційної роботи __________________ Лев РИЖКОВ
(підпис керівника)

Завдання прийняв до виконання __________________ Артем КОГУТ
(підпис випускника)

NATIONAL AVIATION UNIVERSITY

Faculty of Air Navigation, Electronics and Telecommunications

Aerospace Control Systems Department

Specialty: 151 Automation and Computer-integrated Technologies

APPROVED BY

Head of the ACS Department

______ Yurii MELNYK

"____" __________2024

Qualification Paper Assignment for Graduate Student

Kohut Artem Vitaliyovych

1. **The qualification paper title** «Orientation detection system with auto-compensation of measurement errors» was approved by the Rector's order of “ 13 ” April 2024 № 507/ст.
2. **The paper to be completed between: 10.05.24 and 10.06.24**
3. **Initial data for the paper:** a consolidated computer model containing models in accordance with kinematic equations.
4. **The content of the explanatory note:**

Chapter 1. Mathematical model of the orientation determination system; Chapter 2. Study of orientation determination with auto-compensation of measurement errors; Chapter 3. Implementation of computer models in MATLAB-Simulink environment for determining the orientation based on the kinematic equations of body motion.

5. **The list of mandatory illustrations:** Graphs of modeling results and calculations. Presentation materials in PowerPoint.

6. **Timetable:**

№	Assignment	Dates of completion	Completion mark
1.	Receiving a task	10.05.2024 – 11.05.2024	
2.	Formation of the purpose and main objectives of the study	12.05.2024 – 13.05.2024	
3.	Analysis of existing methods of automatic error compensation	14.05.2024 – 22.05.2024	
4.	Theoretical consideration of the problem solution	23.05-2024 – 26.05.2024	
5	Development of a model for an orientation detection system	27.05.2024-06.06.2024	
6	Preparing a presentation and handouts	07.06.2024-13.06.2024	

7. **Assignment issue date: “10” May 2024 y.**

Qualification paper supervisor ______________________ Lev RYZHKOV
(signature)

Issued task accepted ______________________ Artem KOHUT
(signature)

РЕФЕРАТ

Об’єкт дослідження – Стропова система з чутливими елементами (гіроскопами), що обертаються відносно об'єкта

Предмет дослідження – Математична модель похибок вимірювання кутів орієнтації з урахуванням постійних переміщень гіроскопів

Мета роботи – Визначення ефективності автокомпенсації похибок гіроскопів просторовим поворотом чутливих елементів для підвищення точності вимірювання кінематичних параметрів

Методи дослідження – теорія автоматичного керування рухомими об’єктами, математичне моделювання систем визначення орієнтації; моделі динаміки та контролю орієнтації з використанням методів автокомпенсації похибок

У роботі проведено дослідження системи визначення орієнтації з автокомпенсацією похибок вимірювачів. У даній роботі проведено експериментальні дослідження процесу корекції орієнтації з використанням методів автокомпенсації похибок, що виникають у вимірювачах під час роботи супутника. Це дозволило оцінити ефективність алгоритмів корекції та виявити оптимальні умови для стабілізації системи визначення орієнтації.

ABSTRACT

Object of research – A sling system with sensing elements (gyroscopes) that rotate relative to the object

Purpose of the work – A mathematical model of errors in measuring orientation angles, taking into account the constant displacements of gyroscopes

Purpose of the work – To determine the effectiveness of autocompensation of gyroscope errors by spatial rotation of sensitive elements to improve the accuracy of measurement of kinematic parameters

Research methods – Theory of automatic control of moving objects, mathematical modeling of aircraft orientation systems; models of aircraft dynamics and orientation control using methods of auto-compensation of errors.

objects, mathematical modeling of aircraft orientation determination systems; models of aircraft dynamics and orientation control using error autocompensation methods

The paper deals with the study of an orientation determination system with autocompensation of measurement errors. In this work, experimental studies of the process of orientation correction using methods of autocompensation of errors arising in the meters during satellite operation were carried out. This made it possible to evaluate the effectiveness of the correction algorithms and identify the optimal conditions for stabilizing the orientation determination system..

Content

Introduction

The development of modern technologies requires the creation of increasingly accurate and reliable systems for determining the spatial orientation of objects. This is especially true for the aviation, space, automotive, and robotics industries. One of the key aspects of such systems is the accuracy of determining the orientation parameters, which significantly depends on the quality of measurement error correction.

The problem of measurement errors is an integral part of the orientation determination process. These errors can be caused by various factors, including sensor noise, external influences such as magnetic fields, atmospheric interference, and other dynamic changes in the operating conditions of the equipment. Therefore, the development of effective methods for auto-compensating for these errors is an urgent task.

This thesis is devoted to the development of an orientation determination system with autocompensation of measurement errors. We will focus on analyzing the main sources of errors, developing a mathematical model for autocompensating these errors, and verifying the model in practice. An important part of the work is the study and modeling of orientation measurement processes, as well as the development and testing of algorithms for correcting measurement errors.

The aim of the thesis is to create a reliable and efficient system capable of providing high accuracy of orientation determination in various operating conditions. The implementation of this system involves the integration of theoretical developments with experimental studies, which will provide a deep understanding of error compensation mechanisms and increase the efficiency of the system in real conditions.

The results of this work can be used to increase the accuracy of navigation systems, improve motion control systems for robots, cars, aircraft, and other mobile objects, and optimize the operation of industrial automatic control systems.

Thus, the thesis is aimed at solving important applied problems and contributes to the further development of research in this area. In the course of the work, a thorough analysis of existing methods will be carried out, new algorithms for efficient auto-compensation of errors will be developed, and their effectiveness will be investigated on real and test data. Considerable attention will also be paid to verification of theoretical developments using experimental methods, including the implementation of system prototypes and their testing under controlled conditions.

Current trends in space missions increasingly favor the intensive use of small satellites. Although the classification of satellites by mass may vary, satellites weighing up to 500 kg are usually considered small. Remote sensing, broadband communications, and Earth observation are just a few examples of small satellite applications. The main advantage of small satellites over traditional large satellites is a reduction in both time and material costs for development. Their smaller size also allows them to be launched as secondary payloads on launch vehicles, providing a cost-effective way to get them into orbit. The popularity of small satellites is growing even more as satellite constellations have become a viable way to meet mission requirements. SpaceX has launched the Starlink broadband constellation, which consists of thousands of small satellites. It is said that up to 7000 small satellites may be launched between 2018 and 2027.

Small satellites are not without their drawbacks. The overall capabilities of these satellites are limited compared to traditional large satellites. The reduction in available volume and mass limits the choice of components for payloads, communications, power generation and storage, and actuators. Additional difficulties can also arise in predicting satellite performance, as some of the previous assumptions made for larger satellites are no longer valid for a smaller satellite.

Development of the Mathematical Model

One of the main tasks of this work is to develop a mathematical model for autocompensation of measurement errors. The model should take into account all the main sources of errors that can affect orientation measurements. This includes sensor noise, magnetic and electric fields, temperature fluctuations, mechanical vibrations, and errors arising from imperfect signal processing algorithms.

For each type of error, a compensation algorithm will be developed using various methods, including statistical methods, Kalman filters, machine learning methods, and other modern approaches to data processing. An important aspect is the adaptation of the algorithms to real operating conditions, which allows for high accuracy of orientation determination in various operational modes.

Experimental Research and Model Verification

A significant part of the work will be experimental research. We plan to conduct a number of experiments to test the developed algorithms in controlled conditions. This includes the use of specialized equipment to simulate external influences such as magnetic fields, atmospheric interference, mechanical vibrations, and other factors that can affect the accuracy of measurements.

With the help of these experiments, we will be able to determine the real efficiency of the developed autocompensation methods and make the necessary adjustments to the algorithms. Particular attention will be paid to the analysis of the results, which will eliminate possible shortcomings and optimize the system before its practical implementation.

Integration of theoretical developments with practical applications

Implementation of an orientation determination system with auto-compensation of measurement errors requires integration of theoretical research with experimental methods. This approach allows not only to develop new algorithms, but also to adapt them to real operating conditions, taking into account all possible aspects and nuances that may affect the system's operation.

Modeling and optimization of measurement processes

To achieve high accuracy and reliability of the system, special attention should be paid to modeling and optimization of orientation measurement processes. This includes the development of detailed mathematical models that describe the behavior of sensors and measurement systems in various scenarios, including the influence of external noise, temperature changes, and other important factors.

This also includes analyzing possible structural and functional shortcomings of existing systems and suggesting methods to eliminate or minimize them. The use of optimization methods can significantly improve system efficiency by selecting the optimal parameters of its components and data processing algorithms.

Verification and analysis of results

After the development of theoretical models and algorithms, it is necessary to conduct their detailed verification. This includes analyzing the results based on real data obtained from experimental tests and comparing them with theoretically expected results. Such analysis helps to identify and correct possible errors and inaccuracies in models and algorithms, ensuring that the orientation determination system is highly compliant with real-world operating conditions.

The problem of small satellites and its impact on orientation systems

As already mentioned, current trends in space missions indicate the intensive use of small satellites. This puts special demands on attitude determination systems, as the operating conditions of such vehicles differ significantly from those of traditional large satellites. The use of small satellites reduces the available space and weight for system components, which requires orientation systems to be more compact and energy efficient.

Practical application and further prospects

Further application of the developed system may include its integration into car navigation systems to improve the accuracy of autonomous driving, use in robotics for precise control of robotic mechanisms, and use in aviation to improve the navigation and control systems of aircraft.

The work is expected to produce new scientific and practical results that will help improve the efficiency of using modern orientation detection technologies and their adaptation to specific operating conditions. Particular attention will be paid to minimizing the impact of external and internal disturbances on the accuracy of the system, which is key to improving its reliability and efficiency.

CHAPTER 1. Mathematical model of the orientation determination system

1.1 Coordinate systems

Coordinate systems are of paramount importance in systems that determine orientation. These systems are essential for precisely determining and displaying a satellite's location and movement relative to Earth.

Firstly, it is vital to pinpoint a satellite's position accurately relative to a reference point on Earth's surface. This precision is essential for navigation, monitoring, and, if necessary, communication with the satellite.

Secondly, satellites follow specific orbital paths around Earth. Coordinate systems enable us to characterize these orbital paths. These systems permit the computation and prediction of a satellite's future positions, velocities, and orbital characteristics. Third, real-time monitoring of the satellite's current position is possible. This data is essential for satellite operations, including path modification and route adherence.

Fourth, coordinate systems are used to determine the positions of ground stations in relation to the satellite. This capability allows for the establishment of communication links and the direction of antennas to optimize data transmission efficiency between the satellite and ground stations.

Fifth, coordinate systems facilitate coordination and collaboration among various satellite operators, space agencies, and international bodies. This cooperation enhances a shared understanding of satellites' positions and movements.

In conclusion, we have explored the significance of coordinate systems for satellites and highlighted the primary reasons these systems are employed.

The majority of satellites utilize two coordinate systems: the Earth-centered, Earth-fixed (ECEF) system and the geocentric-equatorial system.

The ECEF coordinate system is anchored to the Earth's center and serves as a reference frame for satellite navigation and positioning. The X-axis is oriented towards the intersection of the prime meridional and equatorial lines. The Y-axis is parallel to the prime meridian, while the Z-axis is aligned with the Earth's axis of rotation (north pole). The ECEF coordinate system allows for the precise location of a satellite relative to the Earth's center, typically expressed in meters. This system enables the accurate determination of satellite positions, and it is extensively employed in systems like GPS.

The geocentric-equatorial coordinate system is based on the Earth's equatorial plane and describes the positions of celestial bodies and satellites relative to Earth. It expands on the Earth's latitude and longitude system. The origin (0, 0) is at the Earth's center, the X-axis matches the prime meridian and equator intersection, and the Z-axis is vertical to the equatorial plane, going through the North Pole. The Y-axis completes this right-handed system. In this system, a satellite's position is specified by latitude (Φ), longitude (λ), and altitude (h) or radius (r). The latitude of a satellite is defined as the angle from the equatorial plane, which extends from -90° to +90°. The longitude of a satellite is represented by the rotational angle from the prime meridian, which extends from -180° to +180°. Finally, the altitude or radius of a satellite indicates the distance from Earth's center. These coordinate systems enable satellites to precisely pinpoint their locations, facilitate navigation, enhance communication, and synchronize time accurately. They offer a uniform framework for managing satellite operations and

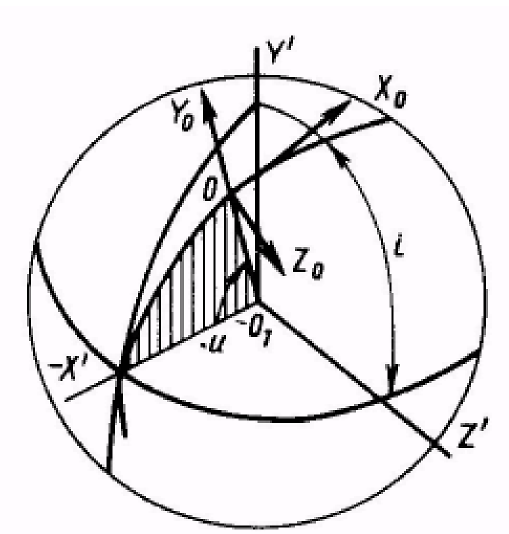


Figure 1.1 Orbital coordinate system

ensure compatibility among various satellite systems and applications.

The fundamental coordinate system is the orbital coordinate system, whose axes are oriented as depicted in Figure 1.1. This figure also illustrates the coordinate system that denotes the satellite's orbit position relative to the geocentric coordinate system, along with the parameters i and Ω, which are referred to as the orbit's inclination and the argument of latitude, respectively. These parameters describe the satellite's position within its orbit. To quantify the angular deviations of the satellite from the fundamental coordinate system, a coordinate system fixed to the satellite is introduced. Both systems have their origins at the satellite's center of mass. If the spacecraft achieves the intended position in space, the axes of the bound and orbital coordinate systems will align.

The discrepancy between the satellite's position and the aforementioned reference point will be quantified by measuring the distance between the axes of the related coordinate system and those of the base coordinate system.

This position is described by three Euler-Krylov angles ϑ, ϕ and φ, which are called pitch, yaw (course) and roll angles (Figure 1.2). Based on this figure, a table of direction cosines can be constructed.

	X	Y	Z
X_0	$\cos\phi\cos\vartheta$	$\sin\varphi\sin\phi\cos\vartheta - \cos\varphi\sin\vartheta$	$\cos\varphi\sin\phi\cos\vartheta + \sin\varphi\sin\vartheta$
Y_0	$\cos\phi\sin\vartheta$	$\sin\varphi\sin\phi\sin\vartheta + \cos\varphi\cos\vartheta$	$\cos\varphi\sin\phi\sin\vartheta - \sin\varphi\cos\vartheta$
Z_0	$-\sin\phi$	$\sin\varphi\cos\phi$	$\cos\varphi\cos\phi$

1.2 Euler's kinematic equations for the orientation determination system

The location of the body in the reference coordinate system $X_oY_oZ_o$ is given by the Euler angles. In Matlab, the arrangement of the axes and the order of rotation is called 'ZYX'.

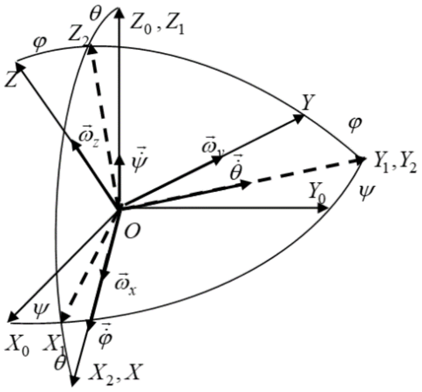


Fig 1.2.1

Let me remind you that in flight dynamics, a different axis designation and a different sequence of turns are often used.

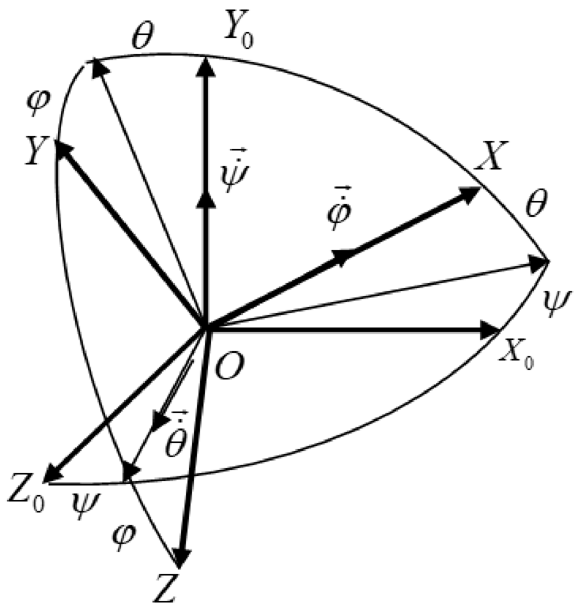


Figure 1.2.2

In Matlab, this is called 'YZX'. To obtain the directional cosine matrix for this rotation sequence for the given angles, enter the following program

psi=0.1; tet=0.2; phi=0.3;

dcm=angle2dcm(psi,tet,phi,'YZX')

The modelling result is as follows:

dcm =

0.9752 0.1987 -0.0978

-0.1593 0.9363 0.3130

0.1538 -0.2896 0.9447

The sequence of turns 'YZX' can be omitted:

```
psi=0.1; tet=0.2; phi=0.3;
dcm=angle2dcm(psi,tet,phi)
```

The modelling result is as follows:

dcm =

 0.9752 0.0978 -0.1987

-0.0370 0.9564 0.2896

 0.2184 -0.2751 0.9363

Let's return to Figure 1.2.1

Let's enter the vectors ,$\vec{\omega}_x,\ \vec{\omega}_y,\ \vec{\omega}_z\ \vec{\dot\psi},\ \vec{\dot\theta},\ \vec{\dot\varphi}$ and write

$$\vec{\omega}_x+\vec{\omega}_y+\vec{\omega}_z=\vec{\dot\psi}+\vec{\dot\theta}+\vec{\dot\varphi}. \quad (1)$$

Using Fig. 1, we obtain the following dependencies (Euler's kinematic equations)

$$\omega_x=\dot\varphi-\dot\psi\sin\theta;$$
$$\omega_y=\dot\theta\cos\varphi+\dot\psi\cos\theta\sin\varphi;$$
$$\omega_z=\dot\psi\cos\theta\cos\varphi-\dot\theta\sin\varphi.$$

We assume that the angular velocities of the body $\omega_x,\ \omega_y,\ \omega_z$ are measured by angular velocity sensors. Since we are interested in Euler angles, we express the angular velocities of $\dot\psi,\ \dot\theta,\ \dot\varphi$ in terms of the angular velocities of $\omega_x,\ \omega_y,\ \omega_z$. This can be done using Euler's kinematic equations, but more simply it can be done as follows.

Let's project expression (1) onto the axes marked with dashed lines in the figure - these are the intersection lines of the planes where the angles of rotation are set. The peculiarity of these axes is that only one of the vectors $\vec{\dot\psi},\ \vec{\dot\theta},\ \vec{\dot\varphi}$ is projected onto each of them.

Projected onto the OZ_2 axis, we get:

$$\dot\psi\cos\theta=\omega_z\cos\varphi+\omega_y\sin\varphi,$$

That is,

$$\dot{\psi} = \frac{1}{\cos\theta}\left(\omega_z \cos\varphi + \omega_y \sin\varphi\right). \tag{2}$$

Projected onto the OY_2 axis, we get:

$$\dot{\theta} = \omega_y \cos\varphi - \omega_z \sin\varphi. \tag{3}$$

Projected onto the OX_1 axis, we get:

$$\dot{\varphi}\cos\theta = \omega_x \cos\theta + \sin\theta\left(\omega_z \cos\varphi + \omega_x \sin\varphi\right),$$

That is,

$$\dot{\varphi} = \omega_x + tg\theta\left(\omega_z \cos\varphi + \omega_y \sin\varphi\right). \tag{4}$$

Equations (2)-(4) are also called Euler's kinematic equations. By integrating these expressions, the orientation angles can be found (Fig. 1.2.2)

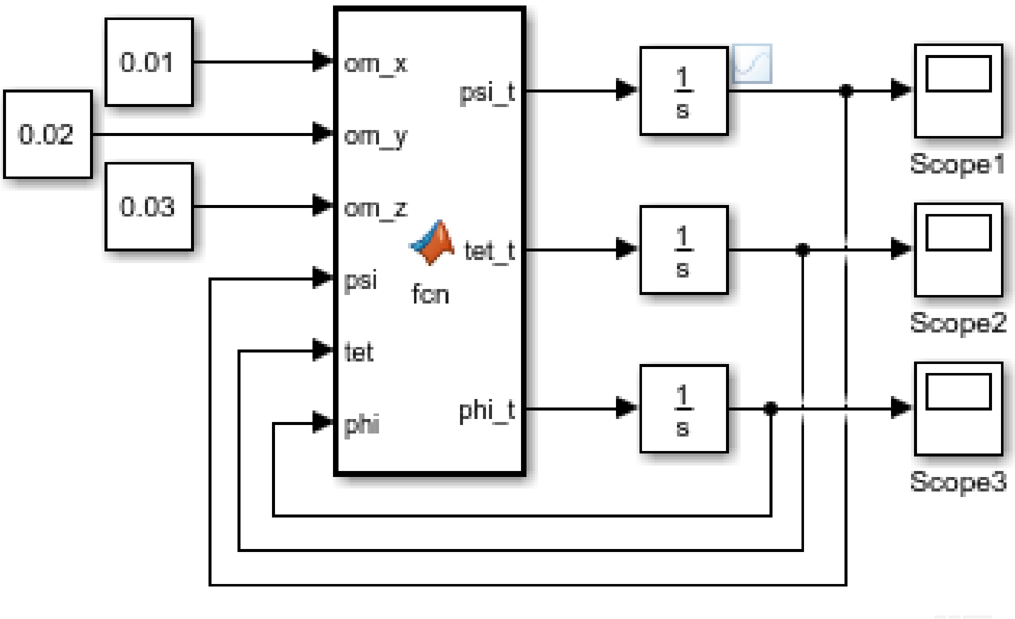


Figure 1.2.3

Implemented in the "fcn" block:

```
function [psi_t,tet_t,phi_t] = fcn(om_x,om_y,om_z,psi,tet,phi)
psi_t=1/cos(tet)*(om_z*cos(phi)+om_y*sin(phi));
tet_t=om_y*cos(phi)-om_z*sin(phi);
phi_t=om_x+tan(tet)*(om_z*cos(phi)+om_y*sin(phi));
```

For small Euler angles, we obtain the obvious relations

$\dot{\psi} \approx \omega_z;\ \dot{\theta} \approx \omega_y;\ \dot{\varphi} \approx \omega_x.$

If the output signals of the gyroscopes contain constant interference (zero offsets), the integration of these signals will result in time-growing errors in the measurement of Euler angles.

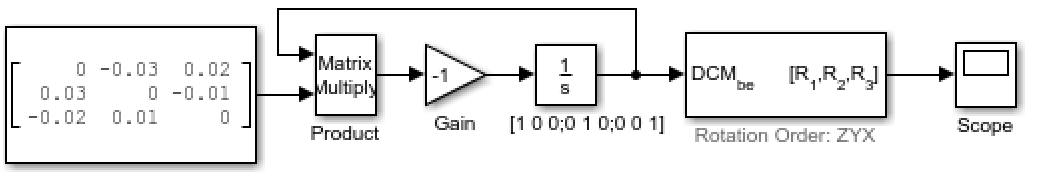
Euler's kinematic equations in matrix form (Poisson's equations)

In matrix form, Euler's kinematic equations (Poisson's equations) are as follows

$$\dot{\boldsymbol{R}} = -\boldsymbol{R}\boldsymbol{\Omega}_E, \qquad (5)$$

where.

$$\boldsymbol{\Omega}_E = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}.$$



The Simulink model for solving equation (5) is shown in Fig. 1.2.3.

Figure 1.2.4

1.3 Quaterions to define body position

According to the Euler-Dalembert theorem, any rotation of a solid having a single fixed point can be obtained by a single rotation about an axis passing through that point.

Thus, the task of determining the position of the body in this case is reduced to establishing the rotation axis, rotation angle and direction of rotation. These elements can be searched both separately and simultaneously, which is the case when using quaternions.

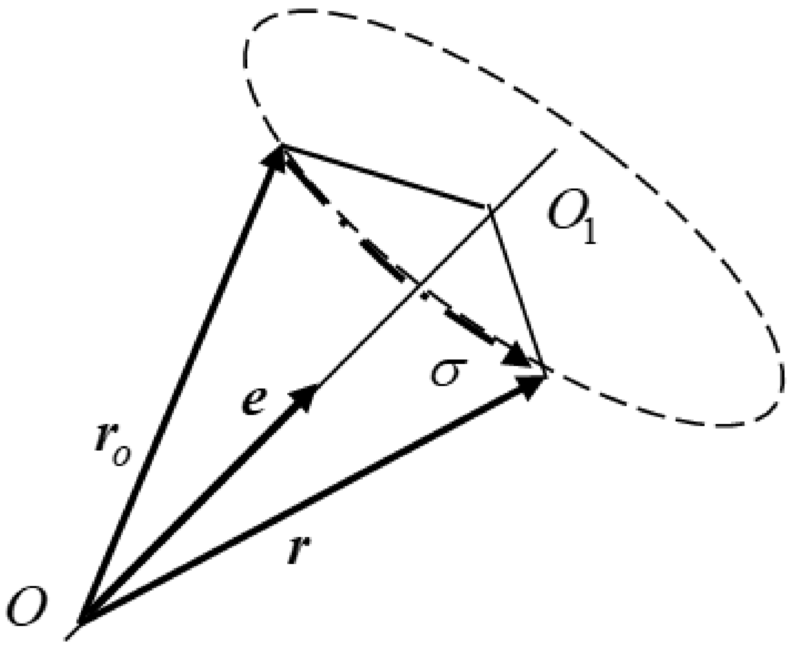


Figure 1.3.1

Fig. 1.3.1 shows the rotation of the vector $\boldsymbol{r}$ around the axis of rotation OO_1 by the angle σ. The direction of rotation is determined by the unit vector $\boldsymbol{e}$.

Quaterion is an operator

$$\boldsymbol{q} = \cos\frac{\sigma}{2} + \boldsymbol{e}\sin\frac{\sigma}{2},$$

which characterises the rotation of the vector $\boldsymbol{r}$ about the axis of rotation with the unit vector $\boldsymbol{e}$ by the angle σ.

The general view of the quaternion that defines the rotation in the coordinate system with the orthos is as follows: $\boldsymbol{i}_o,\ \boldsymbol{j}_o,\ \boldsymbol{k}_o$

$$\boldsymbol{q} = q_0 + q_1\boldsymbol{i}_o + q_2\boldsymbol{j}_o + q_3\boldsymbol{k}_o.$$

A restriction is imposed on the parameters of the quaternion

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1.$$

Multiplication of quaternions is denoted as "$\circ$". The following algebra of quaternions is used

$$\boldsymbol{i}\circ\boldsymbol{j} = \boldsymbol{k};\ \ \boldsymbol{j}\circ\boldsymbol{k} = \boldsymbol{i};\ \ \boldsymbol{k}\circ\boldsymbol{i} = \boldsymbol{j};\ \ \boldsymbol{i}\circ\boldsymbol{j} = -\boldsymbol{j}\circ\boldsymbol{i};\ \ \boldsymbol{j}\circ\boldsymbol{k} = -\boldsymbol{k}\circ\boldsymbol{j};\ \ \boldsymbol{k}\circ\boldsymbol{i} = -\boldsymbol{j}\circ\boldsymbol{k};$$
$$\boldsymbol{i}\circ\boldsymbol{i} = \boldsymbol{j}\circ\boldsymbol{j} = \boldsymbol{k}\circ\boldsymbol{k} = -1.$$

The relationship between the vectors $\boldsymbol{r}_o$ and $\boldsymbol{r}$ is as follows

$$\boldsymbol{r} = \boldsymbol{q} \circ \boldsymbol{r}_o \circ \tilde{\boldsymbol{q}},$$

where $\tilde{\boldsymbol{q}} = \cos\frac{\sigma}{2} - \boldsymbol{e}\sin\frac{\sigma}{2}$ is a conjugate quaternion.

From now on, we will assume that the quaternion characterises the rotation of the basis vectors, i.e. the axes that are invariably connected to the body.

The guide cosine matrix $\boldsymbol{R}$ is written in terms of quaternion components as follows:

$$\boldsymbol{R} = \begin{vmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 + q_0 q_3) & 2(q_1 q_3 - q_0 q_2) \\ 2(q_1 q_2 - q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 + q_0 q_1) \\ 2(q_1 q_3 + q_0 q_2) & 2(q_2 q_3 - q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{vmatrix}.$$

The advantage of the quaternion compared to the matrix of guide cosines is the smaller number of unknowns (four), while for the matrix method we have nine unknown guide cosines.

Suppose that first there is a turn characterised by the quaternion $\boldsymbol{d}_1$, and then there is a turn characterised by the quaternion $\boldsymbol{d}_2$. In this case, the quaternions are written in a fixed base. The quaternion $\boldsymbol{d}$ of the resulting rotation has the form

$$\boldsymbol{d} = \boldsymbol{d}_1 \circ \boldsymbol{d}_2.$$

This rule applies to an arbitrary number of turns.

The transition from the reference coordinate system $OX_0Y_0Z_0$ to the linked coordinate system $OXYZ$ is defined by the quaternion

$$\boldsymbol{q} = \boldsymbol{q}_\psi \circ \boldsymbol{q}_\theta \circ \boldsymbol{q}_\varphi = \left(\cos\frac{\psi}{2} + \boldsymbol{k}_o \sin\frac{\psi}{2}\right) \circ \left(\cos\frac{\theta}{2} + \boldsymbol{j}_o \sin\frac{\theta}{2}\right) \circ \left(\cos\frac{\varphi}{2} + \boldsymbol{i}_o \sin\frac{\varphi}{2}\right) =$$
$$= q_0 + q_1 \boldsymbol{i}_o + q_2 \boldsymbol{j}_o + q_3 \boldsymbol{k}_o,$$

where.

$$q_0 = \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\varphi}{2}\right) + \sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\varphi}{2}\right);$$

$$q_1 = \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\varphi}{2}\right) - \sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\varphi}{2}\right);$$

$$q_2 = \cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\varphi}{2}\right)+\sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\varphi}{2}\right);$$
$$q_3 = \sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\varphi}{2}\right)-\cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\varphi}{2}\right).\#$$

1.4 Structure of the Orientation Determination System

The orientation determination system in space is designed to accurately measure and control the angular position of spacecraft relative to an inertial frame of reference. This system is crucial for the correct functioning of satellites, space telescopes, and other space-based assets. Below is a detailed breakdown of the components and functionalities of this system.

1. Introduction to Orientation Determination

   The orientation determination system is essential for navigating and controlling spacecraft. It provides critical data that helps in aligning instruments, antennas, and solar panels with their target positions. This system uses various sensors and algorithms to determine the spacecraft's orientation in three-dimensional space.

2. Core Components of the Orientation Determination System

   The system's effectiveness depends on the integration of its core components, which include sensors, processors, and control units. Each component has a vital role in ensuring accurate orientation determination.

   - Sensors: These are the primary inputs for the system, providing the raw data needed for orientation analysis.
     - Gyroscopes: Measure the rate of rotation around the spacecraft's internal axes. These sensors are crucial for understanding the rotational dynamics of the spacecraft.
     - Star Trackers: Use observations of stars and other celestial bodies to determine the spacecraft's orientation with high accuracy.
     - Sun Sensors: Provide orientation data relative to the Sun, useful for spacecraft that rely on solar panels for energy.

        - Magnetometers: Measure the magnetic field of the Earth, aiding in orientation determination for low Earth orbit satellites.
        - Inertial Measurement Units (IMUs): Combine multiple sensing technologies, including accelerometers and gyroscopes, to track both rotation and position.
    - Data Processing Units:
        - Onboard Computers: Process the data from various sensors to compute the spacecraft's orientation. These computers run sophisticated algorithms that fuse data from multiple sources to enhance accuracy.
        - Software Algorithms: Include Kalman filters, quaternion estimators, and other mathematical models that integrate sensor data over time to predict and correct the spacecraft's orientation.
    - Actuators and Control Mechanisms:
        - Reaction Wheels: These are flywheels spun at controlled speeds to adjust the spacecraft's orientation using the conservation of angular momentum.
        - Control Moment Gyroscopes (CMGs): Similar to reaction wheels but capable of generating larger torques for faster orientation adjustments.
        - Thrusters: Small rockets that can alter the spacecraft's orientation by firing in short bursts. Used primarily for larger, quick adjustments or when reaction wheels are saturated.
3. Operational Phases of the Orientation Determination System

   Understanding the operational workflow is crucial for maximizing the efficiency and accuracy of the orientation determination system.
    - Initialization Phase:
        - The spacecraft uses coarse sensors like magnetometers and Sun sensors to establish a preliminary orientation after launch.

        - Low-accuracy data are refined using algorithms that compensate for sensor biases and initial uncertainties.
    - Operational Tracking:
        - High-precision sensors like star trackers and gyroscopes take over to provide detailed orientation data.
        - Data fusion algorithms continuously integrate new sensor data to update the spacecraft's orientation.
    - Maneuvering Phase:
        - During orbital maneuvers, thrusters and gyroscopes adjust the spacecraft's orientation to follow the desired trajectory.
        - The system coordinates with the navigation system to ensure that changes in position and orientation are aligned.
    - Stabilization and Drift Correction:
        - Reaction wheels and CMGs adjust the orientation to counteract any undesired drift caused by external forces like gravity gradients and solar radiation pressure.
        - Continuous monitoring helps detect and correct minor misalignments and sensor drift.

4. Key Technologies and Algorithms

   The effectiveness of an orientation determination system depends on the technologies and algorithms it employs.

    - Sensor Fusion Algorithms:
        - Kalman Filtering: A widely used technique that combines data from various sensors to estimate the spacecraft's orientation with reduced uncertainty.
        - Quaternion Algorithms: Use quaternion mathematics to avoid the singularities and complexities of traditional Euler angles in 3D space.
    - Fault Tolerance and Redundancy:

        - Critical for ensuring system reliability, especially in the harsh environment of space.
        - Redundant sensors and backup systems are employed to maintain functionality even if a primary component fails.
    - Machine Learning and Predictive Analytics:
        - Advanced systems incorporate machine learning to predict sensor failures and compensate for them in real-time.
        - Predictive algorithms analyze historical data to optimize orientation adjustments and reduce fuel consumption.

5. Applications and Implications

   The orientation determination system has a wide range of applications in space exploration and satellite operations.

    - Satellite Communications: Ensures that communication satellites are accurately pointed towards Earth to maximize signal strength and coverage.
    - Astronomical Observations: Allows space telescopes to maintain precise orientation to capture high-quality images of celestial objects.
    - Interplanetary Missions: Helps navigate spacecraft through deep space, adjusting orientation for gravity assists and trajectory corrections.
    - Space Station Operations: Essential for maintaining the orientation of the International Space Station and other habitable satellites for solar power optimization and docking operations.

6. Challenges and Future Directions

   While the current systems are robust, ongoing research aims to address several challenges and enhance the capabilities of orientation determination systems.

    - Miniaturization: Developing smaller, more power-efficient sensors and actuators to fit within the constraints of smaller spacecraft and CubeSats.

- Autonomous Operations: Enhancing the autonomy of spacecraft by implementing AI-driven orientation control that can adapt to changing conditions without ground intervention.
- Interstellar Navigation: As missions aim further into the solar system and beyond, orientation systems must evolve to use interstellar navigation cues, such as pulsars and distant quasars.

By understanding the structure and functionality of the orientation determination system in space, engineers and researchers can continue to innovate and improve the safety, efficiency, and effectiveness of space missions.

1.5 Satellite Kinematics

In this article, we analyze four main models of the geomagnetic field: The International Geomagnetic Reference Field (IGRF), the oblique and direct dipole, and the averaged field model. The geomagnetic induction vector will be calculated in several coordinate systems that are commonly used to analyze the angular motion of a spacecraft.

$O_a Y_1 Y_2 Y_3$- inertial system, , where O_a– center of mass of the Earth, axis $O_a Y_3$ is directed along the Earth's rotation axis, $O_a Y_1$ - lies in the plane of the Earth's equator and

Predicting the orbit of a spacecraft is an integral part of determining its orientation. Accurate determination requires information from both the inertial and solid coordinate systems. Satellite kinematics models provide information in the inertial system. is directed to the ascending node of the satellite's orbit (we assume a circular orbit), $O_a Y_2$ - complements the system to the right.

Orbital Elements

The Keplerian orbital elements fully describe the shape and orientation of the orbit. These elements consist of the major axis, eccentricity, inclination, ascending node longitude, pericenter argument, and true anomaly. A visualization of each orbital element can be seen in Figure 1.5

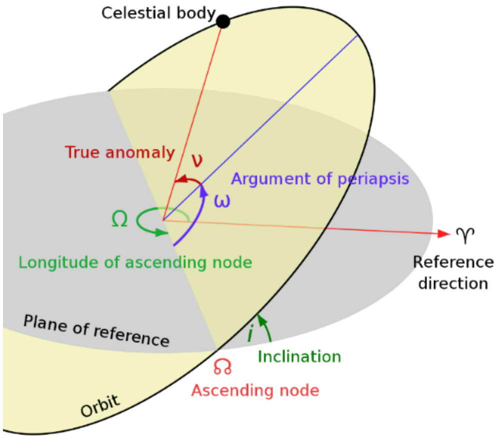


Figure 1.5 Visualization of Orbital Elements

The shape of the orbit is determined by the mission objectives of the satellite. For example, satellites in a constellation may have different inclinations to fully cover the Earth. Orbital elements are used to predict the position of a satellite at a future point in time. This estimated position is important for determining the orientation because the necessary information from the inertial system is predicted from the estimated position of the satellite. For example, when using a solar sensor and magnetometer, an astronomical almanac is used to determine the position of the sun relative to the satellite, and models of the Earth's magnetic field are used to estimate the magnetic force acting on the satellite. This data is combined with sensor readings to calculate the estimated rotation matrix. The future position of the satellite is determined by orbital element propagation.

Orbit propagation

When the satellite's orbit is known, its position and velocity at a given time can be estimated through propagation. This can be done analytically or numerically, and the choice of method depends on the requirements of the satellite. The accuracy of the propagation translates into a better determination of the orientation, which leads to more accurate control of the satellite's orientation. For example, satellites intended for Earth observation require a higher degree of attitude accuracy, while demonstration satellites may only need general estimates of their position.

Disturbances

A propagation model is never accurate due to various disturbances that cannot be accurately accounted for. The four main sources of perturbations for small satellites in low Earth orbit (LEO) in terms of magnitude are: residual magnetic force, aerodynamic drag, solar radiation, and gravity gradient. The equations for determining the magnitude of these disturbances are estimates, and it is assumed that the orientation control algorithms will compensate for any errors.

A residual magnetic force exists due to interactions between spacecraft components and the Earth's magnetic field. This force is difficult to estimate due to the diversity of satellite structures, with typical assumptions for the residual magnetic moment of small satellites ranging from 0.1 Am^2 to 1 Am^2. Aerodynamic drag is significant in low Earth orbit due to the presence of the atmosphere at these altitudes. Torque is calculated through the vector product of the center of pressure and forces. Although the exact center of pressure is difficult to determine for a simulated satellite, it is often assumed to be offset by 5% from the geometric center. More accurate satellite models can improve subsequent iterations of the simulation. The gravity gradient perturbation occurs due to the difference in gravitational pull on different parts of an asymmetric satellite.

Orbit Determination Error Analysis

When analyzing the orbit determination errors, it is important to take into account the uncertainty of the initial conditions, which can significantly affect the accuracy of the prediction. These errors can be caused by unpredictable maneuvers, control errors, and dynamic disturbances. Orbital stability can also be influenced by long-lasting effects such as tidal interactions and resonance phenomena, which require more complex mathematical models to describe them.

Modeling of Orbital Dynamics

To improve the prediction of orbital dynamics, numerical integration methods are used to effectively account for nonlinear effects and disturbances. Methods such as Runge-Kutta, Adams-Bashforth, and Gauss-Legendre provide high computational accuracy for the analysis of long orbital evolutions.

Orbit Correction and Attitude Control

Various techniques are used to correct the orbit and control the attitude of the satellite, depending on the type of mission and maneuvering requirements. The use of reaction wheels, magnetorotor actuators and small motors allows to effectively influence the orientation and trajectory of the satellite.

Use of Satellite Data for Navigation

An important aspect of satellite kinematics is the use of the data obtained for navigation and remote sensing. Satellite systems such as GPS and GLONASS provide the necessary positioning data, which is critical for many aspects of modern technological applications.

Long-term effects and interplanetary missions

For interplanetary missions, where the spacecraft goes beyond the Earth's orbit, it becomes important to take into account effects such as the Euthinga-Leod orientation and the influence of the solar wind. This requires additional corrections in orbit propagation models to ensure mission reliability and accuracy.

1.6 Conclusion

This section presents an analysis of several aspects of attitude determination systems. It considers their mathematical models, coordinate systems used, methods for determining the position of objects in three-dimensional space, and equations modeling the satellite's motion using Euler's dynamical equations. Special attention is given to the structure of orientation determination systems and mathematical models of the Earth's magnetic field.

This information is of great value to scientists, engineering professionals, and space enthusiasts because it provides a comprehensive understanding of the fundamental principles underlying spacecraft orientation technologies. This will enable them to improve existing systems and develop new approaches to explore space more effectively.

CHAPTER 2. Study of orientation determination with auto-compensation of measurement errors

2.1 Analysis of auto-compensation of meter errors

In outer space, the accuracy and reliability of measuring instruments are of particular importance, as they directly affect the operation of satellites, space probes and interplanetary stations. Auto-compensation of measurement errors is a critical technology that improves measurement accuracy by correcting systematic and random errors. This ensures stable operation of spacecraft, improves navigation and increases the overall efficiency of missions.

1. Definition and classification of measurement errors

Measurement errors in spacecraft can be divided into:

Systematic errors, which are usually consistent and repeatable, so they can be modeled and corrected. These errors include instrument calibration, linearity errors, temperature variations, and other factors that vary in a predictable manner.

Random errors that occur due to unpredictable changes in conditions, such as cosmic radiation, micrometeorite impacts, electromagnetic interference, which lead to noise and fluctuations in the data.

2. Technologies of auto-compensation of errors

2.1 Calibration of measuring instruments

Calibration is the main method for correcting systematic errors. In space, this process includes:

Online calibration, where instrument parameters are adjusted in real time using embedded microprocessors.

The use of external calibration standards, which may include the use of signals from stabilized space sources such as stars or quasars.

2.2 Adaptive filtering

Adaptive filters in space applications reduce random errors:

Kalman filters are used to optimize the state estimation of a spacecraft based on sensor inputs and known dynamic models.

Median filters are used to reduce the impact of anomalous outliers in measurements that may occur due to space conditions.

2.3 Self-tuning systems

Self-tuning systems use machine learning algorithms to improve measurement instruments based on collected data:

Neural network algorithms can be trained to identify and correct errors in real time.

Support vector machines (SVMs) analyze large amounts of data to determine the optimal calibration parameters.

3. Impact of autocompensation on the efficiency of space missions

3.1 Improved navigation and orientation

Autocompensation helps to improve the accuracy of spacecraft navigation and attitude control systems. It provides:

Accurate positioning of satellites to ensure stable communication and surveillance.

Efficient motion control, which reduces fuel consumption and increases the duration of active operation of the spacecraft.

3.2 Reduced maintenance costs and risks

Auto-compensation systems reduce the need for frequent maintenance of measuring instruments in harsh space conditions, which

Reduces the cost of support and service operations.

Minimizes the risks associated with equipment failures and measurement errors, ensuring safer and more reliable operation of the spacecraft.

3.3 Optimization of scientific research

Auto-compensation of measurement errors allows for more accurate scientific research in space:

Improving the accuracy of data for space research, including astronomical observations, climate research, and geophysical measurements.

Increasing the amount of useful information obtained from space missions by reducing the impact of errors on measurement results.

Autocompensation of measurement errors in space is an important component of modern space missions aimed at improving the efficiency and safety of research. The use of advanced calibration methods, adaptive filtering, and self-tuning allows to minimize the negative impact of errors on the functioning of spacecraft. This ensures measurement accuracy, optimizes resource utilization, and contributes to the success of long-term space missions.

2.2 Improved orientation accuracy based on auto-compensation of errors

This investigation concerns the enhancement of the precision of determining the orientation of a sling system based on the spatial rotation of gyroscopes relative to an object. The subject of the analysis is a mathematical model of errors in measuring orientation angles, which considers the constant displacements of the gyroscopes. The proposed error model makes it relatively straightforward to assess the effectiveness of autocompensation. The analysis demonstrated that the problem can be simplified by integrating the matrix of directional cosines that define the position of the sensing elements within the reference coordinate system. The outcomes of the theoretical analysis and modeling validated that the proposed method is highly efficient in enhancing the precision of kinematic parameter measurement.

One method of creating autonomous, high-precision tape inertial systems is the autocompensation of sensor errors. A promising approach to implementing this approach is the forced rotation of a block of sensing elements. An essential aspect of the overall problem is the objective of enhancing the precision of determining the angular orientation of an object, which is also of significant independent importance. The choice of the laws of spatial rotation of the sensing elements is of paramount importance; consequently, the construction of a mathematical model of errors is necessary.

Let us assume that the sensing elements (three gyroscopes) are mounted on a platform that rotates in a suspension (Fig. 2.2.1) relative to the object according to the laws , .

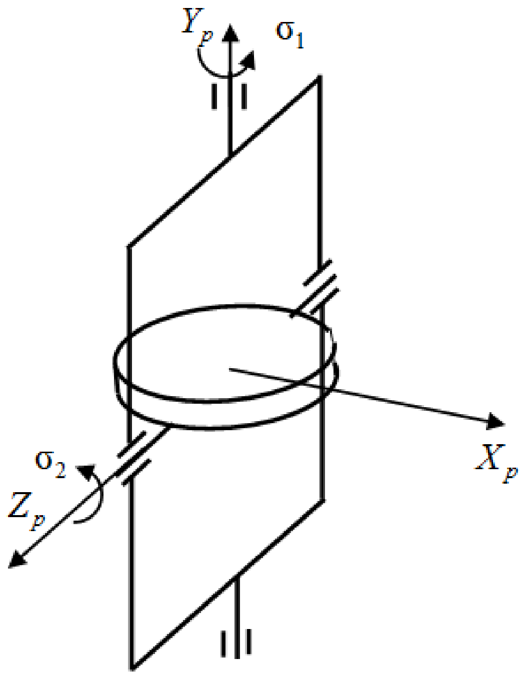


Figure 2.2.1

Let's evaluate the effectiveness of this scheme for auto-compensation of gyroscope errors.

The transition from the coordinate system associated with the object to the coordinate system associated with the platform (Fig. 2.2.2) is determined by the transformation matrix, as shown in the figure.

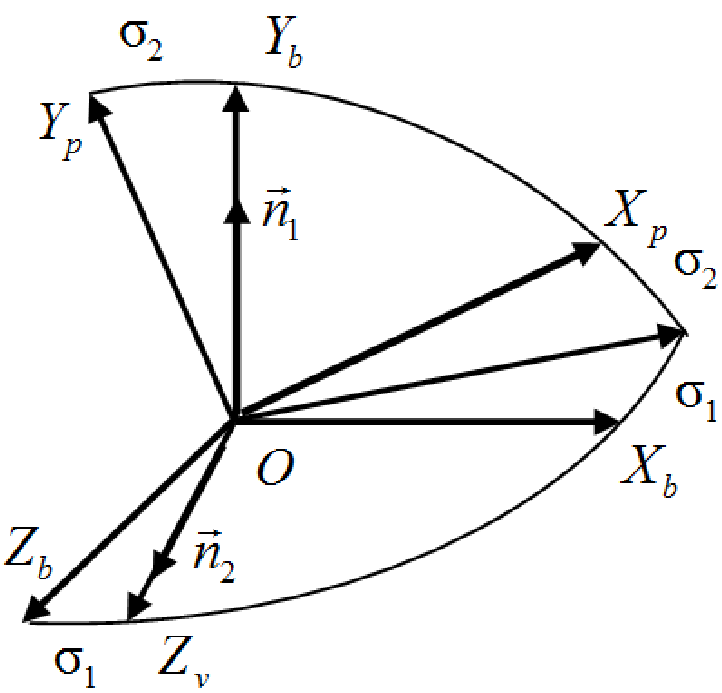


Figure 2.2.2

$$\boldsymbol{B}_p = \begin{bmatrix} \cos\sigma_1\cos\sigma_2 & -\cos\sigma_1\sin\sigma_2 & \sin\sigma_1 \\ \sin\sigma_2 & \cos\sigma_2 & 0 \\ -\sin\sigma_1\cos\sigma_2 & s\text{in}\,\sigma_1\sin\sigma_2 & \cos\sigma_1 \end{bmatrix}$$

We see that the matrix $\boldsymbol{B}_p$ does not contain constant components if the angular velocities n_1 and n_2 are different.

The transition from the coordinate system $OX_bY_bZ_b$ to the reference (fixed) coordinate system $OX_oY_oZ_o$ is defined by the matrix $\boldsymbol{B}_b$. This matrix is found as a solution of Poisson's equation.

$$\dot{\boldsymbol{B}}_b = \boldsymbol{B}_b\boldsymbol{\Omega}_b,$$

where

$$\boldsymbol{\Omega}_b = \begin{bmatrix} 0 & -\omega_{z_b} & \omega_{y_b} \\ \omega_{z_b} & 0 & -\omega_{x_b} \\ -\omega_{y_b} & \omega_{x_b} & 0 \end{bmatrix}.$$

$\boldsymbol{\omega} = \begin{bmatrix} \omega_{x_b} & \omega_{y_b} & \omega_{z_b} \end{bmatrix}^{\mathrm{T}}$ is the vector of body angular velocities.

In addition to the useful angular velocities, the vector of the output signals of the gyroscopes $\begin{bmatrix} \omega_{x_p} & \omega_{y_p} & \omega_{z_p} \end{bmatrix}^{\mathrm{T}}$ also contains the constant biases of gyroscopes $\boldsymbol{\omega}_d = [\omega_{dx}\ \omega_{dy}\ \omega_{dz}]^{\mathrm{T}}$ and the projections of the angular velocities of the additional rotations $[n_1\sin\sigma_2\ n_1\cos\sigma_2\ n_2]^{\mathrm{T}}$. Therefore to find the angular position of the object using the output signals of the gyroscopes, it is necessary to subtract the corresponding angular velocities of rotation of the platform relative to the object and difference project into the coordinate system $OX_bY_bZ_b$:

$$\begin{bmatrix} \hat{\omega}_{x_b} \\ \hat{\omega}_{y_b} \\ \hat{\omega}_{z_b} \end{bmatrix} = \boldsymbol{B}_p \left(\begin{bmatrix} \omega_{x_p} \\ \omega_{y_p} \\ \omega_{z_p} \end{bmatrix} - \begin{bmatrix} n_1\sin\sigma_2 \\ n_1\cos\sigma_2 \\ n_2 \end{bmatrix} \right).$$

The device value of body direction cosine matrix $\hat{\boldsymbol{B}}_b$ is sought as a solution to the Poisson equation

$$\dot{\hat{\boldsymbol{B}}}_b = \hat{\boldsymbol{B}}_b \hat{\boldsymbol{\Omega}}_b ,$$

where

$$\hat{\boldsymbol{\Omega}}_b = \begin{bmatrix} 0 & -\hat{\omega}_{z_b} & \hat{\omega}_{y_b} \\ \hat{\omega}_{z_b} & 0 & -\hat{\omega}_{x_b} \\ -\hat{\omega}_{y_b} & \hat{\omega}_{x_b} & 0 \end{bmatrix}.$$

For an analytical evaluation of orientation errors, we will write this expression in the form

$$\dot{\hat{\boldsymbol{B}}}_b = \hat{\boldsymbol{B}}_b \left(\boldsymbol{\Omega}_b + \boldsymbol{B}_p \boldsymbol{\Omega}_d \boldsymbol{B}_p^{-1} \right),$$

where

$$\Omega_d = \begin{bmatrix} 0 & -\Omega_{dz} & \Omega_{dy} \\ \Omega_{dz} & 0 & -\Omega_{dx} \\ -\Omega_{dy} & \Omega_{dx} & 0 \end{bmatrix}.$$

Considering the estimation error $\Delta_{\boldsymbol{B}_b}$ of the "ideal" matrix $\boldsymbol{B}_b$ to be small, we write

$$\hat{\boldsymbol{B}}_b = (\boldsymbol{I} + \Delta_{\boldsymbol{B}_b}) \boldsymbol{B}_b ,$$

where

$\Delta_{\boldsymbol{B}_b} = \begin{bmatrix} 0 & -\varepsilon_z & \varepsilon_y \\ \varepsilon_z & 0 & -\varepsilon_x \\ -\varepsilon_y & \varepsilon_x & 0 \end{bmatrix}$ is the small skew-symmetric matrix of errors, which is recorded in the coordinate system $OX_oY_oZ_o$.

Then we will write it down

$$\dot{\Delta}_{\boldsymbol{B}_b} \boldsymbol{B}_b + (\boldsymbol{I} + \Delta_{\boldsymbol{B}_b}) \dot{\boldsymbol{B}}_b = (\boldsymbol{I} + \Delta_{\boldsymbol{B}_b}) \boldsymbol{B}_b \left(\boldsymbol{\Omega}_b + \boldsymbol{B}_p \boldsymbol{\Omega}_d \boldsymbol{B}_p^{-1} \right).$$

Neglecting the product of errors, we will have the following expression

$$\boldsymbol{\Omega}_\varepsilon = \boldsymbol{B} \boldsymbol{\Omega}_d \boldsymbol{B}^{\mathrm{T}} .$$

where $\boldsymbol{\Omega}_\varepsilon = \dot{\Delta}_{\boldsymbol{B}_b}$, $\boldsymbol{B} = \boldsymbol{B}_b \boldsymbol{B}_p$ ~ the transition matrix from the coordinate system $OX_pY_pZ_p$ to the coordinate system $OX_oY_oZ_o$.

This expression can be thought of as changing the transformation when the basis changes

Matrix equation $\boldsymbol{\Omega}_\varepsilon = \boldsymbol{B} \boldsymbol{\Omega}_d \boldsymbol{B}^{\mathrm{T}}$ .corresponds to a vector equation

$\dot{\boldsymbol{\varepsilon}} = \boldsymbol{B}\boldsymbol{\omega}_d$,

where $\boldsymbol{\varepsilon} = [\varepsilon_x \ \varepsilon_y \ \varepsilon_z]^{\mathrm{T}}$ is the error vector in the coordinate system. $OX_oY_oZ_o$.

Using an expression $\dot{\boldsymbol{\varepsilon}} = \boldsymbol{B}\boldsymbol{\omega}_d$, simplifies the solution of the problem, since in this case the analysis of the movement of the object is performed before the analysis of errors, while when using an equation $\dot{\hat{\boldsymbol{B}}}_b = \hat{\boldsymbol{B}}_b\hat{\boldsymbol{\Omega}}_b$, these tasks are studied together.

That is, for constant gyroscope errors, the task is reduced to matrix $\boldsymbol{B}$ integration, and for a stationary object – to matrix $\boldsymbol{B}_p$ integration.

The integral of the matrix $\boldsymbol{B}_p$ is equal to

$$J_1 = \begin{bmatrix} \frac{1}{2}\left(\frac{\sin(n_1-n_2)t}{n_1-n_2}+\frac{\sin(n_1+n_2)t}{n_1+n_2}\right) & \frac{1}{2}\left(\frac{\cos(n_1+n_2)t-1}{n_1+n_2}-\frac{\cos(n_1-n_2)t-1}{n_1-n_2}\right) & -\frac{\cos n_1 t-1}{n_1} \\ -\frac{\cos n_2 t-1}{n_2} & -\frac{\sin n_2 t}{n_2} & 0 \\ \frac{1}{2}\left(\frac{\cos(n_1+n_2)t-1}{n_1+n_2}+\frac{\cos(n_1-n_2)t-1}{n_1-n_2}\right) & \frac{1}{2}\left(\frac{\sin(n_1-n_2)t}{n_1-n_2}-\frac{\sin(n_1+n_2)t}{n_1+n_2}\right) & \frac{\sin n_1 t}{n_1} \end{bmatrix}.$$

The matrix of time-independent components is

$$J_2 = \begin{bmatrix} 0 & \frac{n_2}{n_1^2-n_2^2} & \frac{1}{n_1} \\ \frac{1}{n_2} & 0 & 0 \\ -\frac{n_1}{n_1^2-n_2^2} & 0 & 0 \end{bmatrix}.$$

From these matrices, we can see that the angular velocities of rotation n_1 and n_2 must be different.

The presence of irregular rotational frequencies suggests the potential for reverse rotation to enhance the efficacy of this approach to error compensation. It is noteworthy that the matrix comprising constant components assumes significant significance in inertial navigation systems (INS), particularly in the calculation of velocities and coordinates, due to the projection of gravitational acceleration on the sensitivity axes of accelerometers.

In order to ascertain the veracity of this methodology, it is necessary to accept the following.

$\omega_{dx} = \omega_{dy} = \omega_{dz} = 10,^{0}/h, \quad n_1 = 1c^{-1}, \quad n_2 = 2c^{-1}, \quad n_2 = 2c^{-1}, \quad \omega_{x_b} = 1\cos 0.1t,^{0}/s, \ \omega_{y_b} = 2\cos 0.2t.^{0}/s,$ $\omega_{z_b} = 3\cos 0.3t,^{0}/s.$

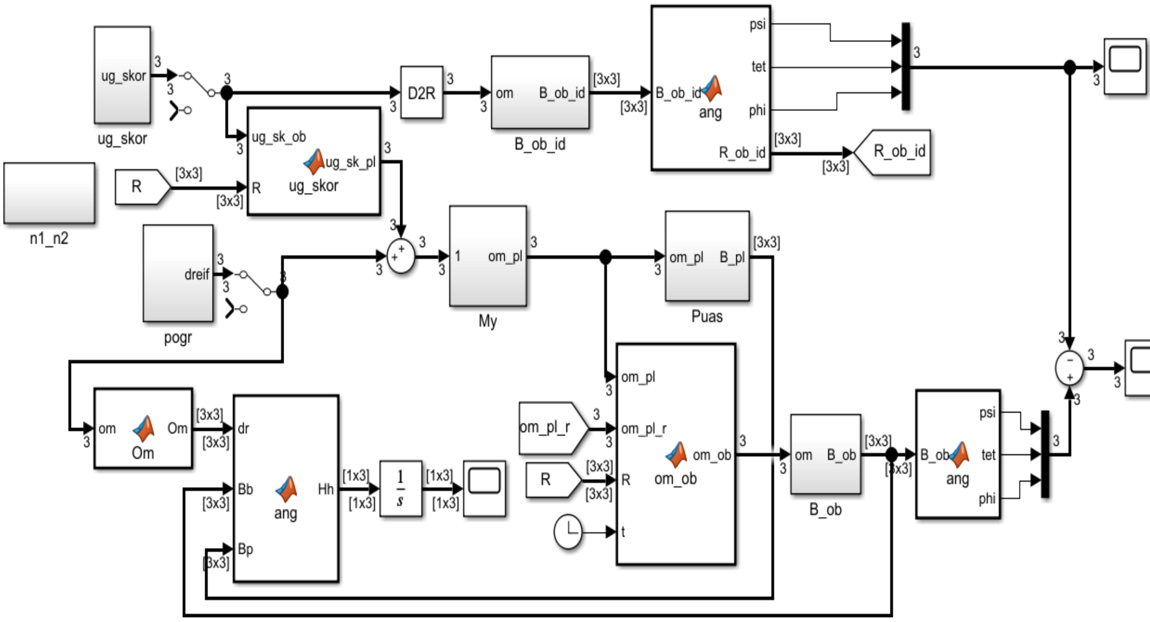


Figure 2.2.3

2.3 Theoretical analysis of the orientation system

Orientation systems are key in many applications, from aerospace engineering to robotics and marine research. These systems use different methods to stabilize the position in space, which are divided into passive and active. Passive stabilization methods are determined by the physical characteristics of the object, while active methods involve external intervention to control the orientation.

Passive stabilization methods

Passive stabilization methods use the intrinsic physical characteristics of an object to maintain its orientation. These methods do not require active intervention and usually provide stabilization in only two axes.

Gravity Gradient: This method uses the Earth's gravitational pull to stabilize an object in space. The center of gravity of the object is critical, as its incorrect positioning can reduce the stabilizing effect. For small satellites like CubeSats, the center of gravity should be located no further than 2 cm from the center to optimize the gravity gradient as a stabilizing factor.

Rotation stabilization: This method utilizes the high moment of inertia of the object by rotating it around its own axis. The correct distribution of the object's mass is key to the effectiveness of this method. Optimal mass placement contributes to rotational stability, while incorrect placement can lead to disturbances. Rotational speed affects stabilization, with increased stabilization as rotational speed increases.

Passive magnets: The use of passive magnets allows the object to be oriented according to the Earth's magnetic field. However, this method may not be ideal for all applications, as it aligns the object with the Earth's magnetic field, which may not be the required orientation for a particular mission. Additionally, the strong magnetic fields of passive magnets can affect magnetic sensors, saturating them and reducing the accuracy of orientation determination.

Active stabilization methods

Active stabilization methods provide the ability to control orientation in three axes by using external forces and moments to correct the position of an object in space.

Magnetic coils: These coils create an electromagnetic field that interacts with the Earth's magnetic field to generate torque. This torque allows you to control the orientation of an object, especially when the magnetic torque sensor and the Earth's magnetic field are oriented in parallel. When they are perpendicular, the stabilization effect is reduced.

Reaction wheel: Flywheels use a rotational impulse to generate torque, which is used to stabilize and control orientation. When the flywheel accelerates or decelerates, it transmits a reverse torque to the object, changing its orientation.

Reactive control system engines: These systems use the release of a substance to create a reactive torque that is used to correct orientation. They are effective for controlling orientation in environments where other methods are less effective or unavailable.

Applications

The theoretical analysis shows that a combination of passive and active methods can provide optimal attitude stabilization for different types of missions. Passive methods are important for basic stabilization and energy reduction, while active methods provide precise and flexible attitude control. The choice of specific methods and their combination depends on the specific mission requirements and operational conditions.

In aerospace engineering, double rotation is used to analyze and control the orientation of spacecraft and satellites. In particular, the concept is applied to:

Orientation stabilization: Changing the rotation angles allows you to control the orientation of the satellite, ensuring the required position for observation, communication or other missions.

Flywheel control: The flywheels use torque to change orientation. Dual rotation helps calculate the necessary parameters to control these systems.

Orientation determination systems are crucial for a variety of applications, from aerospace and satellite navigation to robotics and marine exploration. These systems ensure accurate positioning and directional guidance by incorporating automatic error compensation techniques that address inherent measurement inaccuracies. This theoretical analysis provides an overview of the basic principles, methods, and challenges associated with these systems.

Basic Principles of Orientation Systems

Orientation systems typically use a combination of sensors such as gyroscopes, accelerometers, magnetometers, or a fusion of several different types of sensors to determine the orientation of an object relative to a reference frame. The primary tasks of these systems include:

1. Sensing: Capturing raw data regarding motion and gravitational forces.
2. Processing: Converting raw sensor data into usable orientation information.
3. Compensation: Adjusting the output to mitigate errors and improve accuracy.

Methods of Automatic Error Compensation

Automatic error compensation is vital for enhancing the reliability and accuracy of orientation systems. The common methods include:

1. Sensor Fusion: Combining data from multiple sensors to minimize the effect of individual sensor noise and biases. Techniques such as Kalman filters or complementary filters are often used to blend data from gyroscopes, accelerometers, and magnetometers smoothly.

2. Drift Correction: Gyroscopes, which are used for measuring rotational motion, can accumulate drift over time. Algorithms that detect and correct bias in gyroscopic data are essential for long-term accuracy.

3. Adaptive Algorithms: These algorithms adjust the parameters of the error compensation model in real-time based on observed errors, enhancing the system's ability to react to dynamic changes in the operating environment.

Challenges in Orientation Determination Systems

Despite advances in sensor technology and algorithm development, several challenges remain:

1. Sensor Degradation: Over time, sensors may degrade, leading to a decrease in measurement accuracy. Regular calibration and maintenance are required to ensure ongoing reliability.

2. Environmental Factors: External influences such as magnetic fields, temperature fluctuations, and mechanical shocks can affect sensor performance. Systems must be robust and adaptable to these conditions.

3. Complex Dynamics: Rapid movements or changes in direction can introduce errors that are difficult to compensate for using standard methods. Advanced algorithms are needed to handle these dynamics effectively.

Case Studies and Applications

In practical applications, orientation systems with automatic error compensation are employed in various fields:

- Aerospace: In spacecraft and satellites, orientation systems must operate in a vacuum and zero-gravity environment, where traditional methods based on gravity or magnetic fields are not applicable.

- Automotive: Advanced driver-assistance systems (ADAS) use orientation systems to improve vehicle navigation and safety features.

- Consumer Electronics: Smartphones and wearable devices use miniaturized orientation systems for screen rotation, fitness tracking, and augmented reality applications.

The theoretical framework of orientation systems with automatic error compensation underpins the development and deployment of highly accurate navigational aids in both everyday and critical applications. Ongoing research and development are directed towards refining these systems to handle increasingly complex scenarios and environments, ensuring that they can deliver precise orientation data in real time, under all conditions. This analysis underscores the importance of sophisticated error compensation techniques in maintaining the functionality and reliability of modern orientation systems.

2.4 Conclusion

The objective of this study was to investigate the potential for improving the accuracy of orientation determination through the use of autocompensation techniques. Upon consideration of the aforementioned section, it becomes evident that the meters can be effectively calculated with regard to the errors.

CHAPTER 3. Implementation of computer models in MATLAB-Simulink environment for determining the orientation based on the kinematic equations of body motion

3.1 General characteristics of the consolidated computer model

The creation of computer models for orientation determination with auto-compensation of measurement errors plays a key role in modern technological research and development. This approach allows designers and engineers to efficiently analyze and optimize orientation systems, ensuring high accuracy and reliability in critical applications such as aerospace, robotics, marine research, and automotive.

Advantages of creating computer models

First, the accuracy and detail of modeling: Computer modeling can reproduce the complex interactions in orientation sensing systems with high accuracy. Models can take into account a variety of factors, such as nonlinear dynamics, external disturbances, and internal sensor errors, allowing engineers to gain a deeper understanding of system behavior.

Secondly, it speeds up development and testing: Using models can significantly reduce development and testing time and costs. Engineers can identify and fix potential problems before prototyping, which reduces the risks and costs of experimental research.

Third, optimization of system parameters: Computer models allow for parametric studies, in which key parameters are automatically varied to determine their impact on overall system performance. This helps optimize the design to achieve the best performance.

Fourthly, analysis of operation in real conditions: Simulation allows engineers to analyze how the orientation system will perform in real-world conditions, including extreme temperatures, pressures, vibrations, and other environmental influences.

Fifth, decision support: Computer modeling provides valuable decision support, allowing for the comparison of different designs and control strategies, as well as risk analysis in the early stages of a project.

Disadvantages of creating computer models

First, the complexity of the models: High detail and complexity of models can lead to high computational costs. Sometimes developing and calibrating detailed models requires significant time and resources.

Second, the accuracy of the input data: The accuracy of computer modeling is highly dependent on the quality of the input data. Errors in measurements or incomplete input information can significantly affect modeling results.

Third, anticipation of unpredictable disturbances: Models may not take into account all potential disturbances or inaccurately predict system responses to rare or extreme events due to limitations in knowledge of the environment or material properties.

Fourth, dependence on the experience of developers: The effectiveness of computer models depends on the experience and knowledge of the engineers who develop and analyze them. Insufficient qualifications can lead to misinterpretation of the results.

Fifth, limitations on the use of models: Some aspects of a physical system may be difficult to reproduce in a model due to their nonlinear nature or complexity of interaction with other systems. This can lead to simplifications that reduce the realism of the model.

Fig. 3.1.1 shows a consolidated computer model containing models in accordance with the kinematic equations under consideration.

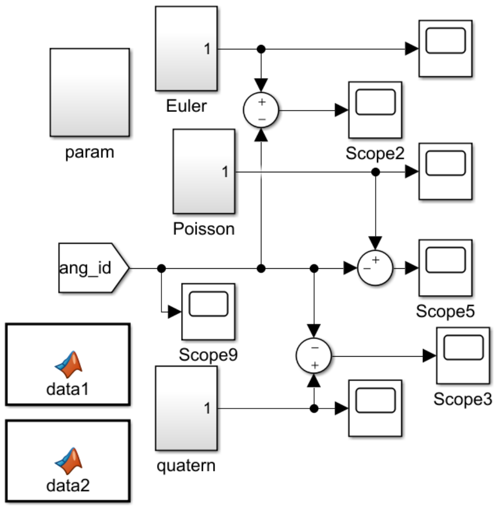


Figure 3.1.1 Main simulink model.

Let's analyze our computer model in more detail. We can distinguish 4 main blocks:

1. Param
2. Euler
3. Poisson
4. Quatern
5. data 1/2

This system does not use the standard block, but instead uses the Embedded MATLAB Function block. This block can be useful in the following situations:

- You have an existing MATLAB function that models specific functionality, or you can easily create one.

- Your model requires specialized functionality that is impossible or impractical to represent using the Simulink graphical language.

- You are more comfortable modeling specialized functionality using a MATLAB function than using a Simulink block diagram.

- The specialized functionality you want to model does not include continuous or discrete dynamic states. To model dynamic states, you should use S-functions.

3.2 Implementation of separate blocks and systems

Let's take a closer look at the structure of the main systems.

In the “param” subsystem, reference Euler angles, reference angular velocities in the linked coordinate system, and angular velocities in the linked coordinate system are generated taking into account measurement errors.

Reference angles are formed in the form

$\varphi = r_{\varphi} t;\ \theta = r_{\theta} t;\ \psi = r_{\psi} t,$

Where

$$r_{\varphi} = \frac{\varphi_{\max}}{t_m};\ r_{\theta} = \frac{\theta_{\max}}{t_m};\ r_{\psi} = \frac{\psi_{\max}}{t_m};$$

$\varphi_{\max},\ \theta_{\max},\ \psi_{\max}$ maximum values of angles;

$t_m = 120\ c$ modeling time.

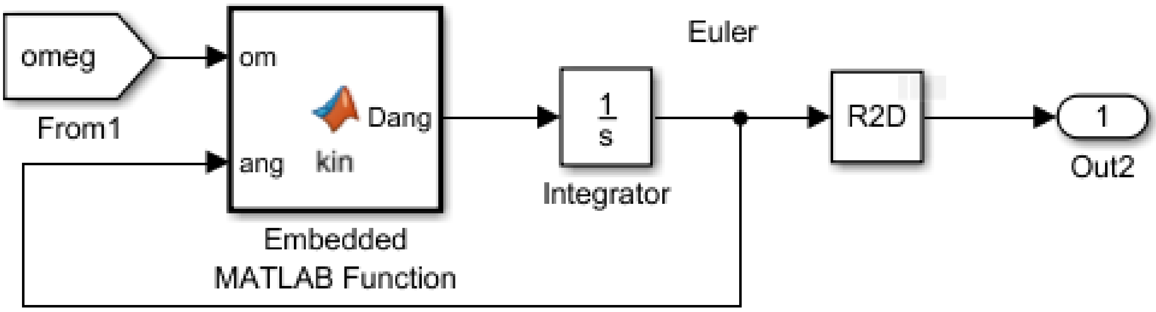


Figure 3.2.1 Subsystem Euler

In the “Embedded MATLAB Function” block, the derivatives of the corresponding angles are calculated

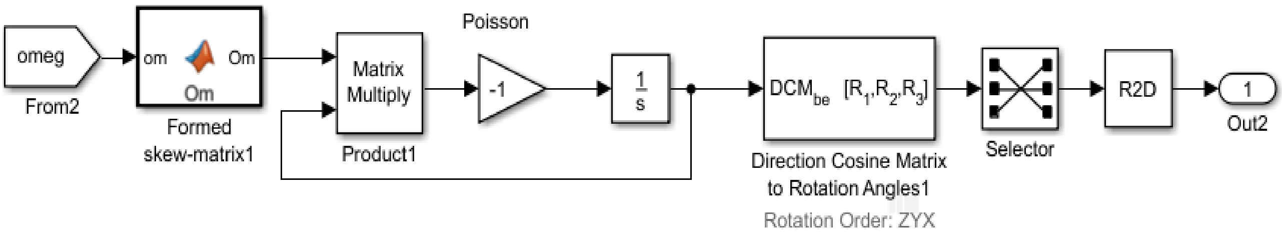
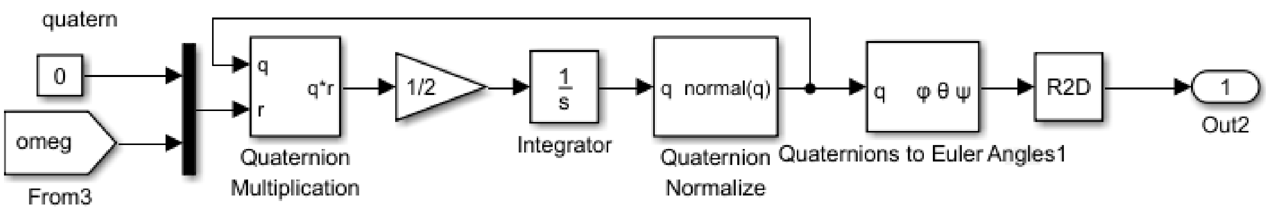


Figure 3.2.2 Subsystem Poisson

The “Formed skew-matrix” block generates a skew-symmetric matrix of angular velocities.

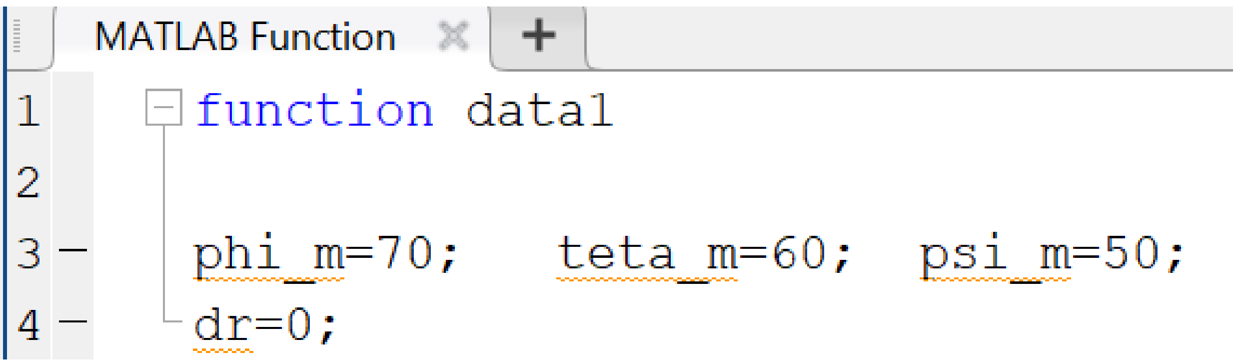
The initial conditions are set on the integrator with a unit matrix (the “eye(3)” function), i.e., a matrix of guide cosines according to the zero initial values of the Euler. The introduction of the “Selector” block is due to the fact that the “Direction Cosine Matrix to Rotation Angles1” block, in accordance with the accepted sequence of rotations "ZYX", forms a sequence of angles "$\psi\theta\varphi$", the angles are displayed on the



graphs in the sequence "$\varphi\theta\psi$". angles.

Figure 3.2.3 Subsystem Quatern

The initial conditions are set on the integrator by an array (quaternion) $[1\ 0\ 0\ 0]$



according to the zero initial values of the Euler angles.

Figure 3.2.4 Data 1

```
function data2

dr_x=1e-4;
dr_y=5e-5;
dr_z=-1e-4;
dr=[dr_x dr_y dr_z];
```

Figure 3.2.5 Data 2

3.3 Determination of orientation without and with measurement errors

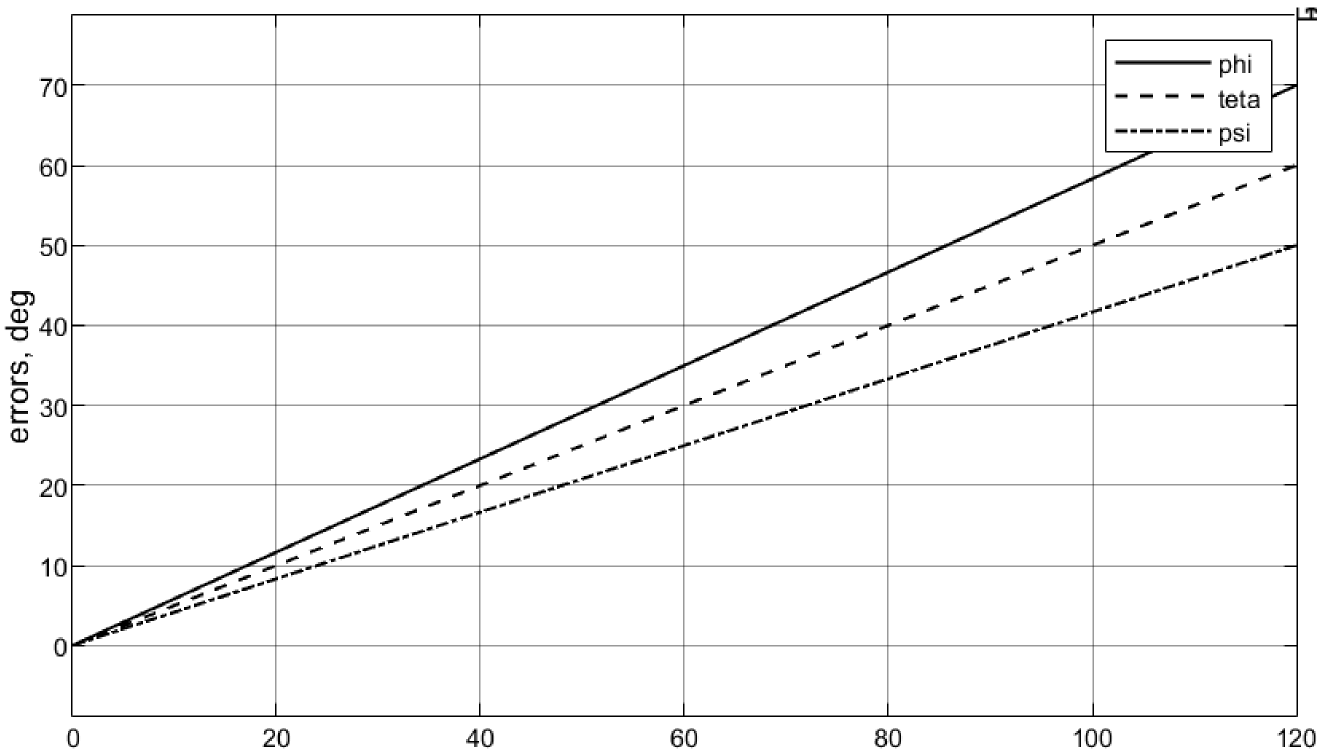
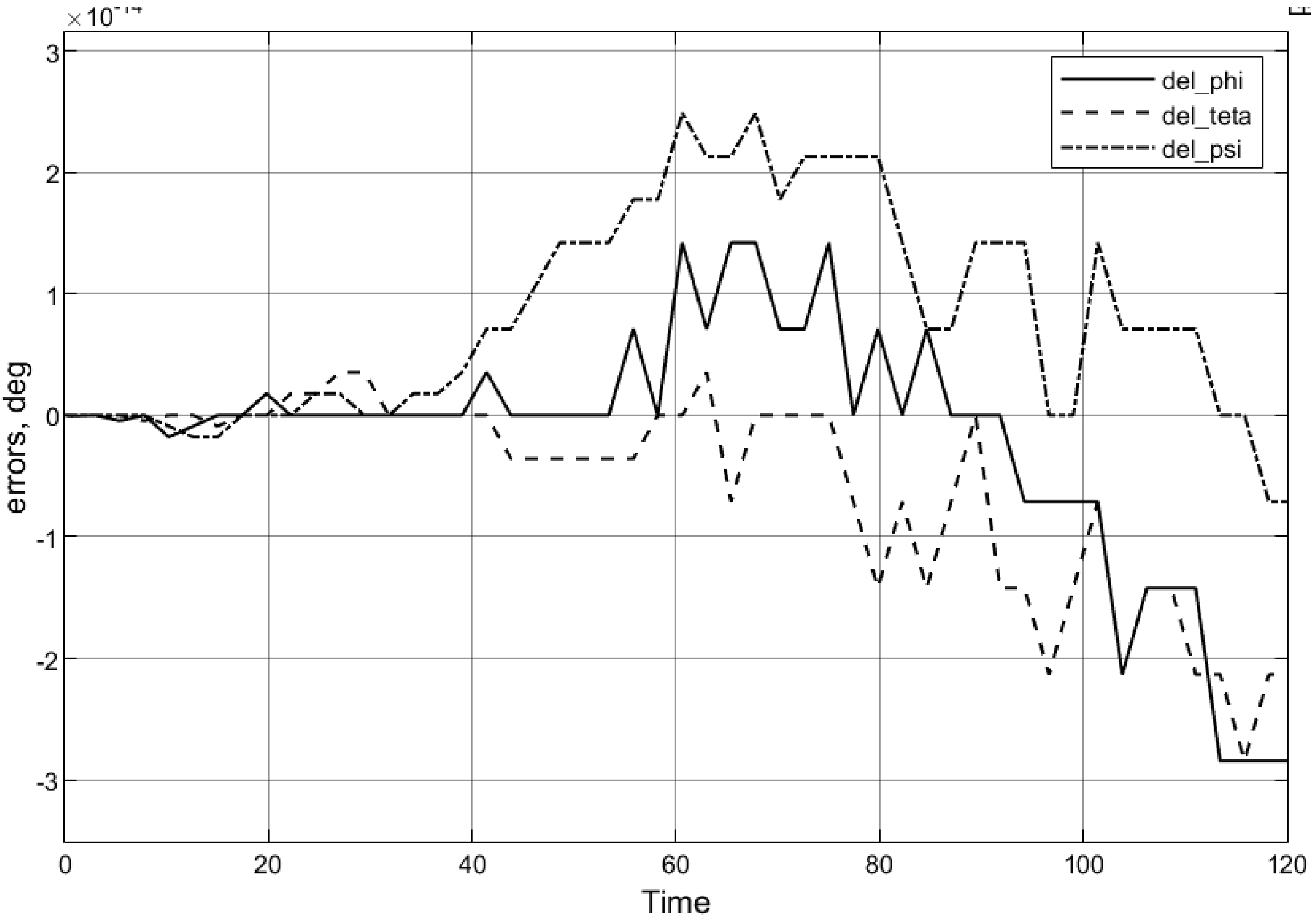
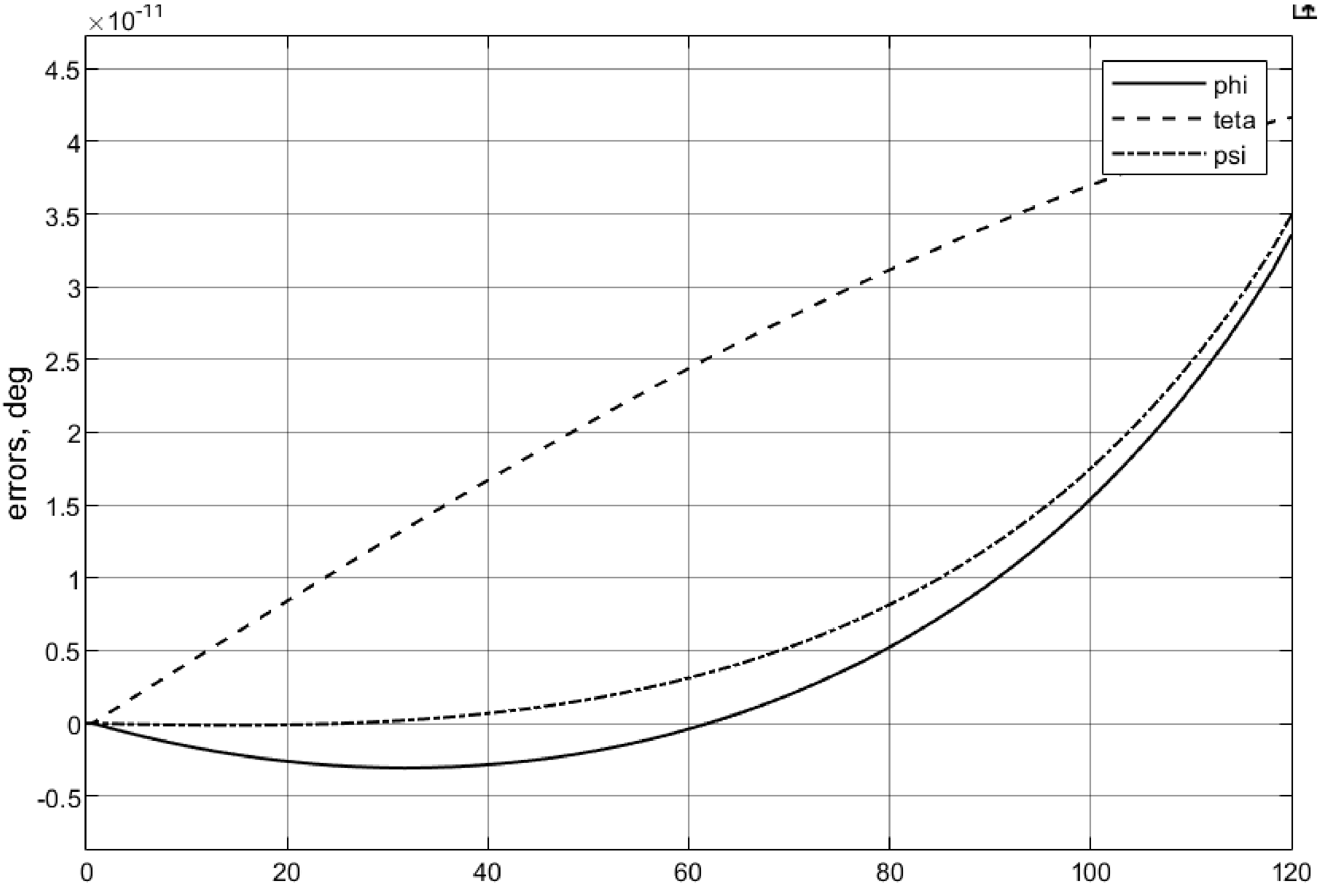


Figure 3.3.1

Figure 3.3.2




×10^-11
errors, deg
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
20
40
60
80
100
120
phi
teta
psi

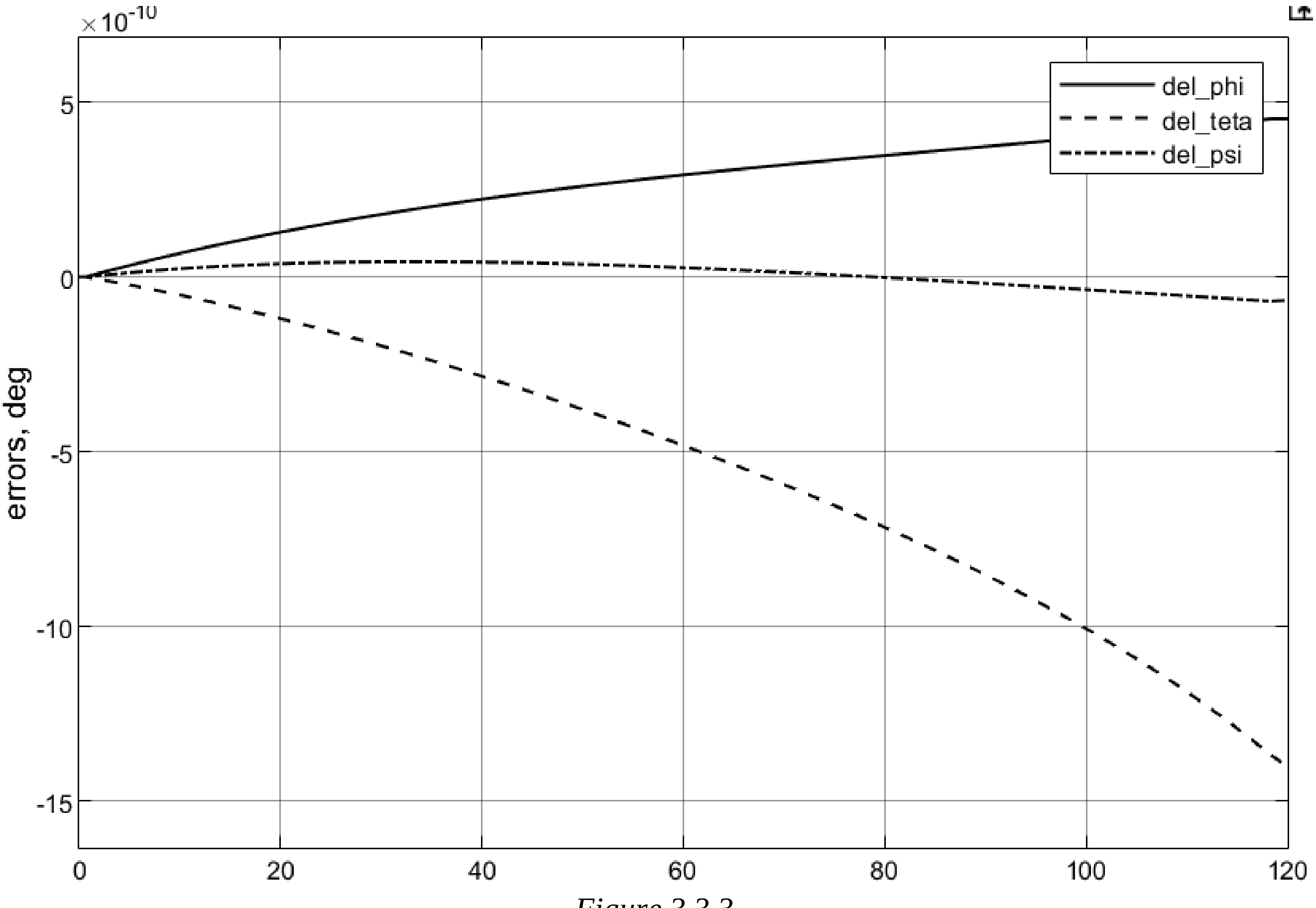
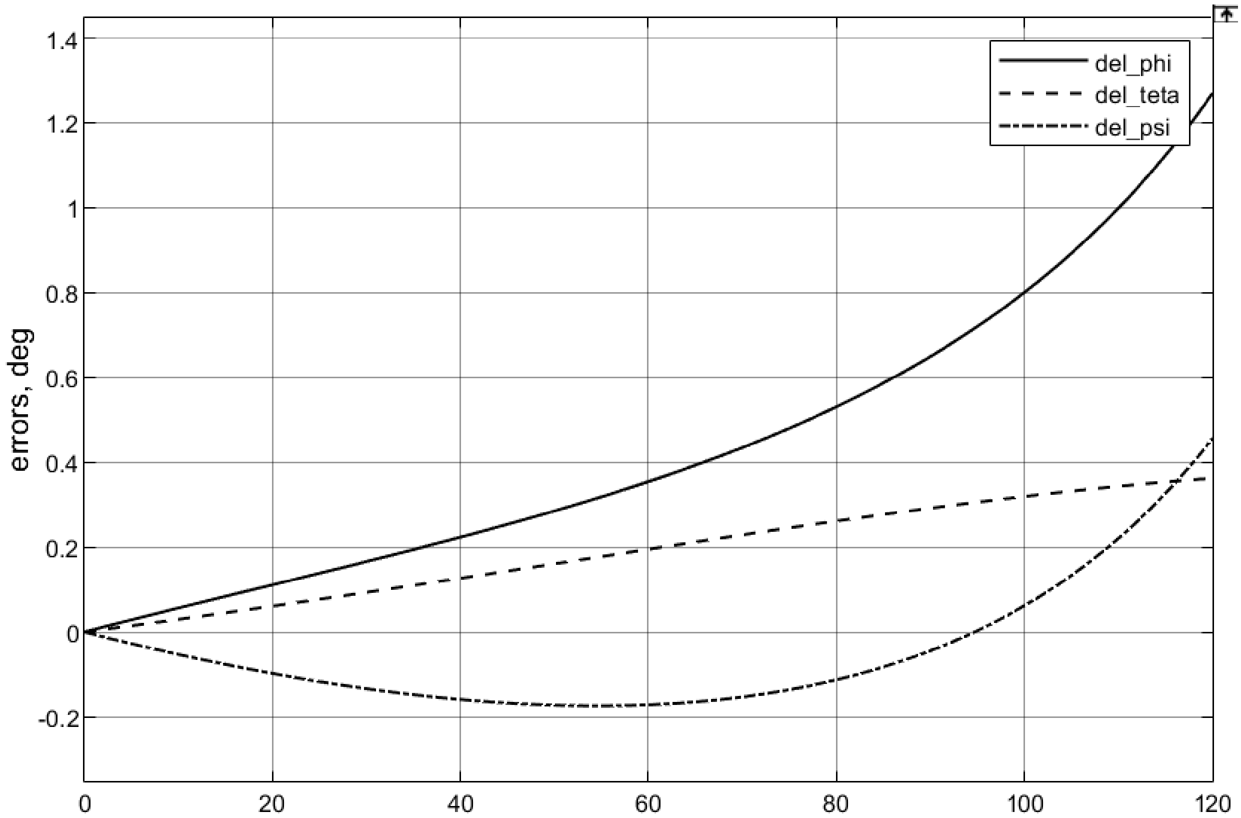

×10^-10
errors, deg
5
0
-5
-10
-15
0
20
40
60
80
100
120
del_phi
del_teta
del_psi

Figure 3.3.3

Figure 3.3.4



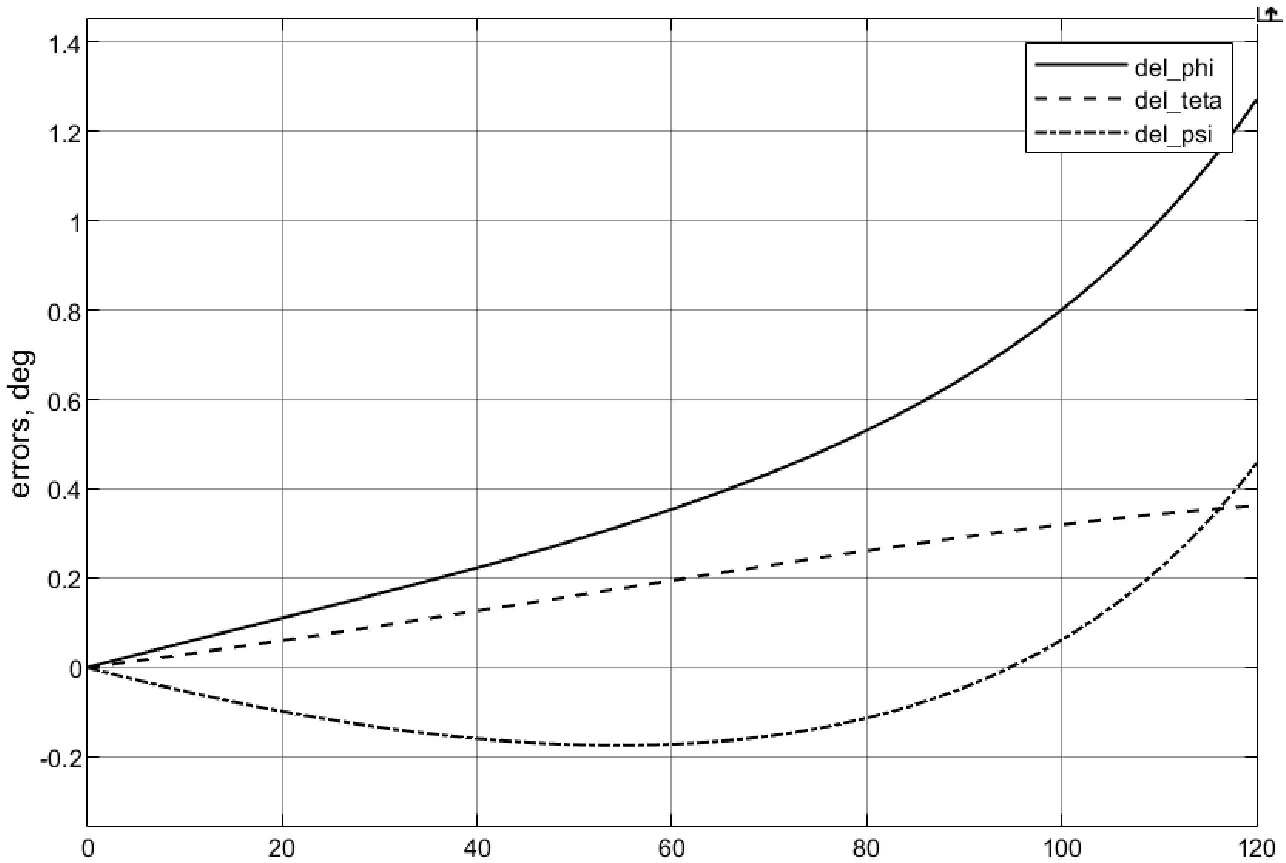


Figure 3.3.5

Figure 3.3.6

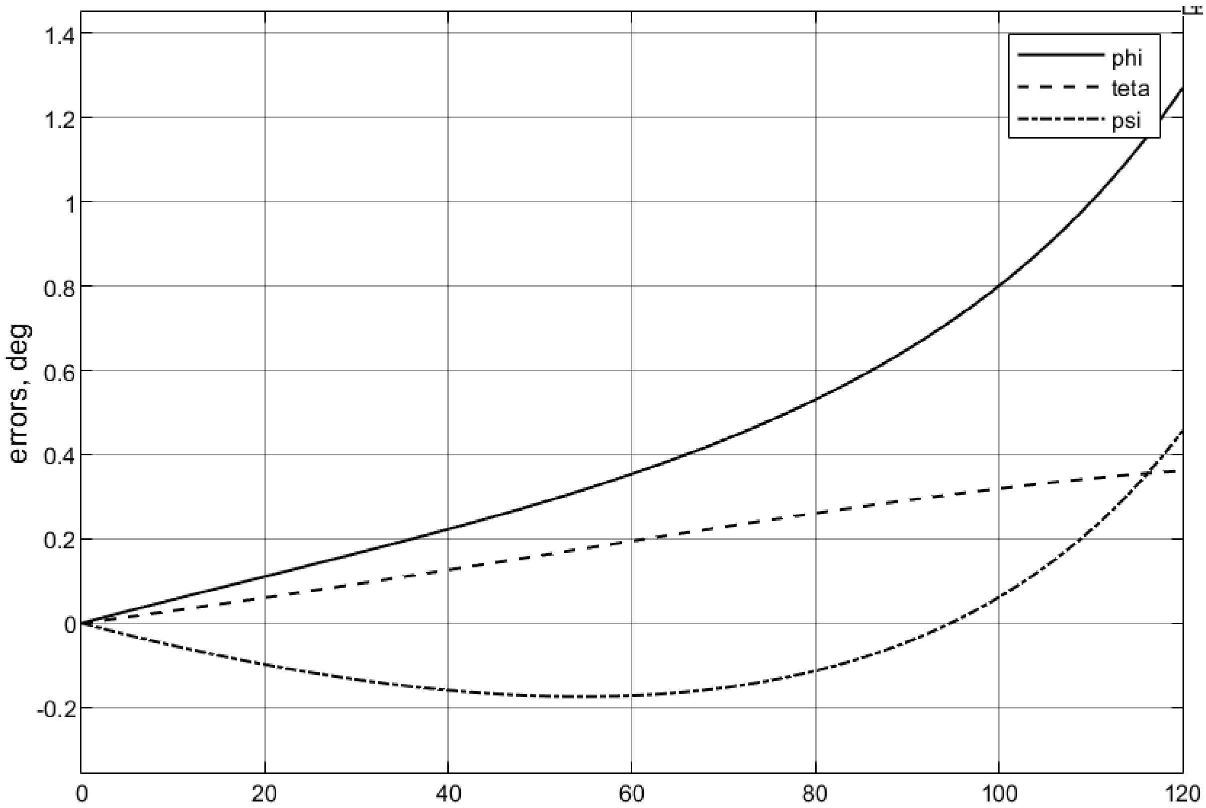


Figure 3.3.7

In Fig. 3.3.5, Fig.3.3.6 and Fig 3.3.7 we see that our reference angles of rotation are formed correctly.

Consider improving the accuracy of orientation detection based on auto-compensation of errors.

Matrix with constant components is of great importance when calculating velocities and coordinates in INS due to the projection of gravitational acceleration on the sensitivity axes of accelerometers.

To assess the accuracy of this method will accept

$$\omega_{dx} = \omega_{dy} = \omega_{dz} = 10,^{0}/h, \quad n_1 = 1c^{-1}, \;\; n_2 = 2c^{-1}, \;\; n_2 = 2c^{-1}, \quad \omega_{x_b} = 1\cos 0.1t,^{0}/s, \; \omega_{y_b} = 2\cos 0.2t.^{0}/s,$$

$$\omega_{z_b} = 3\cos 0.3t,^{0}/s.$$

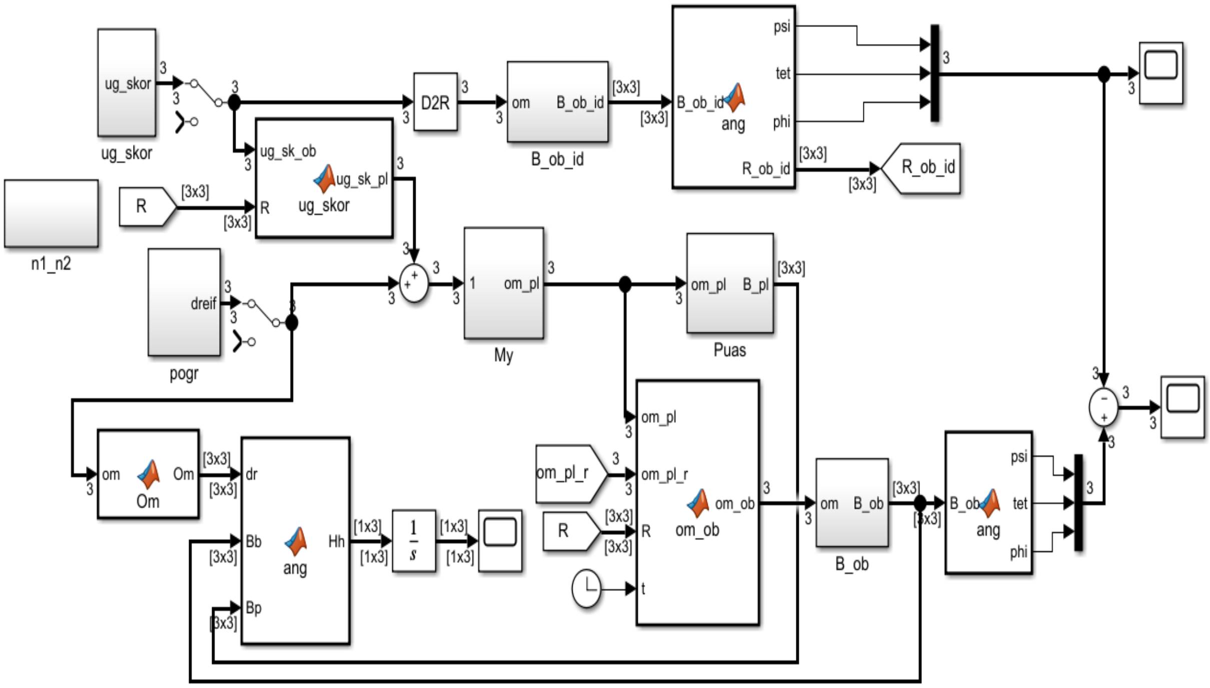


Figure 3.3.8 Simulink Model

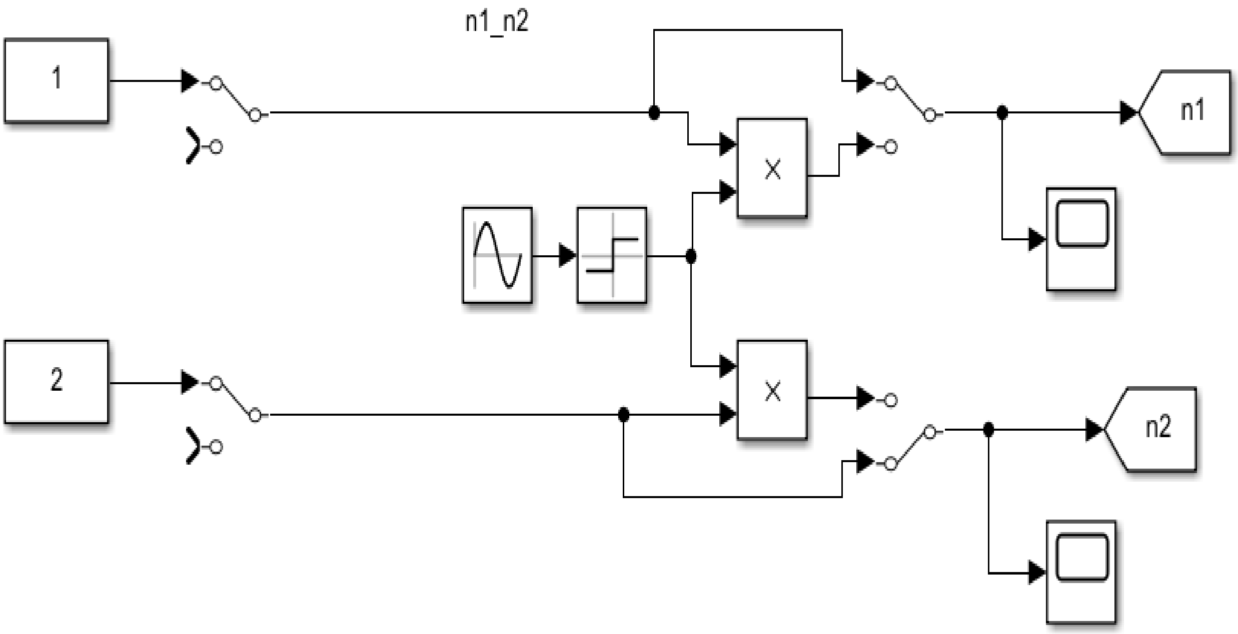


Figure 3.3.9 Block n1_n2

The Fig.3.3.9 shows a block that allows you to switch keys to simulate rotation or no rotation.

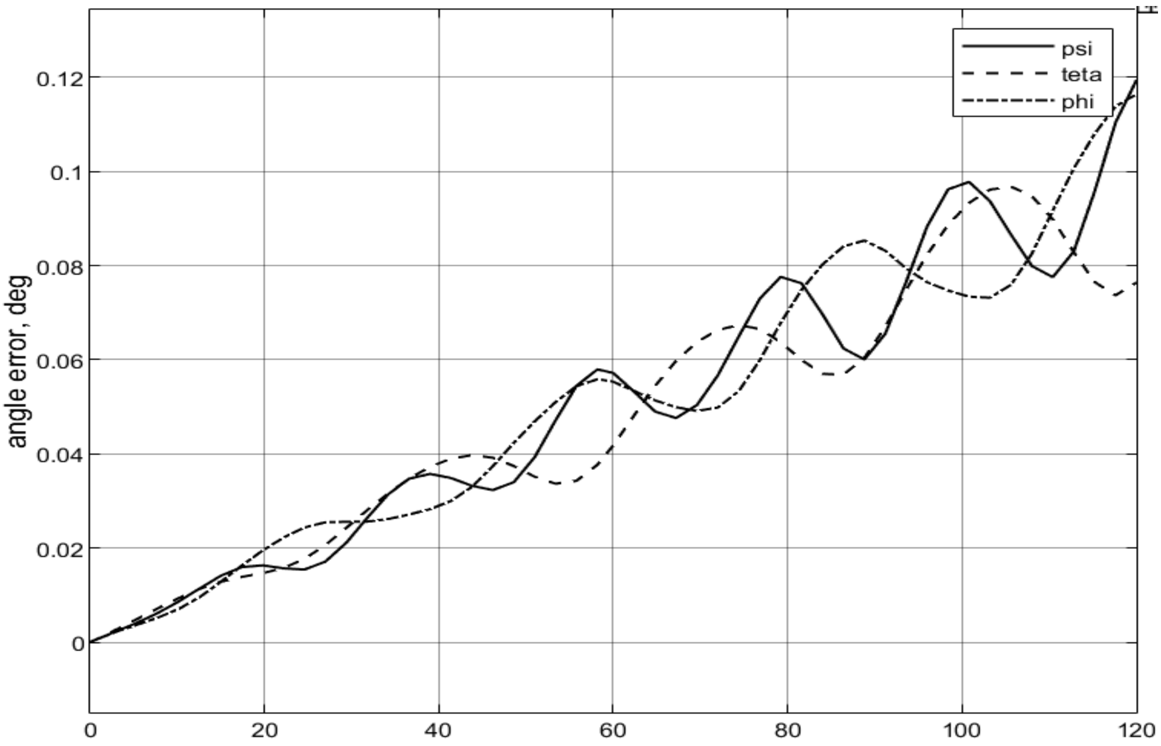


Figure 3.3.10

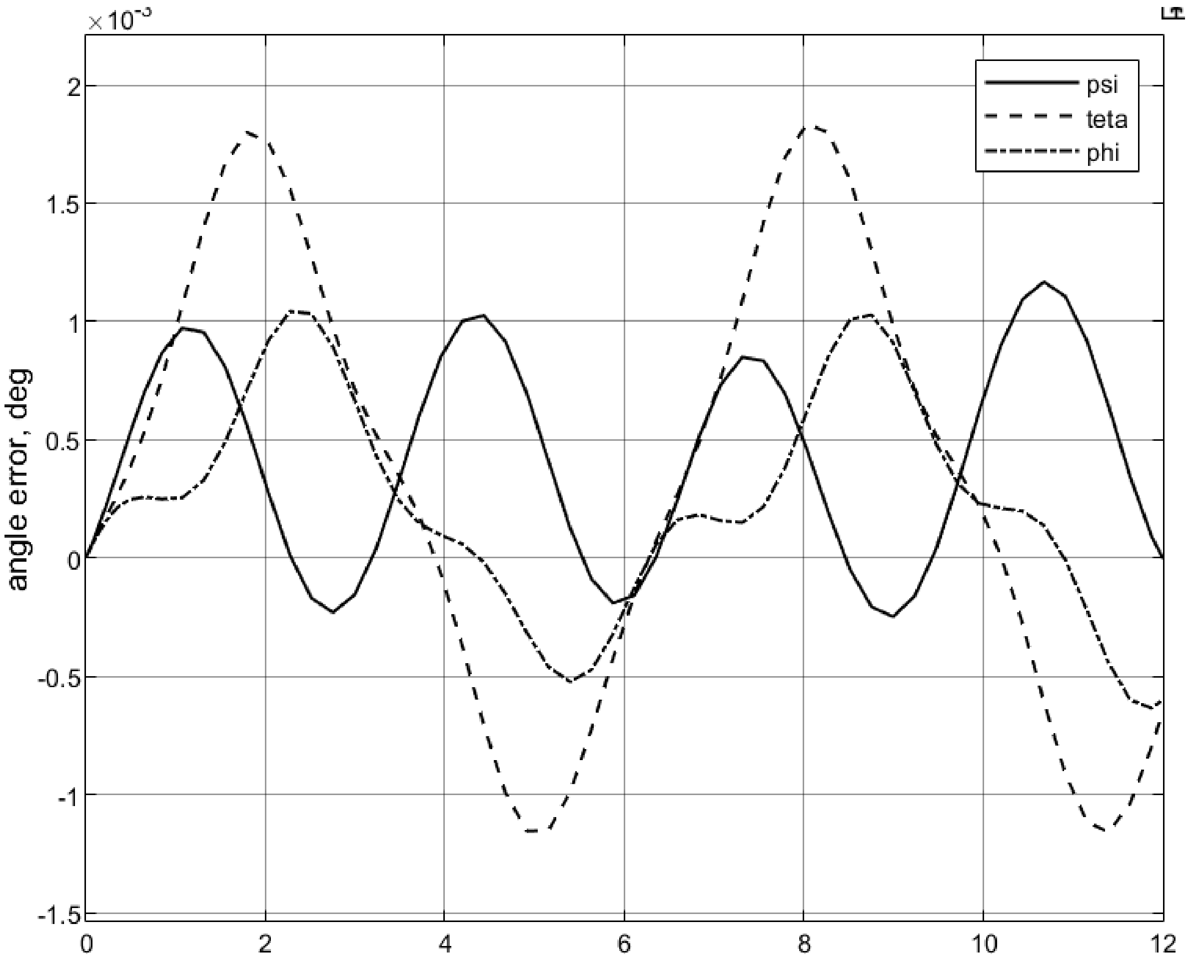


Figure 3.3.11

Errors are defined as the difference between the values of the angles in the presence of gyros errors and the values in their absence.

Errors in the presence of rotations are shown in Fig. 3.3.10

The errors presented in Fig. 3.3.11 are determined in the presence of rotations by the formula $\dot{\boldsymbol{\varepsilon}} = \boldsymbol{B}\boldsymbol{\omega}_d$

We see that during rotation there is no unlimited growth of errors over time.

Also we see expediency of using an approximate formula for performing analysis.

The proposed error model makes it quite easy to evaluate the effectiveness of autocompensation. It is shown that the problem can be reduced to integrating the matrix of directional cosines that determine the position of the sensing elements in the reference coordinate system. The results of theoretical analysis and modeling confirm the high efficiency of this method for improving the accuracy of kinematic parameters measurement.

3.4 Conclusion

The high efficiency of using the rotation of GNSS sensing elements to reduce the influence of their constant errors on the accuracy of determining navigation parameters is shown.

Further improvement of the accuracy can be achieved by increasing the angular speed of rotation and using the reverse of rotation.

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